Contemporary Mathematics and Science Education
2022, 3(2), ep22013
ISSN 2634-4076 (Online)
https://www.conmaths.com/

Research Article

OPEN ACCESS

Affine Functions Using GeoGebra: An Investigation From the Perspective of Conceptual Fields Theory

Paulo Vitor da Silva Santiago [1]*, Francisco Régis Vieira Alves [2]

[1] Federal University of Ceará, Fortaleza, BRAZIL
[2] Federal Institute of Science and Technology of the State of Ceará, Fortaleza, BRAZIL
***Corresponding Author:** pvitor60@hotmail.com**

Citation: Santiago, P. V. da. S., & Alves, F. R. V. (2022). Affine Functions Using GeoGebra: An Investigation From the Perspective of Conceptual Fields Theory. *Contemporary Mathematics and Science Education*, 3(2), ep22013. https://doi.org/10.30935/conmaths/12112

ABSTRACT

This research aims to present the results of a mathematical investigation in the teaching of affine function with the support of the GeoGebra software, analyzed from the perspective of the theory of conceptual fields in the context of hybrid teaching, using the teaching methodology inverted classroom changing the traditional way of teaching. The applied research methodology was the case study, which was structured with a class of 38 high school students from a Brazilian full-time public school. The proposed activity was developed from the construction called affine function (interactive activity), available on the GeoGebra platform, and was structured in two meetings, one remotely and the other in person. The results show us the need to investigate the study of the affine function using digital technology in a more dynamic design in GeoGebra.

Keywords: GeoGebra, affine function, theory of conceptual fields, digital technology, math teaching

Received: 7 Apr. 2022 ◆ Accepted: 15 May. 2022

INTRODUCTION

The need to analyze pedagogical practices for basic education in Brazil persists to the present day. And the teaching of mathematics with the use of technological resources goes through several challenges in the perspective of making this discipline into something more interesting, attractive and close to the daily life in which the student is inserted. Because we are facing these changes, properly modernized, it is clear that the use of educational technologies applied in the classroom as a didactic support in the development of new educational methodologies.

However, currently, learning needs to be necessarily transformative, requiring from the teacher an understanding of new meanings for the class, relating them to previous experiences and student's experiences, allowing the structuring of problems that stimulate, instigate and experiment with new learning. In this context, Bacich (2016) emerges with the perspective of the inverted classroom, with the purpose of a pedagogical approach that makes interaction in the activities developed in the classroom and activities structured through digital technologies. Thus, in this teaching model, the objective is to make a student with an autonomous profile, agile and responsible for their knowledge, thinking and to involve them in learning.

Faced with a context of so many changes, the results of external evaluations inform what societies in general know about certain content. In Brazil, these assessments have been a reference for educational quality indicators and are included in all levels of education. Unlike assessments in Brazil, the *Program for International Student Assessment* (PISA) allows the country to compare student's knowledge and skills, showing the development results of students from the 37 member countries of the *Organization for Economic Cooperation and Development* (OECD), plus 42 partner countries/economies (National Institute of Educational Studies and Research Anísio Teixeira, 2019).

To help students understand more easily the concepts of affine function, the teacher can use digital tools, since GeoGebra helps in visualizing the graphs of the function in question, and its use is justified by the quantity and variability of instruments. technologies to which students show themselves daily (Oliveira et al., 2021). One of the topics that students have difficulty learning in basic education is the study of functions, especially the affine function (Campos et al., 2021), both in understanding the issues and in the relationship between the concepts of the function and the constructed images. for its traditional graphic construction.

According to studies by authors such as Araújo (2021), Jimenez (2021), and Silva (2021), some of the most recurrent problems that students have in relation to teaching related functions are the description of a function from a problem-situation, the structuring of graphs and sketches, through algebraic functions, position of the lines and orientation of the quadrants and orientation of the axes of the Cartesian plane. To better understand the difficulties faced, we analyzed the results of the research presented in this work from the perspective of the theory of conceptual fields, conceived by Vergnaud (1990), mathematician, philosopher, French psychologist, and student in the field of mathematics education.

According to Vergnaud (1990), the central concept of conceptual field is defined by the set of concepts, properties, situations, symbolic representations and theorems linked to the same concept. In this investigation, we are interested in the relationships between problem-situations and the structuring of the concept, the researcher professor considers two moments: diversity (variety of contexts associated with the concept) and history (the progress of the contextual presentation).

With these assumptions and considering that the structuring of the conceptual field of the affine function is developed in the GeoGebra software, making use of a construction called "affine function study", to understand how students relate the concept of affine function, its mathematical elements, and the handling of its graph, through a dynamic didactic situation. GeoGebra is a program designed for teaching and learning, whose main objective is to make mathematical concepts easier and clearer for students to understand (Ziatdinov & Valles Júnior, 2022). The objective of this research is to present the results of an investigation in the teaching of the affine function with the support of the GeoGebra software, analyzed from the perspective of the theory of conceptual fields in the context of blended learning, using the inverted classroom methodology.

