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A proposed framework for incorporating augmented reality in mathematics lesson design

Jeanette Zhi Qi Lim [1], Tin Lam Toh [1]*

[1] National Institute of Education, Nanyang Technological University, Nanyang, SINGAPORE
***Corresponding Author:** tinlam.toh@nie.edu.sg

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ABSTRACT

Augmented reality (AR) has complemented the field of mathematics education and shown effectiveness in enhancing students' learning experiences and outcomes by addressing the gaps and limitations of traditional teaching methods. However, little research has examined how AR can be effectively incorporated into mathematics lessons. The lack of relevant frameworks and guidelines have limited the use of AR classrooms, restricting learners to learn static content passively in traditional teaching methods. In this study, we review existing literature on the benefits and limitations of using AR in mathematics education, and propose the RAMS (Resources, Accompanying materials, Multimedia, Scaffolding) framework with the objective to harness the benefits and mitigate the limitations of using AR. This framework that we propose can serve as a guide for mathematics educators to design and implement AR activities in their lessons. Based on the proposed framework, we demonstrate the application of the framework to the design of a lesson on the topic of three-dimensional (3D) trigonometry, the visualization of which pose much difficulty to many students.

Keywords: augmented reality, mathematics education, 3D figures, trigonometry, RAMS framework

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INTRODUCTION

Augmented reality (AR) as an immersive technology, together with virtual reality (VR), has emerged as ground-breaking technology in the sphere of education, bringing about new possibilities and novel approaches to teaching and learning (Alzahrani, 2020; Alalwan et al., 2020; Koumpouros, 2024). Many studies have found AR and VR effective in addressing the gaps and limitations in traditional teaching methods across different school subjects and levels (e.g., Castellanos & Pérez, 2017; Elkoubaiti & Mrabet, 2018; Hajirasouli & Banihashemi, 2022; Kesim & Ozarslan, 2012; Manisha & Gargrish, 2023).

AR is a technology that overlays digital virtual objects over real-life environments (Jiao et al., 2013). Azuma (1997) defines AR as a system that comprises three characteristics:

(1) a combination of real and virtual worlds,

(2) real-time interaction with users, and

(3) registered in three-dimensional (3D) space.

It provides a seamless interaction between augmented virtual objects and real environments where virtual objects naturally adapt to the user's surroundings as users move around and view the object from different vantage points (Kesim & Ozarslan, 2012). Studies have found that AR has offered a multitude of educational opportunities, ranging from experiential learning, visualization of abstract concepts, gamification and personalization of learning (e.g., Akcayir & Akcayir, 2017; Bower et al., 2014; Koumpouros, 2024).

AR technology has a positive impact on the teaching and learning of mathematics in schools (Ivan & Maat, 2024; Rohendi & Wihardi, 2020; Supli & Yan, 2023). In particular, studies have found that the use of AR is effective in improving students' acquisition of spatial abilities when learning geometry concepts, where students often face difficulties in developing strong conceptual understandings due to the lack of visualization skills (Ismail, 2020; Kaufmann, 2011; Koparan et al., 2023; Supli & Yan, 2023; Zhang, 2021). AR has also been employed in the teaching of other mathematics concepts such as Algebra and Statistics and has yielded positive learning outcomes (Poçan et al., 2022), albeit not as widespread and impactful as learning geometry with AR (Ivan & Maat, 2024).

Studies mostly focused on examining the effectiveness of using AR in the learning and teaching of mathematics. However, studies relating to the effective enactment of lessons implementing AR in the mathematics classroom is still limited due to the lack of relevant conceptual frameworks and guidelines (Rasimah et al., 2011). Without such frameworks, the implementation of AR into lessons is often "superficial and unproductive" (Bower et al., 2014), which restricts the possibility of leveraging AR to reap the potential benefits of AR and create unique classroom learning experiences that traditional methods cannot replicate. In this paper, we aim to conceptualize a framework to guide the use of AR in mathematics lessons and demonstrate the design of an AR-incorporated lesson on the topic of 3D trigonometry based on the proposed framework.

METHOD

In conceptualizing our framework for the use of AR in mathematics lesson, we review existing literature on the use of AR in education and mathematics education from information sources comprising including journal articles and books/book chapters. Keyword and citation searching were two main techniques used to identify and locate relevant sources. Firstly, keyword searching was carried out on the electronic databases Google Scholar and ERIC. In the search process, we made use of Boolean operators (and, or, & not) to combine the keywords: "augmented reality", "mathematics", and "education". Next, citation searching was performed on selected journal articles by tracing their cited references to locate relevant works by other scholars. A total of 55 information sources were consulted in this study.

LITERATURE REVIEW

We examine the affordances of AR, the relevant educational theories that support AR-incorporated learning, and the limitations of AR in conducting our literature review. These three components forms the basis for the development of the proposed framework.

Benefits of AR

AR technology has proved to be a powerful tool that complements the traditional learning approaches for mathematics concepts, particularly in the topic of geometry. Studies have shown that AR-supported teaching approaches have significantly ameliorated students' performance in geometry (e.g., Koparan et al., 2023; Lainufar et al., 2021; Rohendi & Wihardi, 2020). Studies have analyzed the features of AR that have contributed to enhancement of learning outcomes in mathematics (Baharuddin et al., 2020; Kaushik & Jain, 2014; Mayer & Moreno, 1998; Sweller, 2010). Common findings across literature have been broadly categorized below.

