An analysis of semantic structures of addition and subtraction word problems used in primary two mathematics textbooks

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Citation: Wee, T. M., & Yeo, K. K. J. (2024). An analysis of semantic structures of addition and subtraction word problems used in primary two mathematics textbooks. Contemporary Mathematics and Science Education, 5(2), ep24011. https://doi.org/10.30935/conmaths/14690

ABSTRACT

In this study, addition and subtraction word problems based on the semantic structures are analyzed in the sole primary two mathematics textbooks and accompanying workbooks used in Singapore. Based on a conceptual coding framework, the word problems were coded accordingly. The results revealed a significant representation of combine and compare structures across all contents in both the textbooks and accompanying workbooks. In particular, the lack of word problems involving change structure suggests an unequal distribution of the semantic structures. Based on the findings, it is recommended that educators and textbook authors to be aware in providing students the opportunity to be equally exposed to the various semantic structures in the teaching and learning of both the addition and subtraction word problems.

Keywords: textbook analysis, mathematics textbooks, semantic structures, additive word problems, primary level, elementary level

Received: 20 Oct. 2023 • Accepted: 13 May 2024

INTRODUCTION

Background of Study

In Singapore, a good amount of effort and time have been invested by the Ministry of Education (MOE) in the development of curriculum materials including mathematics textbooks used in primary schools. Beginning in 2021, primary school mathematics textbooks are developed by curriculum specialists and curriculum officers who have deep knowledge of curriculum and mathematics content. In 2019, Singapore ranked top of the list in the trends in international mathematics and science study (TIMSS). TIMSS was conducted by the International Association for the Evaluation of Educational Achievement on primary four and secondary two students (MOE, 2020b). This indicates that the high competency of Singapore students could be attributed to the national mathematics curriculum and the use of primary school mathematics textbooks in Singapore, which are an important learning resources for students (Sievert et al., 2021). Such achievement in TIMSS has gained the attention and interests of educational researchers, mathematics educators and policy makers around the world in comparing and adopting the use of Singapore’s mathematics textbooks and teaching practices (Lindorff et al., 2019; Oates, 2014; Vicente et al., 2020).

Rationale of Study

Textbooks are important resources for mathematics instruction and the teaching and learning of the intended national curriculum in most primary schools around the world (Mullis et al., 2020; Oates, 2014; Pepin, 2018). How content is developed and presented within the textbooks will impact students’ learning. Even so, compared to many other research areas in mathematics education, research on textbooks has not presented enough interest from researchers. As Howson (1995) documented, in Grouws’ (1992) handbook of research on mathematics teaching and learning, there was no entry in the subject index under “textbooks”. Usiskin (1999) also stated that researchers stayed away from the studies of textbooks.

Furthermore, numerous educators and researchers have indicated a lack of studies in textbooks. This was supported by Sosniak and Stodolsky (1993) who asserted that systematic consideration to textbooks' research was long overdue. In the same vein, Howson (1995) believed that textbooks were one step nearer classroom reality than a national curriculum. Our view is that textbooks are the component of the intended curriculum that has the closest relationship with the implemented curriculum.

Therefore, textbooks matter as they act as a proxy to the intended curriculum. They show features and approaches in which the intended curriculum can be implemented as opportunities to learn (Schmidt et al., 2001) and has been reinforced that they are a “a major source of provision of these opportunities” (Pepin, 2008, p. 2). Pepin (2008) also clearly clarifies that “textbooks … are used extensively … and they influence to a large extent how students think about mathematics and come to understand its meaning” (p. 1).
The central focus of the Singapore mathematics curriculum is mathematical problem solving (MOE, 2020a). In Singapore, the primary purpose of teaching mathematics is to enable students to solve problems. This aim is dependent on five factors: specifically, skills, concepts, processes, metacognition, and attitudes (MOE, 2020a). An essential component in mathematical problem solving are word problems, which carries significant weightage in tests and high-stakes examinations, especially towards the upper primary levels and reported to be at least 60.0% in primary six (Lee et al., 2014). As teaching and solving word problems is an important feature in the Singapore primary textbooks and accompanying workbooks, little research is available on the semantic structures of the word problems that are shown in the current primary two mathematics textbooks. The aim of this study seeks to examine how the semantic structures of word problems are being represented in the sole primary two mathematics textbooks used in Singapore.