The research methodology used for the development of this research was of a qualitative nature, of the case study type, because according to Yin (2001), the case study takes place in the research methodology from guiding questions such as "how" and "why". For the procedure and data analysis, the content analysis methodology, proposed by Bardin (1977), was used to analyze the written productions and the resolutions of the participating students. In this way, we observed the student's availability in relation to the activity proposed by the teacher, as well as their interests and learning conjectures about the content with the use of GeoGebra.

THEORETICAL FOUNDATION

The study of the affine function requires algebraic knowledge and understanding of the graph in relation to the movement of the line, in addition to the ability to understand problems, which is still an immense challenge for basic education students, as presented by the studies by Correa et al. (2020), Eloi and Andrade (2022), Mello and Colombo (2021), and Oliveira and Romao (2018). According to Correa et al. (2020), work with related functions in the classroom still happens, most of the time, with the use of assessments and through traditional methodologies in solving problems, in which little is suggested, from the perspective of the tasks, the use of questions as a way to improve the mathematical knowledge of this subject.

Mello and Colombo (2021) bring in the research a proposal for teaching affine function with use in the methodology of teaching, learning, and evaluation through the resolution of questions, more specifically in the topics of basic knowledge of affine function and construction of graphs, so that can outline teaching strategies that help them to learn the mathematical elements in question. From this perspective, these authors suggest that.

> "Eloi and Andrade (2022) consider it essential that the concept of function can be understood as an instrument of interaction between quantity and quality in the search for regularities of natural or social phenomena, thus constituting a "powerful" concept. The authors point to the importance of the potential didactic contract for student learning. These authors also suggest, in their work, the use of textbooks, what they call literature from the perspective of reading, which involves knowing the coefficients and terms of the function with the fixed and variable parts. Added to the aforementioned is the need to develop human relations skills such as tolerance and understanding" (Oliveira & Romão, 2018, p. 149).

About learning in algebraic developments, Oliveira and Romão (2018) report professional skills in challenging situations in the concept of affine function. Specifically, when teaching functions, these authors claim that the contents encompass values, attitudes and norms to determine learning. Silva and Pitanga (2018) mention the use of the GeoGebra software from a problem situation, that is, an exploration with the contents of affine function before its appropriation. Therefore, students understand the definition of "[...] affine function is: f(x)=ax+b, with a being a real number different from zero" (Silva & Pitanga, 2018, p. 12).

Faced with the obstacles encountered mainly in the teaching of related functions, we use the theory of conceptual fields (Vergnaud, 2009) as a basis for the process of acquiring knowledge by the subject, especially the student in school, as well as understanding the parameters (coefficients) that are inserted in the affine function. According to Vergnaud (2009), the learning context will always exist in various situations for which the student already has an organized strategy and skills necessary to solve it.

Vergnaud (2009), in his theory, describes that a concept is not structured alone and in isolation. Throughout the work, he carried out studies of the relationships between the concepts present in problem-situations that involve zero of the function, graph and movement of the line, respectively. Also, according to Vergnaud (2009), the explanation of the types of relationships between deductions and visualizations that can be considered for each type of problem.

Alves (2019) also points out, from the perspective of Vergnaud (1990), that "[...] its integration into studies aimed at understanding learning through activities [...]" of students, and in turn it is sustained in the understanding of the plastic and adaptive process of the organization of the subject's task, of the mathematics teacher, of their work environment and, above all, of the development of the competent and active educator.

With regard to the use of digital technology for related functions, the document that determines the Brazilian curriculum matrices – the *Common National Curriculum Base* (BNCC) – reinforces that digital technologies in education are not just about using them only as a support for provide learning or stimulate student's interest, but rather to use them with students to structure their knowledge using these educational technologies (Common National Curriculum Base, 2018), as in the same proposed method of activity using manual paper records and the contribution of an educational digital software, program, platform or application.

With this, we take in this research an activity aimed at teaching the related function with the use of digital technology through GeoGebra, which is a software that performs mathematical constructions aimed at teaching mathematics. Oliveira (2021) points out some possible constructions used by the commands available in GeoGebra, to encourage students in their commitment to mathematics, which are

> "[...] modify functions via the slider; obtaining special points of the functions, such as roots, maximum or minimum value, intersection with the axes; solve derivatives and integrals; among others" (Oliveira, 2021, p. 41).

Thus, the GeoGebra software is a technological resource with great potential to help the student in his learning about the affine function, in which he can analyze the commands in a manipulable and participatory way, relating his concepts with the graphic images structures from the handling of its functions, to be a tool that facilitates learning.

In view of the COVID-19 scenario, we reiterate that the activity described in this work took place in the blended learning teaching modality. According to Bacich (2016, p. 4), this modality is divided into

> "[...] two learning environments, the classroom considered traditional and the virtual learning environment, are gradually becoming complementary."

With regard to the flipped classroom methodology, according to Pavanelo and Lima (2017), the active learning methodology can facilitate actions aimed at interactive learning in the school environment, and develop guidelines based on digital resources outside the classroom environment, with the notable feature that class time is not used in classroom exhibitions, but is used for interactive group or individual activities.

With the inclusion of the active methodology of the inverted classroom, what was previously only a traditional homework task started to be carried out within the classroom, bringing everyday situations to the face-to-face school environment, since the concepts and definitions more difficult can be analyzed by the teacher during the class, as it is assumed that the student arrives in the classroom with previous knowledge about the content in question.

