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Algebra errors and misconceptions: A teaching and learning opportunity

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ABSTRACT

This study examines common algebra errors and misconceptions among grade 11 secondary school students in South Africa and proposes a classroom-based intervention strategy to turn these errors into learning opportunities. A purposive sample of 35 students from a public secondary school in Seshego Township, Polokwane, was selected for the study. Using students' scripts from a district-level algebra test, the research utilized a mixed-methods case study design. Qualitative analysis was employed to categorize types of errors and provide interpretive explanations, while frequency counts and percentages were calculated to determine their prevalence. The results revealed three predominant misconceptions: misapplication of algebra rules (37.04%), illegal cancellation (48.15%), and cancellation errors (14.81%), most of which stemmed from prior learning experiences. Building on constructivist and sociocultural learning theories, the study introduced a collaborative, student-centered intervention using error-analysis worksheets integrated into daily lessons. This approach encouraged peer dialogue, reflection, and correction of misconceptions. A decrease in the frequency of errors and misconceptions was observed in the post-intervention assessment results. The study's novelty lies in linking diagnostic error analysis with pedagogy to provide a replicable model for transforming algebraic mistakes and misconceptions into opportunities for conceptual growth. Despite being limited to one school, the findings offer new theoretical and practical insights into how error analysis can enhance metacognition, resilience, and instructional quality in mathematics. Future researchers are encouraged to conduct experimental studies on the proposed intervention to assess its effectiveness.

Keywords: algebra errors, misconceptions, collaborative learning, error analysis, teaching strategy, constructivism

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INTRODUCTION

Algebra is a branch of mathematics that manipulates variables and mathematical symbols according to established rules (Startup Info Team, 2021). It includes everything from solving basic equations to simplifying expressions, dealing with fractions, exponents, and polynomials, and analyzing abstractions. Algebra permeates all other mathematical topics. For example, students need knowledge of algebra to master mathematics topics such as statistics and calculus (Demme, 2018). There is no doubt that students' performance in mathematics is dependent on their algebraic skills. A solid foundation in algebra ensures a smooth transition from high school mathematics to college or university mathematics (Great Schools Staff, 2021). Furthermore, algebraic skills are beneficial in various sectors, including medicine, engineering, science, technology, accounting, economics, programming, and everyday problem-solving (Russell, 2018). It is no wonder that algebra has become an essential component of secondary school mathematics curricula worldwide. However, teachers and students alike face challenges in teaching and learning algebra worldwide.

Algebra is a highly abstract branch of mathematics that relies on symbolic language, which often makes it challenging for secondary school students to grasp (Maharani & Subanji, 2018). In a study involving grade 9 students in South Africa, Pournara (2020) found that students struggled with solving linear equations and simplifying algebraic terms, particularly when negative values were involved. In another South African study involving grade 10 students, conjoin errors, senseless cancelling, change of sign errors, turning expressions into equations, and oversimplification were among the discovered algebra errors and misconceptions (Makonye & Mashaka, 2016). Satianingrum et al. (2020) found that grade 8 students struggle working with variables, coefficients, constants, and properties of whole numbers in Indonesia. Elsewhere in Kenya, Mulungye et al. (2016) found that form 2 students struggled with adding and subtracting unlike terms and incorrectly applied the distributive property. Similar findings were obtained in Malaysia in a study involving university students enrolled in a mathematics course (Ung et al., 2019). In Zimbabwe, Ndemo and

Ndemo (2018) found that Form 3 students violated distributive property, added unlike terms, disregarded brackets, and engaged in illegal cancellation.

The current study aims to add to existing knowledge by extending the investigation of algebra errors and misconceptions to grade 11 students in South Africa. The following research questions are used to guide the study:

1. What are the most common algebra errors and misconceptions among grade 11 students at the selected school?
2. What are the sources of these errors and misconceptions?
3. How can these errors and misconceptions be transformed into teaching and learning opportunities within regular classroom practice?

LITERATURE REVIEW

Defining Errors and Misconceptions in the Context of Algebra

Common errors in algebra are mistakes that students frequently make while trying to solve algebra problems. Errors can be factual, procedural, or conceptual (Muthukrishnan et al., 2019). Factual errors occur when students fail to recall the facts required to solve a mathematics problem. Procedural errors occur when students fail to follow the correct steps to solve a mathematics problem. Factual and procedural errors are regarded as "slips" and can be easily identified and dealt with (Kshetree, 2018). Conceptual errors ("bugs"), on the other hand, are challenging to deal with as they emerge from misconceptions (Makamure, 2021).

Misconceptions are incorrect assumptions, beliefs, interpretations, or explanations that contradict established mathematical meanings (Mulungye et al., 2016). They are so deeply ingrained in students' minds that they cannot be easily dislodged (Lucariello, 2015; Ndemo & Ndemo, 2018). Additionally, misconceptions lead to repeated or systematic errors in students' work. Misconceptions cannot be corrected simply by crossing them out or underlining and providing the correct solutions (Makonye & Fakude, 2016), a practice commonly observed among many mathematics teachers.

Sources of Algebra Errors and Misconceptions Based on Previous Studies

Several studies have uncovered the sources of common errors and misconceptions in algebra. Mulungye et al. (2016) attributed conjoin errors to the duality of mathematical concepts as processes or objects and the use of operator symbols like the (+) and (-) signs in algebraic expressions as an invitation to act. Conjoin errors occur when students use addition or subtraction to combine unlike terms, for example, merging $2x + 2$ to $4x$ or reducing $3x - 1$ to $2x$. The plus and minus signs prompt students to operate with the terms.

Ndemo and Ndemo (2018) share the view that the abstract nature of algebra, the method of lesson delivery, interference from previous learning, conflicting ideas, inadequate knowledge of integers and signs, and not understanding at all, lead to violations of the distributive property, adding unlike terms, illegal cancellation, and disregarding brackets. Ung et al. (2019) attribute students' errors to a lack of prerequisite algebra facts and concepts, insufficient understanding of arithmetic concepts, ineptitude in dealing with integers and signs, and a lack of basic knowledge of algebraic expressions.