**LITERATURE REVIEW**

**Comprehending Word Problems**

Word-problem solving, which characterizes the interaction between mathematical concepts and reality, is an essential component of mathematical school tasks beginning in early grades. Word problems can develop students’ understanding of the meaning of operations involved in the problem, and consequently, their proficiency with whole number arithmetic (Verschaffel et al., 2007). Word problems is an essential part of mathematical problem solving (Marshall, 1995). They can be defined as textual or verbal narrative of a problem that is introduced, where the application of mathematical operators is used to solve problems (Verschaffel et al., 2020). Word problems can range from simple one-step problems to complex problems, which involve multiple steps. It requires the solver to make sense of the story problem, identify the relationships and apply the correct operator. However, the difficulty level of the word problem is often due to the different ways of how the problem is being framed or structured (Van de Walle et al., 2018). Thompson and Hendrickson (1986) further added that the different types of semantic structures are not at the same level in difficulty for children. They suggest that problems involving the change semantic structure are most difficult as children are not able to mentally create a model or physically enact the context using manipulatives when the initial amount or change amounts are unknown to them. Whereas for combining and comparing semantic structures, it is easier for children to identify and create a model. There have been discussions among researchers that some teachers have employed ineffective teaching methods such as identifying questions using key words, grouping problems by addition and subtraction and through generic strategies like guess and check (Ma et al., 2021; Marshall, 1995; Peltier et al., 2022; Powell & Fush, 2018). They proposed that the teaching of semantic structures is more effective in helping children to solve word problems.

**Semantic Structure of Word Problems**

Understanding the semantic structure of word problems is critical for educators, researchers, and curriculum developers, as it can shed light on the cognitive processes involved in problem solving. Educators and researchers have placed the importance of arithmetic word problems because of their cognitive complexity (Múñez et al., 2013).

<table>
<thead>
<tr>
<th>Semantic Structure</th>
<th>Example of problem</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Structure in pictorial form</strong></td>
<td><strong>Combining problem</strong></td>
</tr>
<tr>
<td>Whole (7)</td>
<td>“Abel has 6 pencils. Bella has 14 pencils. How many pencils do they have in all?” (MOE, 2022a, p.65)</td>
</tr>
<tr>
<td>Part (6)</td>
<td>“Mr. Tan had 381 red apples and 212 green apples. He sold 153 apples. How many apples did he have left?” (MOE, 2022b, p.9)</td>
</tr>
<tr>
<td>Part (14)</td>
<td>“Mr. Lee sold 124 books on Tuesday. He sold 135 books on Wednesday. How many more books did he sell on Wednesday than on Tuesday?” (MOE, 2022a, p.67)</td>
</tr>
</tbody>
</table>

Research on word problem solving revealed that besides the conventional mathematical structure of a word problem (e.g., whether it is addition or subtraction), the semantic classification of the problem strongly affects student’s solution representations (De Corte & Verschaffel, 1987). The structure of word problems has been studied and categorized in many ways. Terms such as semantic structure, problem types, problem structures and schema have at times been used interchangeably in literature (Marshall, 1995; Powell & Fush, 2018; Van de Walle et al., 2018; Vicente et al., 2007). One-step additive word problems can be considered following the well-established classification suggested by Carpenter and Moser (1984) and Heller and Greeno (1978) as change, compare, combine, and equalize problems. Understanding these structures can guide the development of instructional materials tailored to specific problem types. Vicente et al. (2007) provides clear distinctions of the differences when referring to the semantic structure of word problems, which will be used for this study. Semantic structures can be identified through three situations in addition and subtraction word problems. These situations include the combination of two sets of quantity to a whole quantity (combine problems), the change, which results in the increase or decrease of the initial quantity (change problems) and the comparison of two quantities with a quantitative difference between the two quantities (compare problems) (Verschaffel et al., 2020; Vicente et al., 2007). Problem types that increase the level of difficulty in word problems involve specific features such as an unknown quantity (start unknown, result unknown, etc.), which are further distinguished from the three semantic structures. Figure 1 shows the representation of the semantic structure and problems.