Multimedia learning

AR technologies support multimedia learning as information is presented in various representation modes, sensory modalities and realities through AR (Krüger & Bodemer, 2022). With AR technologies, information can be expressed in different representation modes such as textual and/or graphical form, in either physical and/or virtual environments, and learners receive this information through visual and/or auditory modalities. Based on Mayer's (2014) cognitive theory of multimedia learning (CTML), the multimedia design principles describe how different representations can be combined to facilitate learning processes and outcomes effectively by optimizing cognitive load. That is, to reduce intrinsic cognitive load that stems from the inherent complexity of the content, minimize extraneous cognitive load that arises from redundant or poor presentation of instructional materials, and to enhance germane cognitive load, which is a beneficial load imposed during the process of transferring information from working memory into long-term memory (Sweller, 2010).

While Elford et al. (2022) purported that intrinsic cognitive load should not be influenced by any form of learning aid such as AR technology as intrinsic cognitive load is solely dependent on the complexity of the learning topic itself, other studies have suggested otherwise. The study by Ibili and Billinghurst (2019) has shown that AR-assisted geometry divided the content into bite-sized information with simple animations, which have effectively reduced intrinsic load. For instance, animations on AR provides dynamic visualization of the concept of volume of a prism by showing the relationship between the uniform cross-sectional area and the height of the prism (**Figure 1**). This chunking of information aligns with the Segmenting principle, which states that learning is most effective when information is unpacked and presented as smaller chunks, rather than a large and continuous unit of information (Mayer, 2009; Mayer & Fiorella, 2021). AR allows learners to replay the animation repeatedly and process these smaller pieces of information at their own paces before continuing.

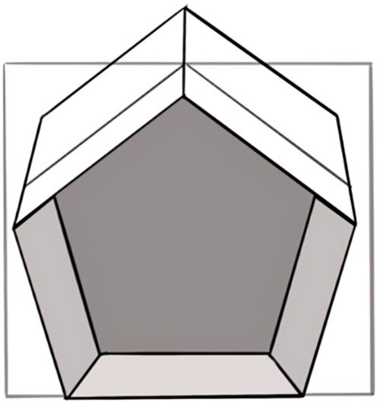


Figure 1. Animation on mathematics AR (Figure conceptualized by the first author)

Furthermore, AR provides a platform for the concurrent representation of textual and pictorial information, which supports the multimedia principle which states that people learn better from words and pictures than from words alone (Mayer & Moreno, 1998). The contiguity principle, which states that learning is maximized when text and visuals are presented contiguously, is also achieved when corresponding text and augmented 3D object are presented adjacent to each other (Mayer & Moreno, 1998). For instance, when textual information and augmented 3D objects are adjacent to each other on the AR handheld device, the augmented 3D object will complement the textual information and form an internal mental image (**Figure 2**).

Mathematics concepts are made less abstract with the use of visual elaboration provided by AR. This facilitates students' internal understanding and development of schemas, thereby reducing intrinsic load and enhancing germane load (Buchner et al., 2021). Therefore, AR's multimedia feature, which supports various multimedia design principles, optimizes cognitive load and enhances the learning of mathematics.

Natural user interface in AR

Learning in a digital environment may impose extraneous cognitive load that is irrelevant to the learning objectives (Sweller, 2010). There is a learning cost associated with learning how to utilize digital tools when a student accesses digital educational content, which imposes additional cognitive load while the student interacts with the application (Bujak et al., 2013). However, as opposed to other computer-based environments, studies have found that the extraneous cognitive load imposed is lower in AR environments due to the natural

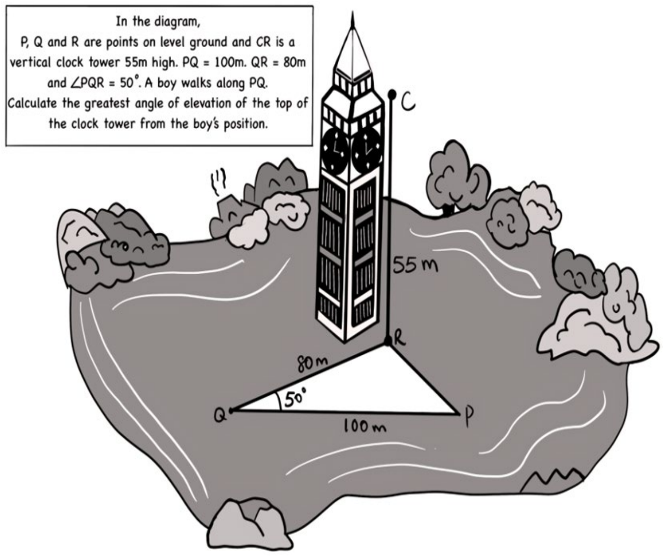


Figure 2. Math VR displaying textual information and 3D object contiguously (Figure conceptualized by the first author. The question was adapted from ACE-Learning)

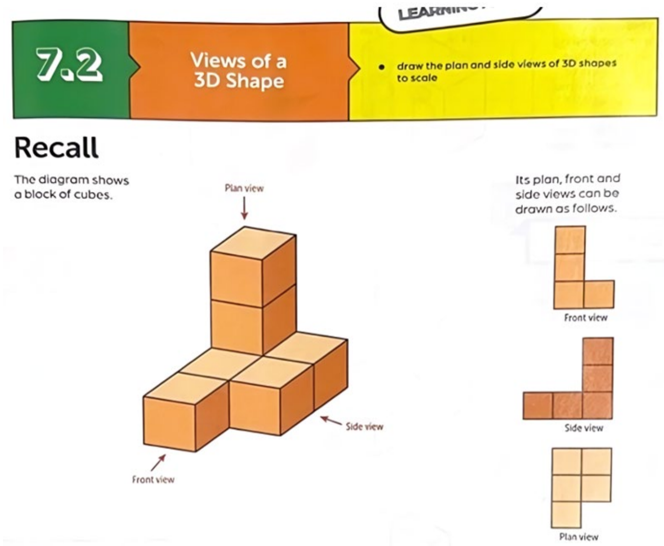


Figure 3. Textbook showing 3D illustrations of model from different perspectives [Figure reproduced from the second author's own book (Toh, 2023), with permission]

user interface (NUI) in AR (e.g., Neumann & Majoros, 2002; Tang et al., 2003).