Many students struggle with new algebraic concepts, which often involve complex symbolic rules and various transformations. This complexity can be overwhelming, leading to cognitive overload when the demands of processing and applying these concepts exceed one's mental capacity (Gupta & Zheng, 2020). As a result, students may make mistakes not due to a lack of understanding but because they feel overwhelmed by the material.

Additionally, repeated experiences of failure in mathematics can worsen this issue. When students continually face difficulties, it can lead to mathematical anxiety, making them more hesitant to engage with algebra (Shields, 2007). This anxiety, combined with the pressure to perform well, can significantly hinder their ability to learn and apply algebraic concepts effectively (Zhang et al., 2019). As a result, these factors can create a cycle of mistakes and misconceptions that make mastering algebra increasingly challenging for many learners.

From the preceding literature, it is evident that researchers have thoroughly explored and understood the types and sources of algebra errors and misconceptions. However, knowledge of how to deal with algebra errors and misconceptions is limited.

Challenges in Dealing with Algebra Errors and Misconceptions in Secondary Schools

The secondary school mathematics curriculum in South Africa is so content-laden that it pressures teachers to rush onto the next topic, leaving many students with knowledge deficits (Zuma, 2021). Similar challenges have been reported in Zimbabwe (Majoni, 2017), Nigeria (Awofala, 2012), Ghana (Mereku & Anumel, 2011), Uganda (Clegg et al., 2008), the USA (Zambo & Cleland, 2005), and Greece (Potari et al., 2019). Apart from having an overloaded mathematics curriculum, many South African secondary schools, like other developing countries, are characterized by large class sizes (Graham, 2023), which makes it difficult for teachers to closely monitor every student's daily written work and offer individualized feedback. In these circumstances, many algebra errors and misconceptions remain unnoticed until summative assessments, resulting in frustration for teachers. At this stage, it is too late to intervene effectively.

Students' misconceptions stay hidden unless teachers actively identify and address them (Askew & Wiliam, 1995). It is important to recognize that it is not possible to teach in a manner that completely eliminates the risk of students developing misconceptions. Therefore, the teacher's role is to reduce the likelihood of students harboring these misconceptions by using regular assessments to identify them (Tobey, 2017). Additionally, teachers should implement intervention strategies that address misconceptions and help students develop a solid conceptual understanding of algebra concepts (Tobey, 2017).

A study conducted by Mulungye et al. (2016) in Kenya's Machakos District found that although teachers were aware of their students' errors and misconceptions when learning algebra, they were unable to utilize this information to develop instructional techniques that addressed these issues. In Nigeria, Zuya (2014) discovered that many teachers struggled to ask questions that would uncover the sources or causes of students' misconceptions. Additionally, some teachers had difficulty understanding algebra problems, which hindered their ability to identify misconceptions or errors in students' solutions. Overall, the teachers' pedagogical content knowledge was found to be generally inadequate (Zuya, 2014).

In Zimbabwe, Ndemo and Ndemo (2018) suggested that instead of trying to avoid algebra errors and misconceptions, teachers should embrace them as a vital component of the teaching and learning process. However, the researchers made no further proposals on how mathematics teachers could practically transform algebra errors and misconceptions into opportunities for teaching and learning.

In Malaysia, Ung et al. (2019) identified students' algebra errors and misconceptions as well as their root causes, but they made no suggestions for correcting the identified errors and misconceptions. In another study involving form 2 students in Indonesia, Maharani and Subanji (2018) found scaffolding effective in resolving algebra errors and misconceptions caused by guessing. However, the intervention was ineffective in addressing mistakes that resulted from the effects of prior learning.

Makonye and Mashaka (2016) investigated how in-depth discussions and interactions could reduce algebra errors and misconceptions in a grade 10 class of low-achieving students in South Africa. The intervention consisted of two 50-minute class discussions and dialogues with participating students, focusing on the misconceptions and errors identified by the researchers. Post-intervention results showed that discussion and dialogue assisted students in overcoming most of their algebra errors and misconceptions, except for the cancelling error.

While discussion, dialogue, and scaffolding were found to be effective strategies for addressing algebra errors and misconceptions, these strategies could not eliminate errors caused by the effects of prior learning and illegal cancelling errors. As a result, further research is warranted to supplement the existing knowledge.

The Research Gap

Despite extensive studies that document the prevalence and types of algebraic errors and misconceptions among secondary school students in various contexts (e.g., Ndemo & Ndemo, 2018; Makonye & Mashaka, 2016; Ung et al., 2019), a significant gap remains in the literature regarding systematic, classroom-based interventions. These interventions need to be inclusive and integrated into everyday instruction. Most existing research focuses on identifying and categorizing misconceptions. However, it often falls short of providing practical, replicable strategies for teachers to address these issues in real-time within mixed-ability classrooms. When interventions have been attempted, they are typically limited to isolated or small groups of struggling students (Maharani & Subanji, 2018) and rely on one-time remedial sessions (Makonye & Mashaka, 2016).

Furthermore, in contexts like South Africa, there is no policy mandate for recording student errors, applying specific remedial strategies, or evaluating their effectiveness, leaving individual teachers to improvise based on their own training and experiences. The lack of institutionalized frameworks or teacher education curricula that support teachers in turning errors into instructional opportunities exacerbates this gap.

Therefore, what is missing is a holistic, teacher-facilitated, learner-centered approach that leverages students' own misconceptions as catalysts for peer dialogue, critical thinking, and conceptual growth within regular classroom settings and without the stigma of isolation. This study aims to fill that gap by proposing, illustrating, and empirically testing an intervention strategy that incorporates error analysis worksheets and collaborative error-correction activities into everyday algebra instruction, with the goal of not only correcting misconceptions but also transforming them into meaningful learning experiences.