METHODOLOGY

The methodology used in this work is of a qualitative nature, of the case study type, in which we observe the student's development in the face of the activity made available through the GeoGebra software, related to the inverted classroom teaching model. Thus, we have a description of the didactic situation from the conjuncture in which the investigation is being developed (Gil, 2002). In the content analysis, we present the proposal of Bardin (1977), to analyze the descriptions and resolutions of the researched students, in which we observe the activities built by the students and the difficulties presented by them.

Data analysis is descriptive, as this type of analysis aims to describe and understand the moments as they happen during the application of the situation. In this way, we describe the interactive moments using GeoGebra in the classes. We sought to structure an interpretation of the data, which according to Gil (2002), is a link between the results obtained and other known results, whether theoretical results or research carried out previously. In this case, we link our interpretation of the data to the theory of conceptual fields and to the learning difficulties on the subject of related functions already described in the research.

The work was carried out with a class of 38 students from the 1st year of high school (age group between 15 and 17 years old) from a public school of integral education in the city of Quixeramobim, Ceará Brazil, in the second half of 2021. The structured activity addresses the topic of affine functions, with the general objective of analyzing the coefficients and graph generated by this type of function. As specific objectives centered on:

(i) understand what happens with the coefficients when moving parameters, a and b in the graph of an affine function in GeoGebra software;

(ii) identify the zero of the affine function and the behavior of the line in relation to parameters a and b of the manipulated function; and

(iii) confront the data of the affine function with the line from a graphical perspective.

The active methodology of the inverted class was separated into two meetings, one virtual by videoconference and the other face-to-face at school. It is worth noting that due to COVID-19 pandemic and the social distancing measures adopted in all school institutions, among the 38 students who are part of the class, all of them interacted at the moment at a distance, but only 27 of them were present in the classroom. meeting at school, while the other students participated in the speech broadcast by the Google Meet platform, interacting through the chat on the virtual platform and using the microphone. However, everyone performed and submitted the proposed activity in Google Classroom.

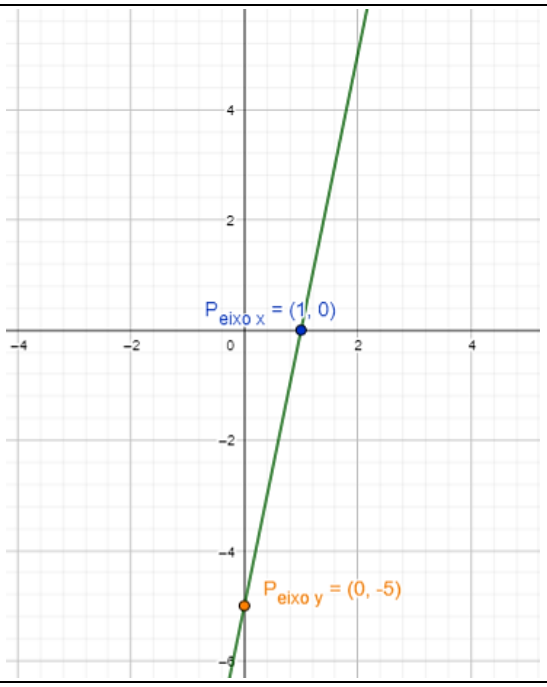
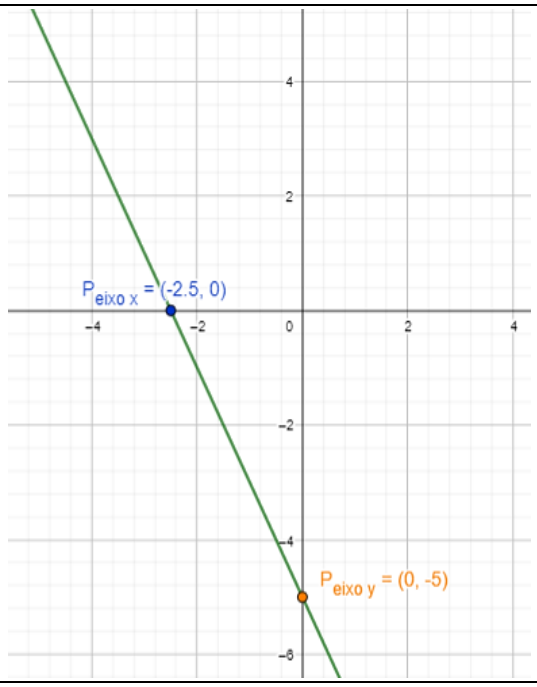
In the virtual interaction, students had access to the activity proposed in GeoGebra through the link provided by the teacher, entitled "affine function (interactive activity)" and had the opportunity to manipulate the graph shown. After handling, they were invited to use the Google Classroom platform to solve an activity on the movement of the line of an affine function, based on their reflections and understanding of the behavior of the graph. The problems proposed in the virtual activity are listed in **Table 1**.

In the face-to-face meeting, the teacher mediated and socialized the student's resolutions, presenting the GeoGebra software and analyzing the authenticity of the answers presented. From there, each student was asked to mathematically structure the movement of the line of an affine function from the handling of its coefficients (a and b), using the activity proposed in GeoGebra, the speech based on Google Classroom resolutions and the textbook of the student as classroom support.