An NUI allows users to interact with technology in a natural and intuitive manner through modalities such as touch, speech and gesture (e.g., Câmara, 2011). Various studies reported that one of the notable strengths of AR technologies is the perception of natural interaction, which increases the usability of the system (Bujak et al., 2013; Elford et al., 2022; Kaushik & Jain, 2014). AR interfaces allow users to interact with the virtual objects as they would with their natural physical environment, which is intuitive and immersive as the augmented virtual objects integrates seamlessly into the user's physical environment. For instance, the user moves around to view an augmented 3D cube from different angles, moves closer or further away from the object to change its scale and rotate the object by swiping it. Kaushik and Jain (2014) highlighted that such intuitive and NUI increases the perceived ease of use of the system and provides user with more freedom and increased usefulness.

To conclude, since the operations of object manipulation in AR are intuitive and natural to most learners, there is less prior knowledge and skills required of users, and the low barrier of entry to AR educational systems averts the extraneous cognitive load imposed on learners.

Interactive and dynamic nature

Many studies have criticized traditional teacher-centered learning and teaching approaches where learners passively receive information and process information through inactive learning such as note-taking and rote memorization (e.g., Baharuddin et al., 2020; Hajirasouli & Banihashemi, 2022). On the other hand, a constructivist approach, where learners learn actively through their own experiences and knowledge is constructed when learners engage in learning activities and reflect on their experiences, is believed to facilitate a deeper understanding of the subject matter taught (Bereiter, 1994; Baharuddin et al., 2020; Hajirasouli & Banihashemi, 2022).

From a constructivist perspective, AR has the potential to create a knowledge-building environment through active interactions with the dynamic software (Afnan et al., 2021). AR can be employed to simulate real-world scenarios, which allow them to apply and practice their theoretical knowledge and skills on virtual objects, without the need to obtain the actual physical objects that might be unavailable to them due to practical or financial constraints (Koumpouros, 2024). Furthermore, the high interactivity between learners and the AR environment enhances student-centered learning (Antonioli et al., 2014). Rather than memorizing textual information in static textbooks, AR allows learners to explore and navigate objects through hands-on experiences in dynamic virtual environments.

In particular, active learning–where learners are experientially involved in the learning activity–is particularly important in the field of spatial Geometry to acquire spatial visualization skills, which can be facilitated by AR technologies (e.g., Koparan et al., 2023; Rohendi & Wihardi, 2020; Supli & Yan, 2023). AR has proven to be effective in overcoming students' difficulties in the learning of Geometry concepts where students often lack spatial visualization skills (Ismail, 2020; Zhang, 2021; Hajirasouli & Banihashemi; 2022). This could be attributed to the traditional teaching approach of using two-dimensional (2D) media such as textbooks.

These static textbooks are unable to offer dynamic content and the presentation of 3D objects in 2D media often poses learning difficulties for students to acquire spatial abilities, which is built up from prior experiences of active manipulation (Clements & Battisa, 1992; Koparan et al., 2023; Zhang, 2021). For instance, textbooks are only able to provide static illustrations of geometry models but are unable to offer interactive manipulation of the models that are required to fully comprehend spatial concepts such as visualization and mental rotation. **Figure 3** is an example of a mathematics textbook on the topic of 3D shapes, where it provides illustrations of the 3D model from the front, side and plan view. Without hands-on manipulation, such static illustrations alone may be inadequate in helping students to spatially visualize and rotate 3D models.

On the other hand, studies have shown that AR allows learners manipulate 3D objects and view it from different angles, which helps them understand the 3D structures and create spatial images that contribute to the construction of mental models of 3D solids (Kesim & Ozarslan, 2012; Moral-Sanchez & Siller, 2022).

With the use of AR, learners internalize abstract concepts through hands-on experiences and interactions with virtual AR objects, rather than solely learning from explanations of abstract concepts in the form of textual or verbal instructions. Therefore, the study by Hajirasouli and Banihashemi (2022) suggested that AR-based immersive technologies mitigates the shortcomings of traditional passive learning approach by offering experiential learning with its highly interactive and dynamic nature, thereby promoting a constructivist approach.

Limitations of AR

Notwithstanding the myriad of benefits of using AR in lessons, educators have also reported some challenges in using AR in lessons. The limitations of the use of AR have been classified below.

Poor instructional design

Some studies found that teachers are resistant to the use of AR systems in class due to the inflexibility of the content in some AR systems (Alalwan et al., 2020; Wu et al., 2012). Given the rigidity of content in AR systems, teachers are unable to tailor content on the AR application according to the local syllabus requirements or accommodate the learning needs of students. Akcayir and Akcayir (2017) found that AR content often lacked the depth required for lasting learning, which is consistent with the study by Bower et al. (2014), who cautioned teachers that solely using AR may potentially limit the cognitive development of students due to the overemphasis on lower order thinking capabilities. Based on these studies, it is clear that unless teachers are given access to authoring tools where they can modify and create their AR content that aligns with their lesson objectives and syllabus, it is unlikely that AR will be significantly integrated into classrooms.