Theoretical Foundation

Errors and misconceptions are essential to the learning process, as they provide opportunities for growth and deeper understanding. When students make mistakes in mathematics, they are prompted to critically analyze their thought processes, which fosters conceptual understanding, problem-solving skills, and resilience (Silver et al., 2023). Research indicates that students who embrace their errors learn more effectively than those who strive for perfection (Shahla et al., 2023).

Teachers play a crucial role in developing a growth mindset by creating a supportive classroom environment where mistakes are valued (Chinn, 2020). By examining students' misconceptions, teachers can identify learning gaps and determine appropriate interventions (Boser, 2024).

This study is grounded in constructivist, sociocultural, metacognitive, and growth mindset theories. Constructivist theory suggests that learners build new knowledge by connecting it to existing understanding (Bruner, 1966; Piaget, 1970). Misconceptions arise when mental models are incomplete. Vygotsky's (1978) sociocultural theory emphasizes learning as a social process, supported by collaboration and peer discussions. From a metacognitive perspective, analyzing and correcting errors enhances reflection and self-regulation (Flavell, 1979). The growth mindset framework promotes perseverance, helping students see errors as a natural part of learning (Chinn, 2020; Dweck, 2006). Overall, this learner-centered approach integrates collaborative error analysis into daily instruction, with the potential to enhance conceptual understanding and resilience in algebra learning.

METHODS

Research Design

This study utilized a case study design. This approach combined both qualitative and quantitative elements to investigate and classify errors and misconceptions in algebra. The case study method was ideal for gaining deep, contextual insights into students' thought processes and error patterns within a natural classroom environment.

Sampling

A purposive sampling technique was used to select participants who could provide valuable insights into the research problem. The study involved 35 grade 11 students from a public secondary school in Seshego Township, Polokwane, South Africa. One of the researchers in this study was responsible for the grade 11 mathematics class that took part in the research. These students were chosen based on the availability of their mathematics test scripts from a district-standard assessment conducted during term 1 of the 2024-2025 academic year.

Research Instruments

The primary data collection tool was the district-level test instrument, which the school had already administered as part of a district-level standard assessment. This test covered essential algebra topics, including simplifying algebraic and exponential expressions, solving quadratic equations, and applying laws of exponents. Although not designed by the researcher, this test was used as a document-based instrument to obtain authentic examples of student work, ensuring ecological validity. The district-level test consisted of ten items: four on simplifying expressions, three on the laws of exponents, and three on solving quadratic equations. Each item was worth between 3 and 5 points.

A post-intervention assessment consisting of six free-response questions was created by the researchers to address areas where errors and misconceptions were identified. The questions aligned with the style and standards of the district-level assessment. This was confirmed by five grade 11 mathematics teachers who independently reviewed and compared the district-level assessment items with the researcher-developed version.

Operational Definitions

1. **Algebra errors:** In this study, these refer to incorrect steps, computations, or symbolic manipulations made by students when solving algebra problems.
2. **Misconceptions:** These are recurring conceptual misunderstandings that lead to systematic errors, such as assuming exponents can be multiplied across terms or incorrectly simplifying rational expressions with multiple terms.
3. **Qualitative data:** Consists of categorized student responses, error types, and illustrative examples.
4. **Quantitative data:** Includes frequency counts and percentages of each identified error type.

Data Collection Procedures

Permission was obtained from the school principal to conduct the study with the grade 11 mathematics students at the school. Data collection took place on school premises, as test papers could not be removed. The researchers photographed student work samples showing algebra errors and ensured strict anonymity. No student names or personal identifiers were recorded. The process involved manually reviewing 35 scripts, identifying algebra errors and misconceptions, photographing representative errors for qualitative analysis, and recording the frequency of each error type.

An intervention strategy was then developed by the researchers after analyzing patterns of errors and misconceptions exhibited by the students' written work. The proposed intervention strategy involves using error analysis worksheets and encouraging learners to confront and discuss mistakes through guided questions. The proposed strategy is learner-centered, enabling students to collaboratively identify and correct misconceptions and errors in groups, compare findings, exchange ideas, engage in argumentation, pose and answer questions, justify their reasoning, and validate answers.

The proposed intervention strategy was implemented in one week by one of the researchers with their own students during normal teaching. The researchers administered a delayed post-intervention assessment two weeks after implementing the suggested strategy to evaluate whether the students benefited from the intervention. The value of a delayed post-test is in measuring knowledge retention and the long-term effectiveness of an intervention (Lee et al., 2024). It demonstrates whether the material learned can be remembered or applied after a certain amount of time has passed. Data were collected over a three-week period during term 1 of the 2024-2025 academic year.

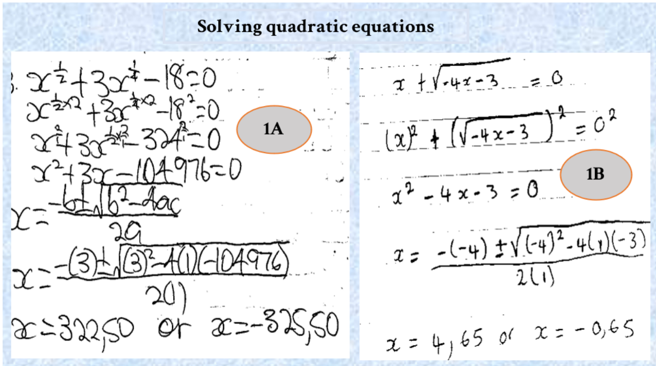

Figure 1. Misapplication of algebra rules (Taken from students' test scripts)

Data Analysis Procedures

The analysis employed a convergent approach, where qualitative coding was used to categorize errors and misconceptions through a grounded interpretation of student work. Categories were validated by cross-checking with algebraic rules and supported by literature (e.g., Maharani & Subanji, 2018; Makonye & Mashaka, 2016). Quantitative analysis involved calculating the frequency and percentage of each error type. This dual method enabled the researcher to understand the nature of conceptual misunderstandings and measure their prevalence.