Van de Walle et al. (2018) emphasized that combining the semantic structure and models (bar model, drawings, manipulatives) is beneficial for students in the mastery of addition and subtraction. Therefore, it is paramount for teachers to have the knowledge of the semantic structures to help students in areas, where they are weak in so as to provide such experiences. In addition, it also provides a learning experience for students to have the knowledge of these semantic structures as they solve diverse real-world problems.
Semantic Structure of Word Problems Research

Studies on the semantic structure of word problems have been conducted at various levels in understanding children’s difficulties in solving word problems. In a study conducted by Cheng (2015) on the error analysis of primary three students in Singapore, it was noted that the knowledge of the semantic structure a student has, is paramount in determining the success of their ability to solve word problems. In her analysis, it was found that students generally performed better in one-step word problems as compared to two-step word problems and concluded that proper scaffolding is required for such transitions. However, the study showed that even though the combine and change problems were thought to be the easiest, the change problems posed greater challenge as compared to compare problems.

Another study conducted by Lim (2020) on primary three students in Singapore indicated that the explicit teaching of the various semantic structures of word problems, combine, change and compare through schema-based instruction and model method approach as an intervention was found to be effective.

Furthermore, the results showed that the intervention was most effective in solving compare word problems, which was identified to be most challenging for students.

In a more recent study, Vicente et al. (2022) analyzed the frequency and various aspects of word problems between Singaporean and Spanish primary school textbooks. One of the areas included the variety of semantic structures presented in the textbooks. The results showed that the distribution of questions based on the semantic structures was unbalanced in both the Singapore and Spanish textbooks.

It was reported that 61.8% of the questions within the Singapore textbooks consisted of combine and change problems that are of low to moderate difficulty. They suggested identifying how the presentation of word problems within the textbooks and understanding the various ways of how teachers use the textbooks, can promote the teaching and learning of problem-solving skills.

**METHODOLOGY**

In this study, a conceptual coding framework was designed for the textbook analysis of the primary 2A and primary 2B textbooks and accompanying workbooks as well as to seek to answer the following two research questions:

**RQ1.** What are the different semantic structures of addition and subtraction represented in the primary two mathematics textbook and accompanying workbooks?

**RQ2.** How are the different semantic structures of addition and subtraction represented in the primary two mathematics textbook and accompanying workbooks?

**Textbooks**

For this study, the primary two levels were chosen to be the focus for the analysis. The reasons for this are that a new syllabus has been implemented for this level, and all Singapore primary schools are using this mathematics textbooks developed by Singapore’s MOE curriculum specialists and curriculum officers. Another reason is that research on the current primary two textbooks is almost non-existent. Therefore, if analysis is done in this series of textbooks, it can help to shed some light on the learning experiences offered to primary two students and provide data for curriculum developers and textbook writers.

**Conceptual Coding Framework**

A conceptual coding framework was developed for the purpose of this study. It will define the problems for the context of textbook analysis and includes coding based on the semantic structures and sections represented in the textbooks and accompanying workbooks.

**Definition of Word Problem**

The operational definition of word problem in this study involves only addition and subtraction word problems, where the solver needs to identify the relationships and apply the correct operations.

**Coding Procedure**

The various word problems identified in the textbooks and accompanying workbooks provides an understanding of what Singapore MOE’s intended curriculum primary two students should experience. Each problem was analyzed and classified under one of the three semantic structures: combine, change and compare (see Table 1).

For this study, a total of 172 problems were analyzed. 71 problems from the textbooks were shown in the following sections: ‘learning tasks’, ‘recall’, ‘examples’, ‘let’s try’, ‘thinking aloud’, ‘mathematics around us’, and ‘what have I learnt?’. Out of the 71 problems, 33 problems appear in the 2A textbook, and 38 problems appear in the 2B textbook. A total of 101 problems from the workbooks were shown in the following sections: ‘recall’, ‘practice’, and ‘review’. Out of the 101 problems, 36 problems appear in the 2A workbook, and 65 problems appear in the 2B workbook.