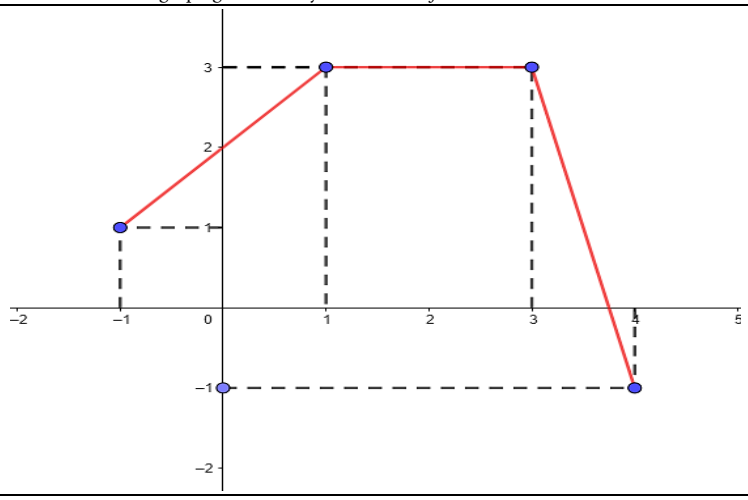
The research tools used in data collection were a virtual questionnaire prepared to collect the records and solutions of the students and observation prepared by the two professors who accompanied the research and data collection. Thus, for data collection, notes of student's resolutions on the Google Classroom virtual platform and photos of the activity developed in the classroom were used. To maintain the identity of the research participants, the students will have their names configured by P1 (participant 1), P2 (participant 2). And so on.

The student's resolutions were analyzed, evidenced and discussed by two mathematics teachers from the aforementioned school where the research was carried out, so that this article could be written. This work involves human beings, therefore, we announce that all identifications of the participants were preserved, as well as we reinforce that the ethical issues that constitute the analysis and interpretation of the data collected in this research were met.

Table 1. Questions directed in the proposed activity

01. The following graphs represent an affine function. Identify whether each exposed function is ascending or descending.	
(a) $f(x)=5x-5$	(b) $f(x)=-2x-5$
	

02. Answers the questions from the visualization of the graph generated by the function f:



(a) What is the domain and what is the image of f?

(b) At how many points does the graph cross the x-axis? and the axis y?

(c) $f(1, 7)$ is greater, less than or equal to $f(2, 9)$?

(d) What is the maximum value of $f(x)$? And the minimum value?

(e) Which point on the graph has an abscissa -1?

(f) The point (4, -1) belongs to the graph of f?

(g) What is the value of x when $f(x)=3$?

In terms of data processing and analysis, a posteriori epistemological reasoning is used to deduce the collected data, interpreting them based on the theoretical references indicated in this study and on reading the findings of research relevant to the topic. When we refer to the quality and reflexivity of the analysis of student's resolutions and interactive moments in the classroom, we are evaluating not only correct or wrong answers, but also the various ways in which they demonstrate knowledge using the GeoGebra software in the classroom. class and also as a way to observe the student's learning difficulties.

In this perspective, the study is based on the reliability of conversations between studies by other authors related to the topic, as well as on the experience of the professors who conducted the study. So, mastering a conceptual field is not a quick process, but when you encounter difficulties, you overcome them (Batista et al., 2021).

Therefore, Batista et al. (2021) state that it is essential to provide opportunities and stimulating moments that allow the student to speak, because through speech the student is able to express his understanding of a certain concept, revealing to the teacher which concepts were assimilated during the application of the activity using digital technology.