Ethical Considerations

Ethical clearance was obtained from school authorities before starting the study. Students' identities were protected through anonymization. No personal data were recorded. Only images of relevant parts of student work were used for analysis and kept confidential. The study followed the principle of non-maleficence, ensuring that no participant was harmed or stigmatized during the process. Since this was a non-intervention exploratory study, there was no direct interaction with students during data collection.

RESULTS AND DISCUSSION

Detection and Analysis of Algebra Errors and Misconceptions

Case 1. Misapplication of algebra rules

Part **1A** and part **1B** in **Figure 1** illustrate students' misconceptions about applying algebra rules, which are influenced by previous learning on the topics of exponents and surds, which precede the solution of quadratic equations in the South African grade 11 mathematics curriculum. When students learned about exponents and surds, they solved problems like $x^{\frac{1}{2}} = 5$ and $\sqrt{x-2} = 3$ by squaring both sides, which was the correct technique and resulted in correct answers. The algebra problems $x^{\frac{1}{2}} + 3^{\frac{1}{4}} - 18 = 0$ and $x + \sqrt{-4x-3} = 0$ are comparable to what students did earlier regarding rational exponents and the square root sign. The students' misapplication of the algebra rules in these two problems is thus linked to the influence of earlier learning.

The student in part **1A** in **Figure 1** understands that the given problem is a quadratic equation and that it has to be transformed into the general form $ax^2 + bx + c = 0$. The student tries to get rid of the fractions in the powers by multiplying each power by 2, until the equation matches the general form $ax^2 + bx + c = 0$, then uses the quadratic formula to solve the equation.

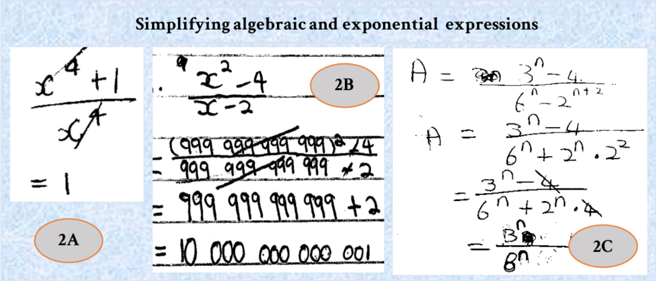

Figure 2. Illegal cancelling misconception and conjoin error (Taken from students' test scripts)

However, multiplying each power by 2 is illegal since, for example, $16^{\frac{1}{2}} + 81^{\frac{1}{4}} - 7 = 0$, whereas $16^{\frac{1}{2}\times 2} + 81^{\frac{1}{4}\times 2} - 7^2 \neq 0$.

Part **1B** in **Figure 1** illustrates another student who recognizes that the problem is a quadratic equation and applies a similar strategy to change it into the general form $ax^2 + bx + c = 0$, by squaring each term in the equation. Again, this is an illegal step because, for example, $-4 + \sqrt{16} = 0$, but $(-4)^2 + (\sqrt{16})^2 \neq 0^2$.

While the quadratic formula is employed correctly in part **1A** and part **1B** in **Figure 1**, the resulting roots do not satisfy the original equations. For example, $(322.5)^{\frac{1}{2}} + 3(322.5)^{\frac{1}{4}} - 18 \neq 0$, and similarly, $-0.65 + \sqrt{-4(-0.65) - 3} \neq 0$.

Another misconception discovered in students' test scripts concerns illegal cancellation. This aspect is presented in detail in the following section.

Case 2. Illegal cancellation and conjoin error

Parts **2A**, **2B**, and **2C** in **Figure 2** demonstrate students' misconceptions about cancellation. Again, this stems from what they learned in earlier grades. In grade 8 and grade 9, students simplified algebraic expressions such as $\frac{3ab\times 4c}{2a}$ by cancelling out identical values in the numerator and denominator, then dividing the remaining values in the numerator by the remaining values in the denominator. The approach was suitable for this kind of algebraic expression because it only has one term in the numerator and one term in the denominator. The students in parts **2A**, **2B**, and **2C** in **Figure 2** are unaware that it is forbidden to cancel out identical terms in circumstances like $\frac{x^4+1}{x^4}$ and $\frac{3^n-4}{6^n+2^n.4}$, when there is more than one term in the numerator and/or denominator. The solution in Exhibit 2A is also influenced by the assumption that simplifying an expression entails reducing it to a numerical value. The student cancels the variable parts (x^4 with x^4) and takes the remaining numerical value or constant as the final answer.

In part **2B** in **Figure 2**, students were required to find the value of $\frac{x^2-4}{x-2}$ without using a calculator for $x = 999\,999\,999\,999$. The student in part **2B** in **Figure 2** used an illegal cancelling procedure and, nevertheless, got the correct answer, coincidentally. The student divided $999\,999\,999\,999^2$ by $999\,999\,999\,999$ to get $999\,999\,999\,999$, then (-) divided by (-) to get (+), and then 4 divided by 2 to get 2. This is a typical example of cases that propagate misconceptions among students, as the student is likely to repeat the same procedure in the future, given that it leads to a correct answer here. If, for example, we replace the 4 by 6, then according to the student's cancellation procedure, the solution would be 999 999 999 999 + 3 = 10 000 000 000 002, which is incorrect.

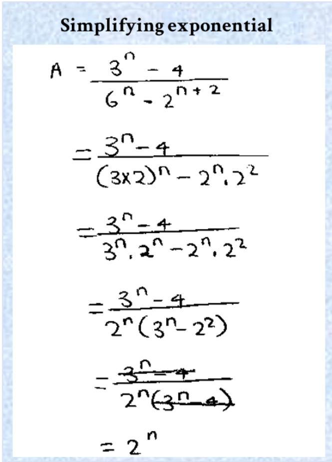

Figure 3. Cancelling error (Taken from students' test scripts)

Table 1. Frequencies of algebra errors and misconceptions

Exhibit	Recorded cases	Percentages (%)
1A	5	18.52
1B	5	18.52
2A	8	29.63
2B	3	11.11
2C	2	7.41
3	4	14.81
Total	**27**	**100**

In part **2C** in **Figure 2**, the student incorrectly cancels out 4 and 4, overlooks the negative sign in the numerator, and then incorrectly adds 6 and 2 to obtain 8. Combining 6^n with 2^n is an example of conjoin error, which occurs when students combine unlike terms because of the (+) sign in between the two terms, inviting students to act.