Researchers also cautioned that careful design of AR is required to ensure effective learning due to AR's exploratory and open-ended nature (Wu et al., 2012). Alalwan et al. (2020) highlighted that AR provides a broad area of learning that lacks meaningful explanations and assessments. Similarly, Ibili and Billinghurst (2019) found that AR's exploratory nature can confuse learners with overwhelming information, where learners are unsure of what they should do next. Hence, learners may lose focus and participate in exploratory activities superficially as there is a lack of structure in AR systems with decentralized information (Klopfer & Squire, 2007). This is in contrast with the traditional structured learning approach with guided instructions. Therefore, Klopfer and Squire (2007) suggested that additional scaffolding and guidance is necessary to help learners navigate the AR application.

Intrusive technology

Sophisticated hardware and software resources negatively affect learners' learning experience with AR systems. Ejaz et al. (2019) found that the use of an intricate AR interface to manipulate virtual digital objects imposes extraneous cognitive load on learners and limits their perception of natural interaction with the AR applications.

Similarly, studies reported that the use of head-mounted displays interrupts the natural interaction with others and cause discomfort (Bacca et al., 2014; Wu et al., 2012). Bujak et al. (2013) found that in an AR-facilitated lesson, students have to juggle between multiple technological devices and perform complex tasks with an overwhelming amount of information.

As such, learners have to devote additional time and attention to understand the know-hows and utilization of AR systems and tools, rather than focusing on the learning target. These elements distract learners and affect their learning experiences adversely as mental efforts are diverted away from learning the subject (Ejaz et al., 2019), which increases their extraneous cognitive load and potentially offsets the cognitive advantages of AR technologies.

OUR PROPOSED FRAMEWORK

Based on the literature findings in the previous section, we synthesize the guiding principles for the design of an AR-incorporated mathematics lesson by incorporating accessibility, pedagogical and didactical considerations. We name the framework with the acronym RAMS:

Resources: AR tools and platforms required for lesson

Accompanying materials: Traditional worksheet to complement AR manipulatives

Multimedia: Elements of AR application

Scaffolding: Demonstration by teacher and guided prompts in AR worksheet

Resources

To carry out an AR-facilitated lesson in class, hardware resources such as AR-compatible devices and software resources such as AR applications are required. It is infeasible for schools to purchase and provide students with technical and sophisticated AR headsets or head-mounted displays (Wu et al., 2012). Hence, assuming the context where all students own digital devices, students will utilize their own mobile phones or tablets as the handheld AR displays while ensuring that the AR application is compatible with these digital devices.

Furthermore, the use of personal devices as handheld displays is also comparatively more elementary than other tools such as headsets and head-mounted displays. As such, the use of personal devices is less likely to cognitively overload students due to the complexity of the AR devices (Ejaz et al., 2019). Hence, using personal digital devices as AR handheld displays increases the accessibility and usability of AR systems.

Accompanying Material

Some researchers suggested that the use of AR alone may be ineffective for students to acquire new mathematical concepts (e.g., Akcayir & Akcayir, 2017; Alalwan et al., 2020; Wu et al., 2012). As noted in the study by Klopfer and Squire (2007), there is a need to balance between the student-centered learning with decentralized information and the traditional teacher-centered, structured learning with guided instructions. Hence, it is proposed that the instructional materials for an AR-incorporated lesson should include a combination of AR and traditional resources such as handouts and worksheets. Other studies have similarly found that the benefits of AR are maximized when integrated with traditional teaching methods, whereas using AR in insolation can result in less productive outcomes due to its broad and open-ended nature (Wu et al., 2012).

As such, the design of AR worksheet is crucial to complement the AR manipulatives by providing a learning structure and allow students to engage in exploratory learning with guided instructions. Using constructivist approach as the underpinning theory, the accompanying worksheet should incorporate and facilitate the three stages of the learning cycle as proposed by Atkin and Karplus (1962): exploration, concept introduction and concept application. The learning cycle provides a structured and active learning process through meaningful activities that guide students from exploration to mastery of content (Marfilinda et al., 2019).

In the exploration stage, students will work in groups or independently to explore and interact with the augmented 3D object based on the guiding instructions given in the worksheet. Following the curated question prompts in the worksheet, students will manipulate the augmented 3D object, discover key features and construct their own understanding (Marfilinda et al., 2019). This guides and engages students in deeper and higher-order thinking during the exploratory activity instead of simply passively observing without thinking or exploring superficially as noted in previous literature findings by Bower et al. (2014) and Akcayir and Akcayir (2017).

In the concept introduction stage, students will refine, clarify and develop their prior understanding obtained in the exploration phase. Students are encouraged to write down observations and explanations of the concepts and definitions from their own understandings in the worksheet. Teacher will then discuss and introduce the lesson content, which eventually leads to the formal definition of the new concept (Marfilinda et al., 2019). Since most teachers are unlikely to be able to alter the content in AR (Wu et al., 2012; Alalwan et al., 2020), the AR worksheet can be utilized to complement the AR application and provide tasks that achieves learning objectives that aligns with the local syllabus.