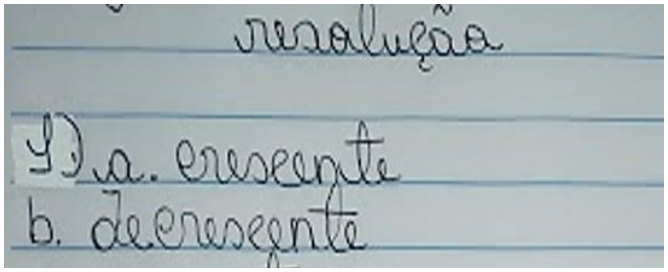


Figure 1. Resolution of P2

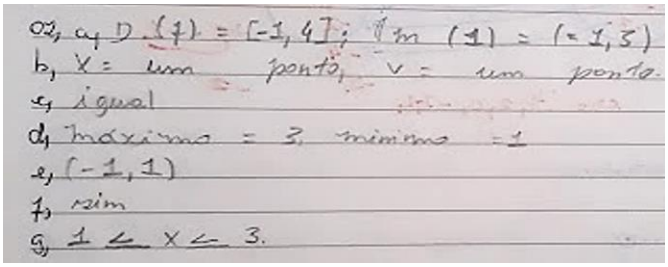


Figure 3. Figure on top of a column

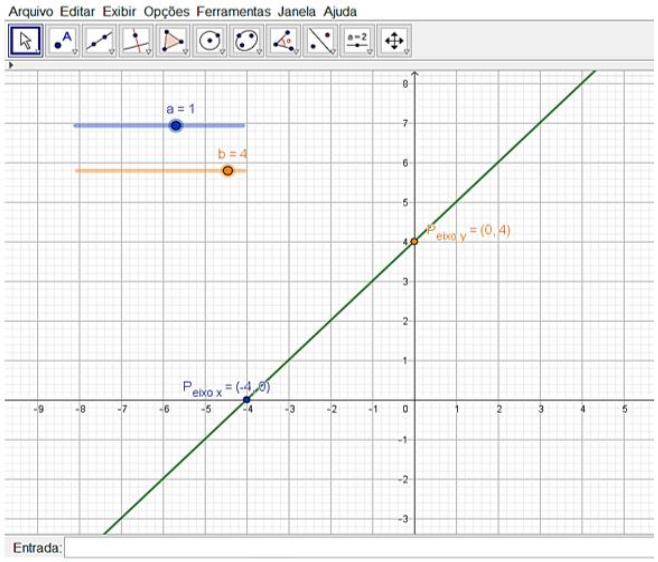


Figure 2. Line visualization in GeoGebra

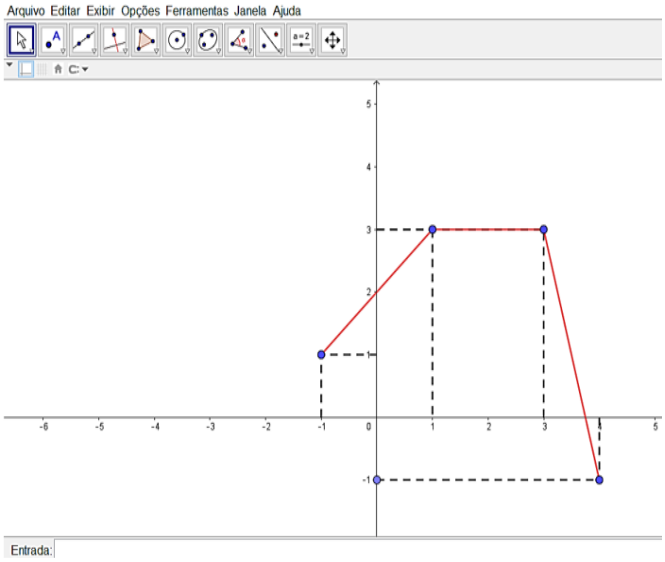


Figure 4. Construction of the graph referring to the second question

RESULTS AND DATA ANALYSIS

In this topic, we will discuss the student's resolutions to the proposed activities, based on the theory of conceptual fields. In addition, we highlight some images as a record and understanding of the use of GeoGebra software in blended learning and combined experiences using the flipped classroom approach. From the construction made available and directed to this activity using GeoGebra, we started the speech with the first question, which asks the student to describe the movement of the line related to the graph of the affine function from the movement of the sliders a and b (**Figure 1**).

In the illustration shown by student P2 in **Figure 1**, they represent what was noted as a resolution of the first question pointed out by most of the class to the movement of the sliders, showing that the line can be directed to the left or right in the horizontal and vertical facing upwards, if a>0 and facing downwards a<0, highlighting the fact that students noticed relationship of coefficients on the line of the function.

In this first question, students were able to analyze and make combinations on the movement of the graph of the function based on intrinsically conceptual characteristics, although the line of the affine function within the GeoGebra software is not a simple learning concept, but an illustration (Vergnaud, 2009). This structured construction in the software is demonstrated in **Figure 2**. From this perspective, we can observe that the second question describes questions directed to the parameter of the function, with some difficulty to verify the information contained in the graph (**Figure 3**).

Student P7 relates the points on the graph from the coefficients visualized in the GeoGebra software. Student P9, on the other hand, states that the sign in the function is changed according to the movement of a and b located in the sliders. We can notice that in **Figure 3**, the students showed resolutions based only on the illustration seen in GeoGebra, but that the relative position of the line had a better understanding and understanding of all.

Based on Vergnaud (2009), the analysis of a mixed question allows a detailed observation of the information contained in the graph and questions relevant to the situation and the relationships defined between the points present in the problem situation (**Figure 4**). Ortega et al. (2020) provide a concept for an affine function, defined as a linear function of type ax+b, as a and b are natural numbers. The question seeks the student's previous knowledge of the zero of the function, numerical sets in the definition of domain, range and image of the function, in addition to requiring a level of conception for which the student is often not prepared to face.

We understand that there was a mistake in the interpretation of the second question by these students in a specific way, possibly because they could not understand the theoretical part mentioned in the textbook, which was explained by the teacher at the time of interaction in the classroom in the face-to-face format. According to Vergnaud (2009), the variety of situations is important for student learning. However, the teacher chose to present similar situations, from the point of view of the theory of conceptual fields, one demonstration after another, a fact that contributed greatly to expanding the student's knowledge of these issues.