Figure 3 illustrates an error committed by students in writing their final answer after cancellation. This type of error is also attributed to the influence of students' past experiences. For instance, $\frac{3^a(a-2)}{(a-2)} = 3^a$ and $\frac{y(y+1)}{(y+1)} = y$. These results give students the impression that after cancelling out identical terms, the final answer is obtained by simply writing down the remaining terms. This explains why the students wrote $\frac{(3^n-4)}{2^n(3^n-4)} = 2^n$, which is incorrect. This error is just a 'slip' and can easily be corrected, unlike the misconceptions ('bugs') described in the preceding paragraphs.

It is crucial to note that **Figure 1**, **Figure 2**, and **Figure 3** only depict a few samples of the errors and misconceptions uncovered by the authors. **Table 1** shows the actual frequencies of the various errors and misconceptions that were found in students' written work. Types **1A**, **1B**, **2A**, and **3** had higher frequencies than type **2B** and type **2C**. However, this does not imply that errors and misconceptions with the lowest frequency should receive less attention. Every case counts.

In a nutshell, the analysis of students' algebra errors and misconceptions led to the categorization and explanations in **Table 2**. The information in **Table 2** suggests that part **1A** and part **1B** in **Figure 1** could be clustered together as they both revealed misapplication of algebra rules due to the influence of past learning experiences. Similarly, parts **2A**, **2B**, and **2C** in **Figure 2** can be grouped together since they all reveal illegal cancelling due to the influence of prior learning. **Figure 3** (cancellation error) is left to stand alone, although it is also linked to past experiences.

Table 2. Categorizing students' algebra errors and misconceptions

Exhibit	Categorization	Explanation(s)	Source(s)
1A	Misconception	Misapplication of algebra rules	Prior learning
1B	Misconception	Misapplication of algebra rules	Prior learning
2A	Misconception	Illegal cancelling	Prior learning
2B	Misconception	Illegal cancelling	Prior learning
2C	Misconception	Illegal cancelling	Prior learning
3	Error	Cancellation error	Prior learning

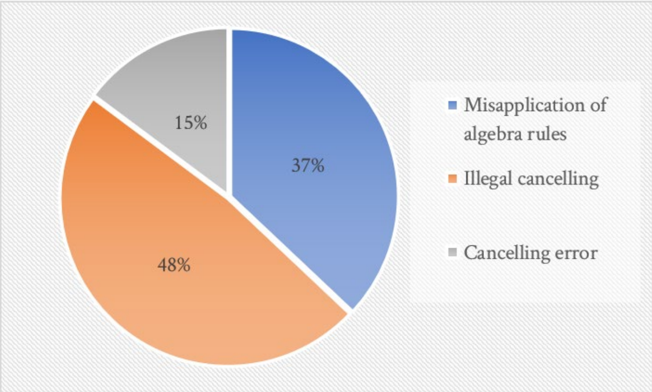

Figure 4. Composition of algebra errors and misconceptions (Authors' own illustration)

Based on the explanations in **Table 2**, the identified algebra errors and misconceptions can be classified into three types: misapplication of algebra rules, illegal cancelling, and cancelling error. **Figure 4** shows a pie chart depicting the percentage compositions of algebra errors and misconceptions depending on these three groupings. The pie chart clearly shows that misapplication of algebra rules and illegal cancelling were the most common algebra misconceptions among grade 11 students at the selected school. Both misconceptions are related to students' previous learning experiences.

A literature analysis found that such misconceptions are challenging to deal with (Maharani & Subanji, 2018; Makonye & Mashaka, 2016). It was also reported that most teachers are aware of the errors and misconceptions made by their students when learning algebra. However, they are unable to use the information to design lessons that would address these challenges (Mulungye et al., 2016). Once-off approaches to intervention, comprising discussion, scaffolding, and dialogue with separated groups of students, could not curb illegal cancelling errors induced by prior learning. Therefore, the critical question is: How can we effectively deal with common algebra errors and misconceptions in the secondary school mathematics classroom?

The following section presents a teaching and learning approach based on the observed algebra errors and misconceptions, which the authors believe will go a long way towards fixing the issue in secondary schools. The suggested approach transforms errors and misconceptions into teaching and learning activities that can be incorporated into regular whole-class teaching and learning. It uses a variety of teaching and learning strategies, including guided discovery, question and answer, group work, peer evaluation, scaffolding, whole class discussion, collaborative learning, argumentation and verification, and self-regulation.

Student X's solution

$$x^{\frac{1}{2}} + 3x^{\frac{1}{4}} - 18 = 0$$
$$x^{\frac{1}{2}\times 2} + 3x^{\frac{1}{4}\times 2} - 18^2 = 0$$
$$x^{1\times 2} + 3x^{\frac{1}{2}\times 2} - 324^2 = 0$$
$$x^2 + 3x - 104976 = 0$$
$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$= \frac{-3 \pm \sqrt{3^2 - 4(1)(-104976)}}{2(1)}$$
$$\approx 322.50 \text{ or } -325.50$$

Student Y's solution

$$x^{\frac{1}{2}} + 3x^{\frac{1}{4}} - 18 = 0$$
$$\left(x^{\frac{1}{4}}\right)^2 + 3\left(x^{\frac{1}{4}}\right) - 18 = 0$$
$$\left(x^{\frac{1}{4}} + 6\right)\left(x^{\frac{1}{4}} - 3\right) = 0$$
$$x^{\frac{1}{4}} + 6 = 0 \text{ or } x^{\frac{1}{4}} - 3 = 0$$
$$x^{\frac{1}{4}} = -6 \text{ (N/A) or } x^{\frac{1}{4}} = 3$$
$$\therefore x = 3^4 = 81$$

Figure 5. Two solutions-1 (Taken from students' test scripts)

Student M's solution

Step 1. $(x)^2 + \left(\sqrt{-4x-3}\right)^2 = 0^2$

Step 2. $x^2 - 4x - 3 = 0$

Step 3. $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Step 4. $x = \frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-3)}}{2(1)}$

Step 5. $x \approx 4.65$ or $x \approx -0.65$

Figure 6. Solution steps (Taken from students' test scripts)

Transforming Algebra Errors and Misconceptions into Teaching and Learning Activities

In this section, the researchers design intervention activities based on the errors and misconceptions identified in the previous section. Activities 1, 2, and 3 are intended to address the misapplication of algebra rules, illegal cancelling, and cancelling errors. Students' solutions are typed to preserve the identities of the students from whose work the misconceptions and errors were made.