In the concept application stage, students will apply their understanding of the concepts learnt through meaningful activities such as problems and practices. This allows students to enhance and consolidate their understanding of the concepts (Marfilinda et al., 2019).

The step-by-step instructions in the AR worksheet streamlines and facilitates the learning process, thereby reducing the extraneous cognitive load imposed on them without compromising the active learning process. This is opposed to allowing students to explore the AR application without any guidance and instructions, which may confuse, overwhelm or distract students away from lesson objectives (Ibili & Billinghurst, 2019), In designing the scaffolding worksheet, the teachers have the autonomy to design the AR worksheets to offer depth, promote high-order thinking and align with local curriculum, which effectively addresses the shortcomings of AR application.

Multimedia

The multimedia design principles based on Mayer's (2014) CTML have implications for the instructional design of AR applications rooted in multimedia technology. In the AR application, elements of text, 3D images and animations have to be presented strategically to maximize the benefits of using AR in the teaching and learning of mathematics.

According to the segmenting principle, AR applications should allow students to control the delivery of information. For instance, students should be able to pause and replay or even adjust the speed of the animation to process bite-size information at a pace suited to their own learning needs. This segmentation of information prevents overwhelming students with complex information and facilitates the construction of schemas, thereby reducing intrinsic cognitive load and enhancing germane cognitive load (Sweller, 2010).

According to the multimedia and contiguity principle, textual information and augmented 3D objects should be presented together and contiguously on the AR application to increase germane cognitive load (Mayer & Moreno, 1998). This feature of AR application also prevents the split attention effect, which arises when mutually referring information is separated spatially (Mayer & Moreno, 1998; Pouw et al., 2019). For instance, students may have to constantly shift their focus between the worksheet and the augmented 3D object if the textual information of the augmented 3D object is printed on a separated worksheet. Hence, AR application is able to have both the textual information and augmented 3D object presented on the same platform. That said, while students will refer to the textual information overlaid in the AR application, it is beneficial to include the same textual information in the accompanying worksheet for students to make annotations and revisit the content for revision in the future.

By conforming to the multimedia design principles, the multimedia elements of AR application can be leveraged to enhance germane cognitive load while reducing intrinsic and extraneous cognitive load.

Scaffolding

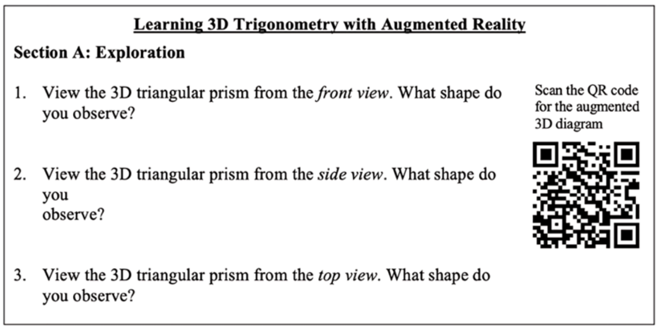
To facilitate an AR-incorporated lesson without cognitively overloading the students, teacher should provide scaffolding to address students' unfamiliarity with AR tools and provide guidance while students engage in exploratory activities with AR application (Klopfer & Squire, 2007).

As students might be first-time users of AR, teacher has to carry out an introductory demonstration on the use of AR handheld displays by demonstrating the foundational manipulation of augmented 3D objects prior to the lesson activity. This includes basic knowledge such as moving about to show how the augmented 3D object changes as the angle of view changes, moving back-and-forth to change the scale of the object and swiping to rotate the object.

However, given the high NUI in AR systems, it is likely that students will find it intuitive and easy to use (Bujak et al., 2013; Elford et al., 2022), and this scaffolding can be removed once students are familiar with the AR interface. Such scaffolding eases students into the use of novel digital technologies like AR and minimizes the extraneous cognitive load imposed on students due to the novelty and complexity of the AR tool (Ejaz et al., 2019).

During the exploratory lesson activity, teacher may have to provide further scaffolding to support students alongside with the AR worksheet provided depending on the class profile (Klopfer & Squire, 2007). Learners may struggle to follow the written instructions on accompanying AR worksheet due to lower reading comprehension abilities or complexity of the mathematics topic (Main et al., 2023) and would require face-to-face guidance and individualized scaffolding such as live demonstration by the teacher.

For instance, teacher can provide live demonstration of manipulating the augmented 3D cube while verbalizing guiding questions such as "What do you observe about the 3D cube as I move from the top view to the side view?" As such, the use of AR worksheet alone may be inadequate for certain class profiles or topics of higher



Learning 3D Trigonometry with Augmented Reality

Section A: Exploration

1. View the 3D triangular prism from the *front view*. What shape do you observe?
2. View the 3D triangular prism from the *side view*. What shape do you observe?
3. View the 3D triangular prism from the *top view*. What shape do you observe?

Scan the QR code for the augmented 3D diagram

Figure 4. Accompanying worksheet for AR activity during exploration stage (Figure created by the first author)

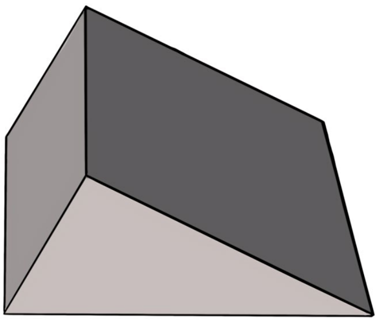


Figure 5. Augmented 3D triangular prism displayed on handheld devices after scanning QR code (Figure conceptualized by the first author)

difficulty, and additional real-time and individualized scaffolding are required to further support learners.

