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An investigation into senior high school students' performance and difficulties in proving Euclidean geometry in mathematics curriculum: Evidence from Ghana

Isaac Davor [1]*, Patrick Dawmah [1], Ernest Larbi [1]

[1] Akenten Appiah Menka University of Skills Training and Entrepreneurial Development, Kumasi, GHANA
***Corresponding Author:** davorisaac70@gmail.com

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ABSTRACT

This study aims to investigate the performance and difficulties of senior high school (SHS) students in proving Euclidean geometry in the Ghanaian mathematics curriculum. The target population was 240. 62 students participated in the study, which was determined using a purposive sampling technique, and data were collected using performance tasks and analysis of students' strategies for geometric proofs. The finding indicated that learners struggled in producing correct proofs, particularly in the application of logical reasoning and connecting geometric concepts. In addition, the results indicated that females' and males' performance had no statistically significant difference, implying that both genders can perform equally well in Euclidean geometry if they have equal learning opportunities. The study underscores the need for a particular pedagogy to address the students' challenges in proof construction and improve the teaching of geometry at the SHS level. Moreover, it highlights the urgent need to enhance students' abilities in deductive reasoning and the development of mathematical proofs, particularly in geometry, based on new evidence.

Keywords: Euclidean geometry, proof difficulties, students' performance, gender comparison, Ghana

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INTRODUCTION

Geometry has historically been regarded as the study of measuring the earth and shapes. Proof and proving are fundamental components of advanced mathematics education, and the main objective of geometry courses is to help students develop their proof skills (Stylianides et al., 2024). Although proof is at the core of doing and knowing mathematics and geometry, research indicates that students preparing to study mathematics often struggle with understanding and writing proofs. Additionally, it has been noted that students' proofreading skills are generally not adequate (Armah & Kissi, 2019). Research on learners confirms that difficulties with proof appear at multiple stages of the education pipeline. A classic study was conducted by Duval (2006), who describes particular obstacles such as limited conceptual grasp of underlying definitions, problems with mathematical language and notation, and difficulties in getting started on a proof. Subsequent studies have repeated these findings, reporting that many students interpret proof differently from mathematicians, rely on empirical examples instead of general arguments, or lack robust strategies for proof construction and evaluation (Almeida, 2000; Stylianides et al., 2007). This body of literature indicates that weaknesses related to proof are enduring and widespread, rather than just temporary.

In the Ghanaian context, Euclidean geometry remains pedagogically and institutionally significant. Recent curriculum reforms for teacher education have reintroduced formal Euclidean geometry into the Bachelor of Education mathematics program with explicit expectations that pre-service teachers engage with deductive reasoning and formal proof (Akosah et al., 2024) to establish the importance of proof skills in students. This is because senior high school (SHS) students are the primary recipients of geometry instruction; understanding their performance and the specific obstacles they face when proving Euclidean results is both timely and necessary. Baah-Duodu et al. (2020) argued that Ghanaian mathematics facilitators lacked the expertise required to instruct Euclidean geometry effectively and efficiently. However, the new mathematics curriculum gives students more in-depth instruction in Euclidean geometry, along with innovative teaching strategies necessary to close this knowledge gap. The mathematics curriculum implied that all mathematics pre-service teachers must perform fully at a higher level (level 4-deduction: with formal proof) of the van Hiele levels in major aspects of Euclidean geometry. Accordingly, this study examines the performance of SHS students on Euclidean proofs, characterizes their prevailing difficulties, and considers the implications for curriculum planning and teacher education in Ghana.

Problem Statement

In recent years, Euclidean geometry has played a vital role in mathematics education in both primary and secondary education in Ghana and various other countries (Heinesen Højsted & Mariotti, 2024). Proof is important in mathematics because it organizes mathematical knowledge and encourages logical reasoning among SHS students. The fact that proofs are important in mathematics education, both teaching and learning, continues to encounter ongoing difficulties. A study conducted by Sunzuma and Maharaj (2019) revealed that teachers face challenges with their teaching methods and have insufficient knowledge of the subject, which adversely impacts geometry instruction and student learning. The drawbacks experienced by Ghanaian mathematics teachers are significant because they determine how proof is presented, practiced, and assessed in classrooms for learners. Examination monitors reported that candidates continuously and frequently skip questions involving geometric proof in the West African senior school certificate examination, which negatively impacts their overall performance in the final assessment (GNA, 2023). Again, the over-reliance on memorization instead of reasoning and understanding limits learners' chances to develop and evaluate proofs. In combination with variable teacher preparation, these instructional patterns are likely to shape whether and how students engage with proof tasks.

Again, although the van Hiele model has been widely applied to explain students' reasoning in geometry, much of the research has focused on identifying the levels students attain rather than examining how specific proof-related difficulties emerge within these levels. Studies such as Bashiru and Nyarko (2019) provide detailed evidence of students' errors in constructing proofs. However, these analyses are often divided and not consistently connected to developmental progressions in reasoning. Also, the international literature on students' difficulties with mathematical proof is heavily weighted toward Western contexts, creating an important gap in evidence from African secondary schools, especially in Ghana (Azrou & Khelladi, 2019). This shortage of context-specific research leaves policymakers and teacher educators without robust, locally generated evidence to guide interventions. This gap leaves unanswered questions about how local curricular emphases, classroom practices, and examination requirements interact with developmental stages to shape students' difficulties in Euclidean geometry. Addressing this gap is important for advancing theory by clarifying the explanatory power of the van Hiele model, and for practice and policy by informing instructional strategies that more effectively support students in developing deductive reasoning.