Activity 1. Worksheet 1–Solving quadratic equations (day 1)

1. *Student **X** and student **Y** were asked to solve the equation: $x^{\frac{1}{2}} + 3x^{\frac{1}{4}} - 18 = 0$, given that x is real. Their solutions are presented below.*

Study the two solutions in Figure 5 and answer the questions that follow.

   a. *Which solution is correct?*
   b. *How can Student **X** and student **Y** verify their solutions?*
   c. *Explain what is wrong with the incorrect solution?*
   d. *How many marks would you allocate to the incorrect solution out of 5? Justify your answer.*

2. *Student **M** was asked to solve the equation: $x + \sqrt{-4x-3} = 0$, given that $x < 0$. The procedure followed by the student is outlined in the steps below.*

   Study the steps in Figure 6 and answer the following questions:

   a. *Is student M's solution, correct?*
   b. *How can the student verify his/her solution?*
   c. *If your answer to (a) is "no", explain what is wrong with the student's solution.*
   d. *If you were asked to mark the student's work out of 5, how many marks would you allocate the student? Justify your answer.*
   e. *Rewrite the student's solution showing the correct procedures that the student could have taken.*

Student A wrote

$$\frac{x^4+1}{x^4}=1$$

Student B wrote

$$\frac{x^4+1}{x^4}=1+\frac{1}{x^4}.$$

Figure 7. Two solutions-2 (Taken from students' test scripts)

Student P's solution

$$\frac{x^2-4}{x-2}$$
$$=\frac{(999\,999\,999\,999)^2-4}{999\,999\,999\,999-2}$$
$$=999\,999\,999\,999+2$$
$$=10\,000\,000\,000\,001$$

Figure 8. Student **P**'s solution (Taken from students' test scripts)

Student V's solution

$$A=\frac{3^n-4}{6^n-2^{n+2}}$$
$$=\frac{3^n-4}{(3\times 2)^n-2^n.2^2}$$
$$=\frac{3^n-4}{3^n.2^n-2^n.2^2}$$
$$=\frac{3^n-4}{2^n(3^n-4)}$$
$$=2^n$$

Student W's solution

$$A=\frac{3^n-4}{6^n-2^{n+2}}$$
$$=\frac{3^n-4}{6^n+2^n.2^2}$$
$$=\frac{3^n-4}{6^n+2^n.4}$$
$$=\frac{3^n}{8^n}$$

Figure 9. Two solutions-3 (Taken from students' test scripts)

Activity 2. Worksheet 2–Simplifying algebraic expressions (day 2)

1. *During a grade 11 mathematics lesson, the teacher asked students to simplify the expression* $\frac{x^4+1}{x^4}$*,* $x \neq 0$*. Student **A** and student **B** wrote two different solutions, which are shown below.*

Study the two solutions in Figure 7 and answer the questions that follow.

   a. *Which solution is correct?*
   b. *How can the two solutions be verified?*
   c. *Write down all the steps that you think should be followed to arrive at the correct answer.*

2. *Grade 11 students at school K were asked to simplify the expression* $\frac{x^2-4}{x-2}$*,* $x \neq 2$*, if* $x = 999\,999\,999\,999$*. Student **P** presented the solution in Figure 8 but was not allocated full marks:*

   a. *What do you think is the reason why the student was not allocated full marks for the presented solution?*
   b. *Rewrite the student's solution, showing the correct procedures that the student could have followed to earn full marks.*

Activity 3. Worksheet 3–Simplifying exponential expressions (day 3)

*Student **V** and student **W** were asked to simplify the expression:* $A = \frac{3^n-4}{6^n-2^{n+2}}$*, for* $n \in \mathbb{N}$*. Their solutions are presented below.*

Study the two solutions in Figure 9 and answer the questions that follow

   a. *Are the students' solutions correct?*
   b. *How can the two students verify their answers?*
   c. *Which student would earn more marks than the other?*
   d. *Explain what is wrong with each solution?*
   e. *If you were asked to mark the students' work, how many marks would you allocate to each solution out of 4? Justify your answer.*
   f. *Write down the correct steps that the students could have followed to simplify the expression.*

The design of the proposed intervention activities was directly informed by an analysis of students' written work, which revealed common patterns of conceptual misunderstanding. For example, the first activity on solving quadratic equations with fractional exponents was specifically created to address the misapplication of exponent rules–a misconception stemming from prior exposure to surd manipulation techniques. Similarly, activity 2 and activity 3 were developed in response to frequent errors involving illegal cancellation and conjunction errors, where students improperly simplified algebraic expressions due to a misunderstanding of the structure of rational expressions with multiple terms.

Each worksheet was carefully designed to reflect the specific errors made by students, promoting peer evaluation and guided correction in small groups. This approach allows students to address familiar yet flawed reasoning processes in a collaborative setting, fostering cognitive conflict and promoting conceptual change. By grounding teaching methods in real classroom evidence, the intervention ensures that instructional efforts are tailored to the actual challenges learners face, rather than relying on generic remedial strategies.