APPLICATION OF RAMS FRAMEWORK TO DESIGN A LESSON ON 3D TRIGONOMETRY

Based on the proposed RAMS framework, in this section, I demonstrate the design of a lesson on the topic of 3D trigonometry at a secondary mathematics level. This topic is an extension of 2D trigonometry, where students are required to visualize 2D triangles from 3D shapes and apply trigonometric concepts such as trigonometric ratios (sine, cosine, and tangent), Pythagoras theorem, sine rule, cosine rule, and angles of elevation/depression to solve for unknown lengths or angles (MOE, 2024). Studies by Niranjan (2022) found that students often face difficulties in solving 3D trigonometric problems, mostly stemming from the lack of spatial visualization skills to extract triangles in various planes and confusion with 3D orientation of the shape which resulted in the application of wrong rule, sides and angles.

The use of AR aims to overcome the shortcomings of using static 2D textbooks to teach the topic of 3D trigonometry and serves as a digital manipulative where students actively engage in exploratory lesson activities that develops their spatial visualization abilities. As such, this section outlines a lesson design based on the proposed RAMS framework.

Resources

For software resources, this lesson on 3D trigonometry utilizes an AR application and students will be instructed to download the AR application on their personal mobile phones or tablets prior to the lesson. Students will use their personal digital devices as the handheld

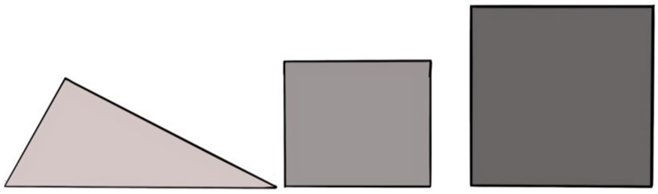
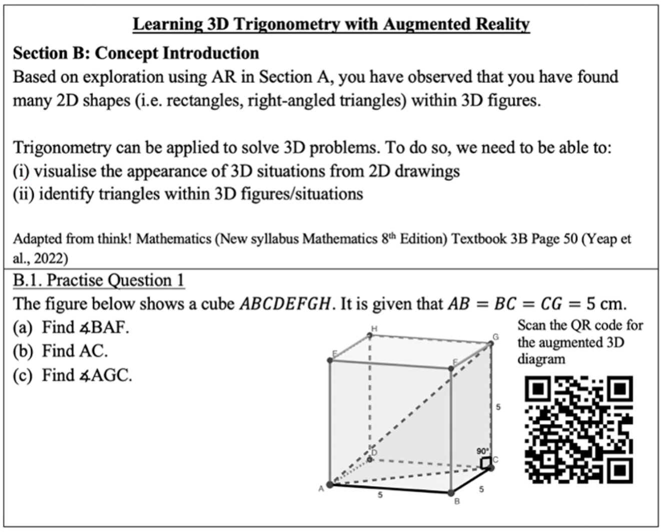


Figure 6. 3D Triangular prism from different views (front, side, and top view) (Figure conceptualized by the first author)



Learning 3D Trigonometry with Augmented Reality

Section B: Concept Introduction

Based on exploration using AR in Section A, you have observed that you have found many 2D shapes (i.e. rectangles, right-angled triangles) within 3D figures.

Trigonometry can be applied to solve 3D problems. To do so, we need to be able to:
(i) visualise the appearance of 3D situations from 2D drawings
(ii) identify triangles within 3D figures/situations

Adapted from think! Mathematics (New syllabus Mathematics 8th Edition) Textbook 3B Page 50 (Yeap et al., 2022)

B.1. Practise Question 1

The figure below shows a cube $ABCDEFGH$. It is given that $AB = BC = CG = 5$ cm.

(a) Find ∡BAF.
(b) Find AC.
(c) Find ∡AGC.

Scan the QR code for the augmented 3D diagram

Figure 7. Accompanying worksheet for AR activity during concept introduction stage (Figure created by the first author)

AR devices during the lesson. Students may have to work in pairs or groups to share their digital devices if the application is incompatible with some of their devices. Since students are utilizing their personal devices in class, the teacher may have to constantly monitor the usage of their devices to prevent any misuse or distractions away from the lesson tasks.

Accompanying Material

An accompanying worksheet will be designed to complement the AR lesson activity. At the exploration stage, students will follow the guiding instructions in the worksheet to carry out a sequence of activities. Students will use their handheld devices to scan the QR marker at the right column of the worksheet (**Figure 4**) to overlay an augmented 3D triangular prism in their handheld devices (**Figure 5**). Following the instructions in the worksheet, students will use AR to manipulate the 3D triangular prism by moving around and viewing it from different angles. Students will observe the different 2D shapes (i.e., rectangles and right-angled triangles) found in the 3D triangular prism based on different perspectives and answer the guiding question prompts in the worksheet (**Figure 4**). **Figure 6** illustrates the different 2D shapes observed when students look at the 3D triangular prism from different angles.