Figure 5. Face-to-face moment using GeoGebra

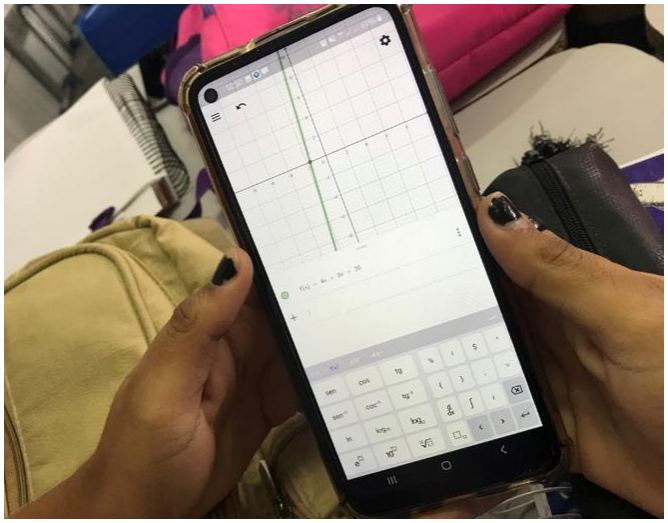


Figure 6. Records of the face-to-face meeting in the classroom

In the face-to-face meeting, **Figure 5**, **Figure 6**, and **Figure 7** show records of students using the GeoGebra software divided into teams, where the contents explored in the software were discussed and doubts about the activity proposed by the teacher were clarified. The use of GeoGebra in the classroom by the teacher after the development of the activity at home, following the teaching methodology of the inverted classroom, allowed a positive learning, in which the student's resolutions to the proposed activity happened in an organic way.

The main differences between this study and other studies read and presented in the theoretical framework of this work are: conducted in mixed instruction; the use of GeoGebra software as a teaching resource in conjunction with a flipped classroom approach; and analysis of the theory of conceptual fields.

However, we can report that the research by Silva and Pitanga (2018) inspired us to use the GeoGebra software with related functions, as well as the works of Correa et al. (2020), Mello and Colombo (2021), Oliveira and Romão (2018), and other works cited throughout the research helped us to understand the main difficulties of students in understanding the affine function. In addition, Vergnaud (1990, 2009) research presented us with the means to understand these difficulties and interpret them based on his theory of conceptual fields.

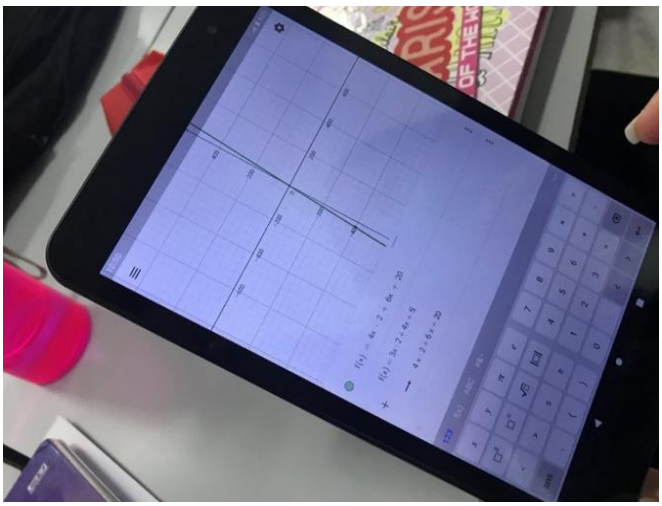


Figure 7. Use of mobile devices to resolve the issue

DISCUSSION

Based on the theoretical framework, we can find the reasons for the errors pointed out by the students and understand the way in which mathematics textbooks approach the topic very succinctly, creating a learning gap (Silva & Pitanga, 2018). Thus, as shown in the works of Correa et al. (2020), Mello and Colombo (2021), and Oliveira and Romão (2018), the study of the affine function can be motivated by application problems where it is necessary to find a line with declination for vertical and horizontal, for example in problem of finding the zero of the function. According to these authors, based on the results collected, we can understand the study of this type of function must be structured so that the student can understand the relationship between its coefficients and the graphic aspects, avoiding the rules of memorization.

In addition, as the research by Eloi and Andrade (2022) shows, the methodology of teachers in the Brazilian context has some gaps, such as traditional classes and little use of technology or recreational resources, and thus, little student development and a narrow and simplified affine functions. In addition, a more in-depth discussion of the concept of affine function in the classroom environment is lacking, as well as the exploration of the concepts of functions, considering the relationship between concepts and mathematical thinking, as explained by Vergnaud (1990), explained in theoretical foundation of this research.

The observations and resolutions presented by the students allow us, as teachers, to think about our practice in order to bring students the materials and methods related to mathematics (especially algebra) in a clearer and more concrete way, strengthening theoretical concepts for answers to the questions that require practical calculations. The research result suggests that mathematics teachers should modernize their classes, choosing new strategies for the teaching of a related function in Brazilian basic education, since only the problem-situation method and the use of the textbook are not acceptable enough for students to students learn to calculate algebraic questions.

Finally, we highlight the importance of including the BNCC, making use of technological educational resources as a way of including prior knowledge and strengthening student's mathematical thoughts, given that we have new generations of students connected to the internet, who need methodologies of teaching that interact with their abilities in solving mathematical problems.