**RESULTS & DISCUSSION**

**Distribution of Word Problems in Textbooks and Workbooks**

Out of 125 problems, 65.0% of the addition and subtraction word problems are from the accompanying workbooks and 35.0% are from the textbooks (Table 2). The high percentage of problems in the workbooks suggests that students require more practice in concept development and skill mastery. Such emphasis is also reflective in the learning cycle–readiness, engagement, and mastery (MOE, 2020a). A higher percentage of word problems appear in the 2B textbook and

<table>
<thead>
<tr>
<th>Table 1. Explanation of each semantic structure</th>
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<tbody>
<tr>
<td>Semantic structure</td>
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<tr>
<td>Combine problem</td>
</tr>
<tr>
<td>Change problem</td>
</tr>
<tr>
<td>Compare problem</td>
</tr>
</tbody>
</table>

<table>
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<tr>
<th>Table 2. Distribution of problems in textbooks &amp; workbooks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (%)</td>
</tr>
<tr>
<td>Number of problems in textbooks</td>
</tr>
<tr>
<td>Number of problems in workbooks</td>
</tr>
<tr>
<td>Total number of problems</td>
</tr>
</tbody>
</table>
Table 3. Distribution of semantic structures of addition & subtraction word problems in textbooks & workbooks

<table>
<thead>
<tr>
<th></th>
<th>Textbooks</th>
<th>Workbooks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Addition</td>
<td>Subtraction</td>
</tr>
<tr>
<td>Combine problems</td>
<td>16 (72.7%)</td>
<td>1 (4.6%)</td>
</tr>
<tr>
<td>Change problems</td>
<td>1 (4.6%)</td>
<td>8 (36.3%)</td>
</tr>
<tr>
<td>Compare problems</td>
<td>5 (22.7%)</td>
<td>13 (59.1%)</td>
</tr>
</tbody>
</table>

Table 4. Distribution of semantic structures in textbooks appearing in different content

<table>
<thead>
<tr>
<th>Content</th>
<th>Semantic structure of addition &amp; subtraction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Combine (%)</td>
</tr>
<tr>
<td>Chapter 2: Addition &amp; subtraction within 1,000</td>
<td>44.4</td>
</tr>
<tr>
<td>Chapter 3: Length</td>
<td>40.0</td>
</tr>
<tr>
<td>Chapter 6: Mass</td>
<td>50.0</td>
</tr>
<tr>
<td>Chapter 8: Addition &amp; subtraction</td>
<td>31.3</td>
</tr>
<tr>
<td>Chapter 10: Money</td>
<td>50.0</td>
</tr>
<tr>
<td>Chapter 12: Volume</td>
<td>25.0</td>
</tr>
</tbody>
</table>

Table 5. Distribution of semantic structures in workbooks appearing in different content

<table>
<thead>
<tr>
<th>Content</th>
<th>Semantic structure of addition &amp; subtraction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Combine (%)</td>
</tr>
<tr>
<td>Chapter 2: Addition &amp; subtraction within 1,000</td>
<td>37.5</td>
</tr>
<tr>
<td>Chapter 3: Length</td>
<td>25.0</td>
</tr>
<tr>
<td>Chapter 6: Mass</td>
<td>50.0</td>
</tr>
<tr>
<td>Chapter 8: Addition &amp; subtraction</td>
<td>45.5</td>
</tr>
<tr>
<td>Chapter 10: Money</td>
<td>50.0</td>
</tr>
<tr>
<td>Chapter 12: Volume</td>
<td>33.3</td>
</tr>
<tr>
<td>Review one to six</td>
<td>40.0</td>
</tr>
</tbody>
</table>

accompanying workbook as there is a sole chapter focused only on addition and subtraction word problems.

Addition Verse Subtraction of Semantic Structures in Textbooks

Table 3 shows the distribution of semantic structures of addition and subtraction word problems in both the textbooks and workbooks.

Within the word problems found in the textbooks, the data shows that only one word problem is represented in combine semantic structure involving subtraction and one word problem is represented in change semantic structure involving addition. There is an equal distribution of addition and subtraction word problems found within the textbooks. The semantic structure that appears most in addition word problems is the combine structure, while the semantic structure that appears most in subtraction word problems is the compare structure.

This frequency is consistent in both the textbooks and workbooks as seen in Table 3. The findings also revealed that none of the change problems involving addition semantic structure is being represented in the workbooks and only one such problem is represented in the textbooks.

Representation of Semantic Structure in Different Content (Textbooks)

Table 4 shows the distribution of semantic structures in the textbooks appearing in different content.

The data in Table 4 shows that the compare structure appears most frequently in the contents of chapters 3, 8, and 12. None of the change structure appear in the content for chapter 3 and chapter 6, which involve the content of length and mass, respectively.

Out of all the content chapters, change problems appeared most in chapter 10: chapter 10 on money has more applications of the context of the increase or decrease in an initial quantity.