At the concept introduction stage, teacher will lead students in the discussion of the insights gained from the exploration activity, before formally introducing the main lesson content. The earlier portion of **Figure 7** shows an example of a section for lesson notes in the AR worksheet. Thereafter, students will use AR to attempt basic 3D trigonometric questions involving 3D solids such as cube, cuboid, triangular prism and pyramids. The QR markers at the right column of

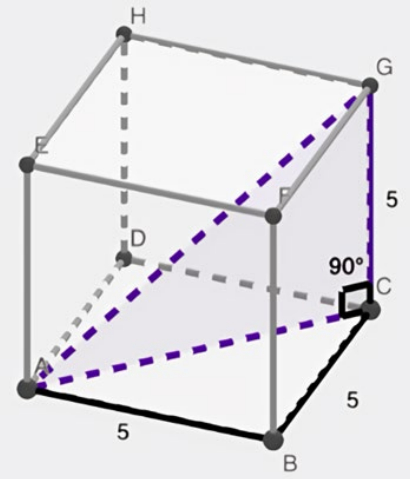


Figure 8. Augmented 3D cube displayed on handheld devices after scanning QR marker (Figure created by the first author using GeoGebra and pieced together using Canva)

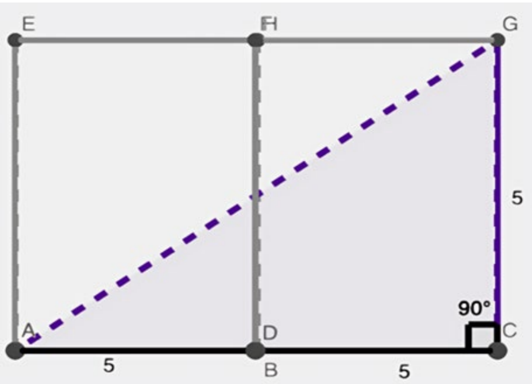
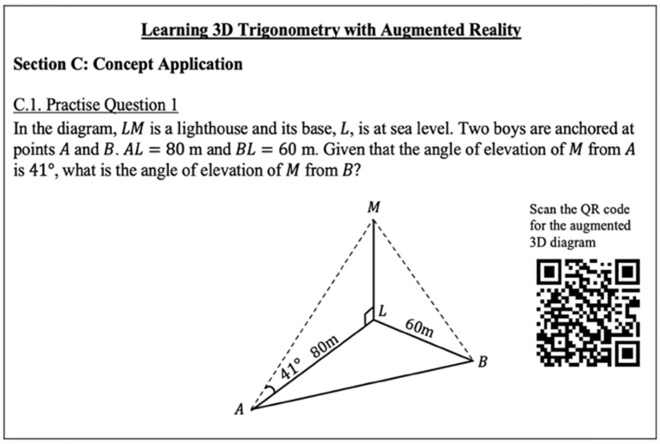


Figure 9. Augmented 3D cube from the side view (Figure created by the first author using GeoGebra and pieced together using Canva)



Learning 3D Trigonometry with Augmented Reality

Section C: Concept Application

C.1. Practise Question 1

In the diagram, LM is a lighthouse and its base, L, is at sea level. Two boys are anchored at points A and B. $AL = 80$ m and $BL = 60$ m. Given that the angle of elevation of M from A is 41°, what is the angle of elevation of M from B?

Figure 10. Accompanying worksheet for AR activity during concept application stage (Figure created by the first author using GeoGebra and pieced together using Canva. The question is adapted from a question used in ACE-Learning Math VR application)

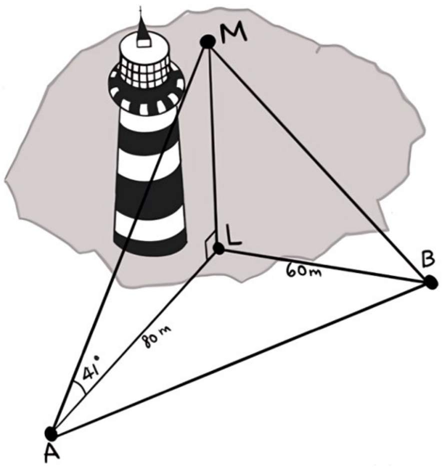


Figure 11. Augmented 3D model displayed on handheld devices after scanning QR code (Figure conceptualized by the first author)

each question will direct students to the corresponding augmented 3D solid to facilitate the seamless integration between the AR worksheet and AR application.

The later portion of **Figure 8** shows an example of a 3D trigonometric question with an accompanying QR code that allows students to scan and view the corresponding augmented 3D cube (**Figure 8**).

Since most learners struggle with the visualization and extraction of 2D triangles from the different planes within 3D figures (Niranjan, 2022), the augmented 3D model allows hands-on manipulation for better visualization of abstract concepts. For instance, in the context of the question in **Figure 8**, many students may struggle to identify triangle ACG as a right-angled triangle from the 2D illustration of cube ABCDEFGH. As such, the augmented 3D cube will allow students to view it from different angles and visualize triangle ACG as a right-angled triangle when viewed from the side (**Figure 9**).

At the concept application stage, students will apply the concepts learnt to attempt higher-order 3D trigonometric problems that involves 3D real-world scenarios. Similarly, students will use their personal devices to scan the QR marker at the right column of each question for the corresponding augmented 3D figure (**Figure 10**).

While students will refer to the augmented 3D solid/figure in the AR application (**Figure 11**), its corresponding 2D form will be included in the accompanying worksheet (**Figure 10**). This develops students' skills of constructing mental models of objects from textual descriptions in textbooks, questions or from their corresponding 2D form. This facilitates the acquisition of spatial abilities as students learn to construct mental 3D spatial images from 2D diagrams, mentally rotate the 3D model and understand their spatial relationships.