FINAL CONSIDERATIONS

This research presents the description of an activity carried out in the hybrid teaching model with high school students in the study of the affine function, analyzed from the perspective of the theory of conceptual fields, in which we seek to observe the relationship between the concept and figure demonstrated by the students through of a construction in GeoGebra software. The study of the affine function with the support of GeoGebra can provide the teacher with a better view of the student's progress. The student's interest becomes efficient in the structuring of activities with the support of the GeoGebra software and with the use of the inverted classroom methodology.

The theory of conceptual fields based the analysis of the results, showing that there is a certain difficulty in the association between the parameters of a concept of an affine function and its graphic representation in the software, highlighting the relevance of working with this content to arouse student interest and meaning in everyday situations involving this subject, which became clearer due to the use of GeoGebra.

The GeoGebra software has several tools and commands for the development of the student's mathematical thinking and reasoning. In this research, we observed that the moment of interaction of the resolutions was productive, in which the students had the opportunity to analyze their mistakes and think about them, as well as internalize their successes and expose the understanding of the affine function in a natural way. The transformations in the educational scenario caused by COVID-19 require new work methodologies, different from the traditional expository class. From this practice, we consider the need for a new attitude towards the classroom, in which we clearly reflect the role of digital technology for the transformation and learning of students and improvement of the teacher's pedagogical time.

When structuring this work, we realized that one of the most provocative ways to modify the study of related functions for the student is to use technological educational resources as support. In this experience with the GeoGebra software, we learned that using this technological tool we can structure not only activities on the functions described in this research, but also other themes and practical activities that direct the student to analyze, reason, develop and mainly structure their own constructions to solve problems. mathematicians.

Among the obstacles of this study, we have the adversity that occurred during the virtual class related to the access to the internet by the students, as well as the lack of technological equipment compatible with the software for the development of household tasks. Unfortunately, this is common in the reality of many Brazilian public educational institutions. In addition, we also consider the fact that part of the students participated in the two virtual meetings, in view of the social distance imposed by the COVID-19 pandemic, as we have students who belong to the risk group.

We propose the reproduction and reading of this work, as well as the use of the GeoGebra software and the inverted classroom teaching methodology for high school mathematics teachers. Future developments in this line of research can insert an appropriate analysis that can compare the use of GeoGebra with other technological tools or even expand the use of GeoGebra to other areas of education and in other high school grades.

In addition, as a conception of the future, it is relevant to note that other mathematics teachers in Brazil and in other countries teaching with digital educational technologies, as well as influencing them to use the flipped classroom method, which yielded good practice in school and home environment. Finally, we hope that this research can encourage other teachers to work with active methodologies, using digital technology beyond the classroom and using GeoGebra software to explore other methodological possibilities for teaching mathematics.

Author contributions: All authors were involved in concept, design, collection of data, interpretation, writing, and critically revising the article. All authors approve final version of the article.

Funding: A pesquisa foi desenvolvida no Brasil e apoiada financeiramente pelo Conselho Nacional de Desenvolvimento Científico e Tecnológico [The research was developed in Brazil and financially supported by the National Council for Scientific and Technological Development].

Declaration of interest: Authors declare no competing interest.

Data availability: Data generated or analyzed during this study are available from the authors on request.

REFERENCES

Alves, F. R. V. (2019). A vertente Francesa de estudos da didática profissional: Implicações para a atividade do professor de matemática [The French branch of professional didactics studies: Implications for the activity of the mathematics teacher]. *Vidya, 39*(1), 255-275.

Araújo, J. R. de. (2021). *Conversão entre os registros de representação gráfico e algébrico da função afim: Análise a partir da interpretação global de propriedades figurais* [*Conversion between graphical and algebraic representation records of the affine function: Analysis from the global interpretation of figural properties*] [Master's thesis, Universidade Federal de Pernambuco, Recife].

Bacich, L. (2016). Ensino híbrido: Relato de formação e prática docente para a personalização e o uso integrado das tecnologias digitais na educação [Blended teaching: Report of teacher training and practice for the personalization and integrated use of digital technologies in education]. In *Proceedings of the Simpósio Internacional de Educação e Comunicação-SIMEDUC* [*International Symposium on Education and Communication*].

Bardin, L. (1977). *Análise de conteúdo* [*Content analysis*]. Edições 70.

Batista, G. E., Staudt, E., Zabadal, J. R. S., & Tauceda, K. C. (2021). Resolução de problemas abertos considerando a aprendizagem significativa e teoria de campos conceituais: Uma proposta para ensinar física quântica no ensino médio [Open-ended problem solving considering meaningful learning and conceptual field theory: A proposal for teaching quantum physics in high school]. *Experiências em Ensio de Ciências* [*Experiences in Science Teaching*], *16*(3).