The teaching strategies suggested, such as group analysis, argumentation, and structured reflection, were chosen because they align with constructivist learning principles and have been proven effective in enhancing conceptual understanding (Barbieri & Booth, 2020; Chinn, 2020; Costa & Kallick, 2000). The connection between diagnosis and pedagogy highlights the innovative nature of the proposed approach: it is not merely a theoretical framework, but a practical teaching model developed in direct response to the specific algebraic challenges faced by students in real educational settings.

The role of the mathematics teacher in implementing the suggested strategies is to prepare the worksheets, assign students to groups, create an environment where students feel free to express their ideas without fear of being ridiculed, facilitate the proceedings, and provide probing questions as needed. The strategy is based on asking purposeful questions that encourage the students to think deeply. The questions are carefully prepared in accordance with Bloom's taxonomy. Unlike past interventions, the approach described in the current study relieves the mathematics teacher from being the sole bearer of the responsibility of addressing errors and misconceptions. Instead, it makes every individual in the mathematics classroom a teacher and a learner. Based on the suggested activities, mathematics no longer consists solely of problem-solving. Instead, it seeks to develop habits of the mind such as independent thinking, communication, strategic reasoning, metacognition, persistence, flexibility, empathy, data collection, imagination, applying past knowledge to new situations, accuracy, creativity, and openness to continuous learning (Costa & Kallick, 2000). The suggested remedy is also consistent with constructivist learning theories, which emphasize the significance of allowing students to develop their own knowledge with minimal teacher interference.

Post-Implementation Review of Students' Progress

The questions provided to the students, along with the targeted error types and misconceptions, are recorded in **Table 3**. The results show a reduction in algebra errors and misconceptions compared to **Table 1**. Among the 35 scripts analyzed, there were no instances of misapplication of algebra rules, only one case of an illegal cancellation misconception, and one case of a cancellation error. The cases noted under item **2C** and item **3** involved a student who was absent from school when those cases were discussed in class. These findings suggest

Table 3. Frequency of errors and misconceptions after the intervention

Code	Review Question	Targeted error/misconception	Recorded cases
1A	Solve the equation: $x^{\frac{1}{3}} + 2x^{\frac{1}{6}} - 15 = 0$	Misapplication of algebra rules	0
1B	Solve the equation: $x + \sqrt{-3x - 4} = 0$	Misapplication of algebra rules	0
2A	Simplify: $\frac{x^5+1}{x^5}$	Illegal cancelling	0
2B	Evaluate the expression: $\frac{x^2-9}{x-3}$ if $x = 999\,999\,999\,998$	Illegal cancelling	0
2C	Simplify: $\frac{5^n-2}{2^n.5^n-2.2^n}$	Illegal cancelling	1
3	Simplify: $\frac{5^n-2}{2^n(5^n-2)}$	Cancellation error	1

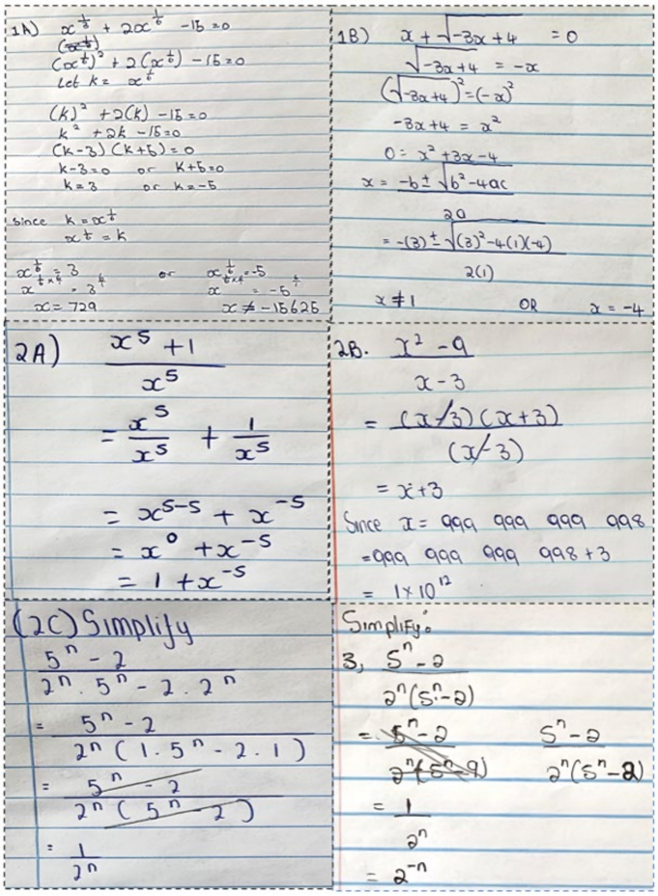

Figure 10. Samples of students' solutions after the intervention (Taken from students' test scripts)

a potentially positive impact of the proposed strategy on students' learning. However, the researchers do not intend to draw any causal inferences, as that is not the purpose of the study.

Figure 10 shows examples of students' written work after applying the proposed strategy. These samples come from some of the cases listed in **Table 1**. They demonstrate that errors and misconceptions can be effectively corrected. However, the final answer for item **2B**, as seen in **Figure 10**, indicates a new error that arises from the student's reliance on calculators for computation: $999\,999\,999\,998 + 3 \neq 1 \times 10^{12}$. This result supports the idea that error analysis and remediation should be an ongoing, cyclical process.

DISCUSSION

The findings of this study confirm that algebraic errors and misconceptions remain a persistent challenge among secondary school learners. Consistent with previous studies by Makonye and Mashaka (2016), Mulungye et al. (2016), and Ndemo and Ndemo (2018), this study found that students often misapply algebraic rules, perform illegal cancellations, and make cancellation errors mainly due to the influence of prior learning. These results reinforce earlier conclusions that misconceptions often originate when students overgeneralize rules learned in earlier mathematical contexts (Maharani & Subanji, 2018).