Multimedia

Multimedia elements such as textual information, 3D images and animations should be strategically presented in accordance with Mayer's (2014) CTML. According to the segmenting principle, the AR application should break down complex information into smaller and manageable chunks that are easier to grasp. One example would be the use of animations to demonstrate the extraction of 2D triangles from the 3D solid (**Figure 12**). The use of such animations directs students' attention away from other redundant elements to focus on the key elements, thereby making the abstract concepts clearer and more comprehendible. Students can control the speed and pace of the animation, where they can choose to play, pause or rewind whenever they want to.

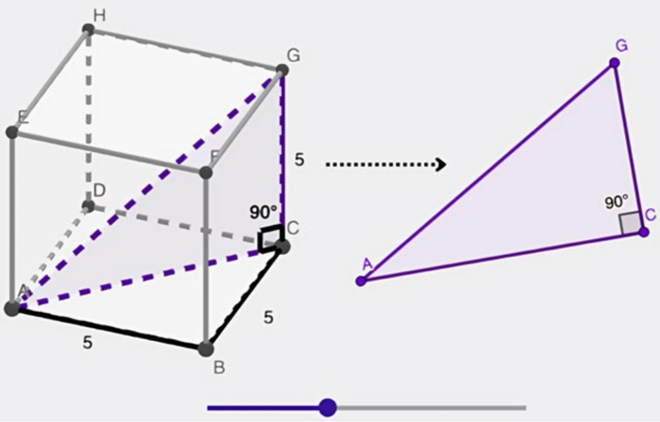


Figure 12. AR application showing an animation of the extraction of triangle (Figure created by the first author using GeoGebra and pieced together using Canva)

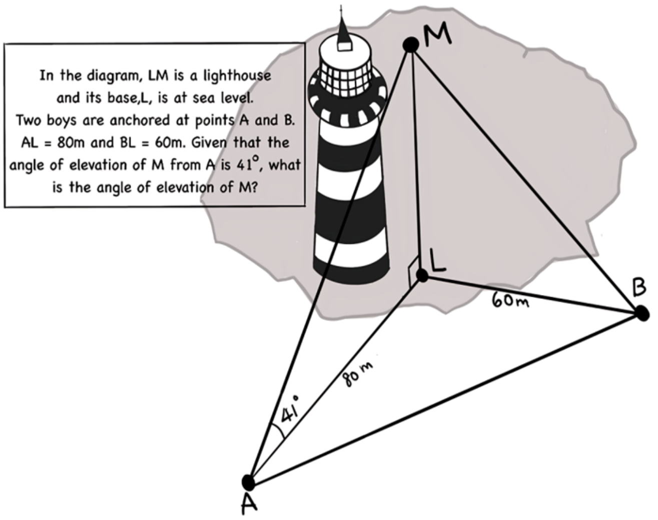


Figure 13. AR application displaying contextual question and augmented 3D object (Figure conceptualized by the first author)

According to the multimedia and contiguity principles, the AR application should present a combination of text and augmented 3D images together. For instance, the 3D trigonometry question can be displayed next to the augmented 3D object (**Figure 13**). This also reduces split attention effect as there is no need for students to shift their attention between the question on the worksheet and the 3D object on the AR application.

Scaffolding

In the first AR-incorporated mathematics lesson, teacher has to provide sufficient scaffold to guide the students. Since the AR application might have different functions and tools, it is crucial for the teacher to guide students to the right platform step-by-step to avoid confusion and distraction which may cognitively overload them. Thereafter, teacher can demonstrate the scanning of the QR markers to overlay the augmented 3D object, as well as the basic operations of the AR application such as moving around and back-and-forth to vary the angle and scale.

To further support low readiness students, teacher can provide additional scaffolding in the form of live demonstration. For instance, during the exploration stage, low readiness students may struggle with comprehending the guiding instructions in the AR worksheet together with the manipulation of the augmented 3D triangular prism. Hence, teacher demonstrate how to move the handheld display to observe the augmented 3D triangular prism from different angles while asking additional guiding questions such as "What do you observe about the 3D triangular prism when you look at it from the top view? What kind of shape is it? What happens if you now look at it from a side view? Do you see the same shape as before?" On top of the guiding questions in the AR worksheet, teacher will ask additional follow-up questions to better support low readiness students.

CONCLUSION

While AR technologies have been around for decades, the use of AR in a mathematics classroom remains limited due to various limitations of AR. Therefore, this paper has reviewed existing literature on the various benefits and limitations of using AR in the teaching of mathematics, and developed a conceptual framework that aims to harness the benefits and alleviate the limitations of using AR. The proposed RAMS framework guides the lesson design of an AR-incorporated mathematics lesson and this paper has demonstrated the application of the RAMS framework to the design of lesson on the topic of 3D trigonometry. However, due to various time and resource constraints, we were unable to implement this proposed lesson plan in an actual classroom and assess the effectiveness of the proposed framework. Hence, future studies could experiment the proposed RAMS framework with Secondary school students and evaluate its effectiveness by collecting quantitative and qualitative data about students' performance and experience with the AR-incorporated lesson. The insights gained from the experiment will greatly contribute to the refinement of the proposed framework to enhance its effectiveness and credibility.

Nonetheless, this paper has gathered findings on the advantages of using AR in the teaching and learning of mathematics, such as reducing students' cognitive load, providing visualization aid for abstract concepts, increasing students' engagement with interactive tools and promoting active learning. With the proposed RAMS framework, it is hoped the framework will encourage more educators to incorporate the use of AR into mathematics classrooms, and leverage on the benefits to enhance students' learning experiences and instill the joy of learning mathematics.

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