Campos, M. S., Altoé, R. O., & Silva, L. S. F. da. (2021). Formulação de problemas de função afim: Preferências, saberes e vivências dos estudantes [Formulation of affine function problems: Preferences, knowledge and experiences of students]. *Revista de Educação Matemática* [*Mathematics Education Magazine*], *18*. https://doi.org/10.37001/remat25269062v18id531

Common National Curriculum Base. (2018). *Base Nacional Comum Curricular*. http://basenacionalcomum.mec.gov.br/

Correa, M. M., Meneghetti, C. M. S., & Poffal, C. A. (2020). Resolução de problemas envolvendo função afim e semelhança de triângulos [Solving problems involving affine function and similarity of triangles]. *Ensino da Matemática em Debate* [*Teaching Mathematics in Debate*], *7*(3), 28-46. https://doi.org/10.23925/2358-4122.2020v7i3p28-46

Eloi, Q. C., & Andrade, V. L. V. X. de. (2022). Lei de formação da função afim: um estudo à luz do contrato didático potencial [Law of formation of the affine function: a study in the light of the potential didactic contract]. *Vidya, 42*(1), 55-73. https://doi.org/10.37781/vidya.v42i1.3805

Gil, A. C. (2002). *Como elaborar projetos de pesquisa* [*How to design research projects*]. Atlas.

Jimenez, J. B. (2021). *Uma sequência didática para o ensino e aprendizagem da função afim com o uso da robótica estrutural* [*A didactic sequence for the teaching and learning of affine function using structural robotics*] [Master's thesis, Universidade Católica de São Paulo].

Mello, A. L. de., & Colombo, J. A. A. (2021). Ensino de função afim através da resolução de problemas: Uma intervenção no ensino médio [Affine function teaching through problem solving: An intervention in high school]. *Revista Ensin@ UFMS* [*Ensin@ UFMS Magazine*], *2*, 67-89. https://doi.org/10.55028/revens.v2iEsp..13906

National Institute of Educational Studies and Research Anísio Teixeira. (2019). Programa internacional de avaliação de estudantes [Program for international student assessment]. *Brazilian Ministry of Education*. http://portal.inep.gov.br/pisa

Oliveira, F. de. F. (2021). *Modelagem matemática e a calculadora gráfica geogebra no estudo da função afim* [*Mathematical modeling and the geogebra graphing calculator in the study of the affine function*] [Master's thesis, Universidade Federal Rural do Semiárido, Mossoró].

Oliveira, S. L. de., & Romão, E. C. (2018). Sequência didática para o ensino de função afim utilizando aprendizagem baseada em projetos [Didactic sequence for teaching affine function using project-based learning]. *ACTIO: Docência em Ciências* [*ACTIO: Teaching in Science*], *3*(3). https://doi.org/10.3895/actio.v3n3.7485

Oliveira, T. S. P. de., Silva, D. C. S., & Lima, A. C. D. S. (2021). O software GeoGebra no ensino da função quadrática [The GeoGebra software in teaching the quadratic function]. *Boletim Cearense de Educação e História da Matemática* [*Ceará Bulletin of Education and History of Mathematics*], *8*(23), 861-876. https://doi.org/10.30938/bocehm.v8i23.4954

Ortega, M. V., Rodriguez, G. A. A., & Nieto Sanchez, Z. C. (2020). Transposición didáctica para apoyar la enseñanza de la función lineal y afín para estudiantes de cálculo usando las NTIC [Didactic transposition to support the teaching of the linear and affine function for calculus students using NICTs]. *Revista Aglala* [*Aglala Magazine*], *11*(2).

Pavanelo, E., & Lima, R. (2017). Sala de aula invertida: A análise de uma experiência na disciplina de cálculo I [Flipped classroom: The analysis of an experience in calculus I]. *Bolema, 31*(58), 739-759. https://doi.org/10.1590/1980-4415v31n58a11

Silva, C. F. da. (2021). *Ensino aprendizagem de função afim via exploração, resolução e proposição de problemas com o uso do aplicativo Desmos em contexto remote* [*Teaching affine function learning via exploration, problem solving and posing with the use of the Desmos application in a remote context*] [PhD dissertation, Universidade Estadual da Paraíba, Campina Grande].

Silva, F. dos. S., & Pitanga, J. S. (2018). Sequência de ensino: Uma proposta de resolução de problemas na integração do software geogebra no estudo da função afim no 9º ano [Teaching sequence: A proposal to solve problems in the integration of geogebra software in the study of the affine function in the 9th grade]. *Revista Sergipana de Matemática e Educação Matemática* [*Sergipe Magazine of Mathematics and Mathematics Education*], *3*(1). https://doi.org/10.34179/revisem.v3i1.7293

Vergnaud, G. (1990). La théorie des champs conceptuels [Conceptual field theory]. *Recherches en Didactique des Mathématiques* [*Research in Didactics of Mathematics*], *1*(23), 133-170.

Vergnaud, G. (2009). O que é aprender [What is learning?]. In M. Bittar, & C. A. Muniz (Eds.), *A aprendizagem matemática na perspectiva da teoria dos campos conceituais* [*Mathematical learning from the perspective of conceptual fields theory*] (pp. 13-35). CRV.

Yin, R. K. (2001). *Estudo de caso: Planejamento e métodos* [*Case study: Planning and methods*]. Bookman.

Ziatdinov, R., & Valles Júnior, J. R. (2022). Síntese de modelagem, visualização e programação em GeoGebra como uma abordagem eficaz para o ensino e aprendizagem de tópicos STEM [Synthesis of modeling, visualization and programming in GeoGebra as an effective approach to teaching and learning STEM Topics]. *Mathematics, 10*(398). https://doi.org/10.3390/math10030398