While identifying students' errors is an important first step toward improving algebra learning outcomes, the findings indicate that detection alone is not enough. Many teachers, as seen in earlier studies (Mulungye et al., 2016; Zuya, 2014), are aware of students' misconceptions but lack the pedagogical tools to address them effectively during lessons. Simply reteaching or marking incorrect answers without guided reflection does not lead to conceptual change. Similar frustrations have been reported among teachers who find that repeated correction and reteaching yield little progress (Krall, 2018; Samuel & Warner, 2021). Therefore, there is an urgent need to implement systematic, classroom-based intervention strategies that turn errors into intentional learning opportunities.

The current study advances this field by introducing a collaborative error-analysis method that allows students to identify, discuss, and correct misconceptions using structured worksheets and peer dialogue. This method aligns with the constructivist and sociocultural learning theories that form the foundation of the study (Bruner, 1966; Piaget, 1970; Vygotsky, 1978). When students confront their misconceptions in small groups, they engage in cognitive conflict, a process that encourages deeper understanding and long-term retention (Barbieri & Booth, 2020; Tullis & Goldstone, 2020). Additionally, when students collaboratively analyze and justify mathematical reasoning, they develop metacognitive skills that improve self-regulation and error monitoring (Flavell, 1979; Silver et al., 2023).

The study also supports findings by Siller and Ahmad (2024), who showed that collaborative learning encourages positive attitudes toward mathematics. Similarly, Shin et al. (2017) found that peer-led discussions can motivate learners more effectively than instructor-led explanations. In the current study, the proposed whole-class engagement model reflects these principles by promoting active participation, mutual support, and collective problem-solving. This inclusive approach helps eliminate the stigma linked to remedial instruction, which can occur when struggling students are isolated for special interventions (Krzyzaniak et al., 2021).

From a theoretical perspective, the findings support the core idea of constructivist, metacognitive, and growth mindset frameworks. Students who reflect on their mistakes build resilience, persistence, and a stronger understanding of algebraic structures (Chinn, 2020; Dweck, 2006). The results demonstrate Vygotsky's (1978) concept of the zone of proximal development, as students improve their understanding through peer scaffolding and teacher guidance. Viewing mistakes as opportunities for learning rather than failures encourages curiosity and helps develop a positive mathematical identity (Kazemi & Hintz, 2014).

Pedagogically, the proposed intervention has several implications. First, it shifts algebra instruction from a teacher-centered activity to a student-driven, reflective process. Teachers serve as facilitators who ask purposeful questions, guide reasoning, and support discovery instead of simply giving corrections. This method not only reduces teacher fatigue but also encourages inclusivity by making sure all students benefit from collective reflection and feedback. Second, incorporating error-analysis worksheets into daily lessons helps connect diagnosis with remediation. Instead of viewing misconceptions as isolated issues, teachers can address them systematically, reinforcing correct reasoning across multiple lessons.

The findings also emphasize the importance of teacher education and curriculum development. Pre-service and in-service training should include diagnostic assessment and error-analysis skills to help teachers interpret learners' reasoning from their written work. Incorporating these strategies into mathematics teacher preparation programs will enable educators to anticipate and address common misconceptions proactively. Additionally, aligning the national curriculum to encourage reflective learning and collaborative reasoning can foster sustained conceptual understanding instead of just rote mastery of procedures.

Although the study offers valuable insights, it has limitations that must be recognized. The sample consisted of only 35 grade 11 students from a single school in Seshego Township, which limits the ability to generalize findings. Additionally, only written test scripts were analyzed; classroom observations or interviews could have provided deeper insights into students' cognitive processes. Future research should use mixed-methods or quasi-experimental designs to assess the long-term effectiveness of collaborative error-analysis interventions. Longitudinal studies could examine whether students sustain improved conceptual understanding over time.

In summary, this study offers empirical and theoretical support for incorporating structured error analysis into daily algebra teaching. When teachers see errors as opportunities for growth, they not only improve students' understanding of algebra but also foster reflective, self-driven learners who approach mathematics with confidence and curiosity.

CONCLUSION

This study aimed to examine common algebra mistakes and misconceptions among grade 11 students in South Africa and to develop a practical, classroom-based strategy for turning these misconceptions into valuable learning opportunities. Using qualitative error analysis and descriptive statistics, the study identified three main categories of misconceptions: misapplication of algebra rules, illegal cancellation, and cancellation errors; most of which originated from prior learning experiences. By designing and demonstrating structured, collaborative error-analysis worksheets, the study directly addresses the research questions concerning the types, sources, and potential solutions for algebra misconceptions.

The novelty of this study lies in its integration of diagnosis and pedagogy, moving beyond simply identifying misconceptions to proposing a systematic, replicable, and inclusive classroom intervention. While previous research has analyzed algebraic errors descriptively, few studies have turned those findings into practical, whole-class teaching models. This research adds new knowledge by showing how collaborative error analysis, through peer dialogue, reflective questioning, and verification, can be incorporated into daily mathematics instruction to foster conceptual understanding, metacognition, and a growth mindset.

Academically, the study enhances the discussion on mathematics education by connecting constructivist, sociocultural, and metacognitive theories within a unified framework for addressing misconceptions. The findings show that students' errors reflect cognitive transitions rather than failures, supporting Vygotsky's (1978) concept of learning within the zone of proximal development and Piaget's (1970) idea of conceptual restructuring through cognitive conflict. By demonstrating how reflection and peer collaboration promote conceptual change, the study provides a transferable pedagogical model that can be applied to other mathematical areas such as geometry, functions, or calculus, where symbolic manipulation and abstract reasoning similarly challenge learners.

Practically, the findings support incorporating error-analysis practices into teacher training, textbook design, and curriculum policy. Incorporating diagnostic and reflective components into regular lessons can help teachers identify misconceptions early and involve students as active participants in correcting them. Future research should use quasi-experimental or longitudinal methods to assess the long-term effects of this collaborative approach on student achievement and attitudes across various mathematical topics and educational settings.

In summary, this study makes both theoretical and practical contributions: it redefines algebraic misconceptions as teachable opportunities rather than barriers, offers a specific, classroom-tested intervention approach, and broadens the understanding of how social and reflective learning processes can improve mathematical understanding.

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