Representation of Semantic Structure in Different Content (Workbooks)

Table 5 shows the distribution of semantic structures in the workbooks appearing in different content.

The findings show that the highest frequency of addition and subtraction word problems appearing in workbooks were of the compare semantic structure, which are found in the contents of chapters 2, 3, 12, and within the reviews. None of the change structure appears in the content for chapter 6, which involves the content of mass.

The findings are consistent with the representation of semantic structure found in the textbooks, where a higher frequency of compare semantic structure is being represented. The high frequency on the compare structure suggests that more focus is given to expose students in experiencing and practicing various contexts of the word problems that were of the compare structure.

IMPLICATIONS FOR TEACHING & LEARNING

The results of the study have revealed some interesting findings. Among the addition and subtraction word problems, there is a stronger emphasis on the combine and compare semantic structures as seen in Table 3. It is also evident that none of the change problems involving addition appears in the workbooks. It appears that textbook authors could pay more attention to a balanced distribution of different semantic structures of word problems. It is important for teachers to be aware of the lack of word problems involving the change semantic...
structure, and to provide students with related contexts of word problems related to change. Secondly, only the change structure was represented in the “thinking aloud” section of the textbooks.

This suggests that the change structure has a higher level of difficulty for children to grasp as the section seeks to engage students in cognitive reasoning, creative and critical thinking, and to verbalize what it means to them (MOE, 2022a, 2022b, 2022c, 2022d). As supported by Thompson and Hendrickson (1986), they suggest that the change semantic structure is the most difficult as children are not able to mentally create a model or physically enact the context using manipulatives when the initial amount or change amounts are unknown to them. The dynamic increase or decrease and unknowns of the change semantic structure would seem to be more abstract to young children as compared to the combine or compare problems, which are more static in quantity. However, this should not prevent teachers from exposing students to the change structure and should in fact allow students to experience and compare the different semantic structures.

Besides being the least represented structure, none of the change problems appear in the content for length and mass as seen in Table 4. This could be due to the fact that students are newly introduced to learning the concepts and the focus is not on solving word problems that are related to length and mass. Teachers should however be mindful of such absence of change problems and provide students with the opportunities to be exposed to such problems by providing examples as a learning activity related to the change semantic structure at the end of the chapter. Textbook writers need to search for ways how to increase the different semantic structures of word problems. One possible way is to set problems in various realistic situations.

CONCLUSIONS & RECOMMENDATIONS

Textbooks are important resources for instruction in the teaching and learning of mathematics, it is therefore important for authors of mathematics resources to be aware of such gaps and could pay more attention to provide a balanced distribution of the semantic structures of word problems across different contents. This would provide students the opportunity to be equally exposed to the various semantic structures in both addition and subtraction word problems.

While the primary two textbooks used in schools do not classify word problems according to the semantic structure, teachers should provide students the opportunity to describe the various semantic structures by articulating it in their own words. Primary two students with difficulties in word problems are likely to gain from this approach as it allows them to understand that word problem and thus raises the likelihood of better retention. It also allows a process of cognitive reasoning and sense making when solving different semantic structures of addition and subtraction word problems. Furthermore, a side-by-side comparison of the various semantic structures should also be presented to students as part of the learning experience when teaching word problems. Students could be encouraged to express in their own words the differences of the semantic structures as a form of journal writing. This allows teachers to visually see the thought process of each student and to correct any misconceptions.

This study analyzed the various semantic structures of addition and subtraction word problems found in the sole textbooks and accompanying workbooks used in Singapore schools at the primary two level. Building upon the findings from this study, future research could further examine the reasons why certain semantic structures appear less in textbooks and accompanying workbooks. Another possible future research is to observe how teachers re-enact the lesson in teaching the three semantic structures could be studied. Finally, it is hoped that the research has catalyzed the call for more academic inquiry including action research on the area of textbooks analysis and its relationship with the intended curriculum and particularly, teaching practices in the classroom.

Author contributions: TMW: undergraduate educational studies research & YKK J: editing & proofreading. Both authors approved the final version of the article.

Funding: The authors received no financial support for the research and/or authorship of this article.

Ethics declaration: The authors declared that the study does not need ethics clearance as it does not involve any live subjects.

Declaration of interest: The authors declare no competing interest.

Data availability: Data generated or analyzed during this study are available from the authors on request.

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