This study, therefore, responds to that space. Its primary purpose is to investigate SHS students' performance on Euclidean geometry proofs, to identify and categorize the conceptual, procedural, and affective difficulties they experience, and to examine how teacher preparedness, instructional approaches, and language of instruction contribute to those difficulties. By generating context-specific evidence, the research aims to inform curriculum design, teacher education, and classroom interventions that can strengthen evidence-based teaching and improve student outcomes in Ghana.

Theoretical Framework

This current study is anchored in two related theoretical frameworks: van Hiele's theory of geometric thought (Stiff & Curcio, 1999) and Bandura's (2006) social cognitive theory, which focuses on the concept of self-efficacy in the learner. These frameworks provide a combined perspective for understanding the cognitive and emotional factors that influence students' performance and difficulties in proving Euclidean geometry problems.

The van Hiele model discusses the five levels of geometric understanding that learners go through, and these are:

(1) visualization,

(2) analysis,

(3) informal deduction,

(4) formal deduction, and

(5) rigor (Stiff & Curcio, 1999).

The development of students through these levels occurs in a specific order, and the progress relies on suitable instructional experiences of the teacher rather than just age or development. In Euclidean geometry proofs in the mathematics curriculum, students who have not reached a certain level of geometric reasoning are unlikely to create proofs that are logically sound and applicable in general. van Hiele's concept was employed to determine whether students' performance in proofs corresponds to their current level of geometric reasoning. This study aims to identify the conceptual difficulties of students that may hinder the construction of proofs. As students move from informal deduction to the formal deduction stage of geometric reasoning, they develop the capability to understand concepts and use formal geometry concepts to solve geometric proofs. Formal proofs with shape properties that follow logical reasoning are now introduced to recent students; however, they could find it difficult to advance their original arguments. Students can compare and analyze different mathematical systems, including Euclidean and non-Euclidean geometry, without relying on physical models in the most advanced step of van Hiele's geometric learning theory, known as axiomatization. According to the researchers, altering the way geometry is taught in schools can aid kids in comprehending different geometric representations and fostering abstract system thinking. This study aims to examine how students understand geometric proofs through the lens of van Hiele's levels of geometric thinking.

According to Bandura (2006), self-efficacy of the student is defined as the capability the student has to perform and carry out actions necessary to reach particular goals in life. Self-efficacy affects the selection of tasks, the determination to overcome obstacles, and the level of mental involvement (Davor et al., 2025). In mathematics education in Ghana, a higher sense of self-efficacy is associated with better performance in problem-solving and an increased readiness of students to engage with difficult tasks, such as proofs. This current study also examines how students' confidence in their ability to prove geometric expressions impacts their involvement in proof tasks and their overall performance in Euclidean geometry in Ghanaian SHS. The application of van Hiele's theory and Bandura's (2006) concept of students' self-efficacy acknowledges that students' successful performance solely relies on their mental readiness and emotional outlook rather than on development. Students might understand the concepts well taught by a teacher, but their lack of confidence in their skills can impact on their readiness to approach proof questions. Again, students who are very confident but lack a small understanding of concepts may attempt proof tasks but end up creating incorrect or incomplete arguments.

In Ghanaian SHS, curriculum changes focus on formal reasoning in Euclidean geometry, corresponding with the advanced levels of the van Hiele model. Learners' understanding of concepts and confidence in their capability to prove mathematical theorems may result from inadequate facilitators' preparation, the persistent use of a rote learning approach, and mathematical language use in the classroom. The proposed framework facilitates an evidence-based assessment that considers both cognitive and sociocultural dimensions of mathematics education in Ghana by allowing an organized investigation of the interacting factors.

Research Questions

1. What is the level of students' performance in proving problems in Euclidean geometry?
2. Is there a significant difference between male and female students' performance in Euclidean geometry proofs?
3. What specific difficulties do students encounter when attempting Euclidean geometry proofs?

EMPIRICAL REVIEW

Students' Performance in Geometric Proofs

Geometric proof plays an important role in the SHS mathematics curriculum in Ghana. However, students' performance in this subject area remains poor (Chand et al., 2021). Achieving success in proof relies on several important skills. One of the most important abilities is understanding that a proof confirms truth only within its defined area and not outside of it. Studies consistently indicated that high school students face ongoing difficulties with proofs (Bieda, 2010). The difficulties are abstract reasoning, logical thinking, and differentiating examples from deductive arguments. Moreover, an empirical study with the van Hiele model of geometric reasoning at a SHS showed that only 1 percent achieved level 3, i.e., informal deduction. Most students remain at level 1, i.e., recognition, i.e., they are not prepared for formal proof (Selden, 2012). Another research with the SHS 3 students found that 42.5 percent of them had not yet reached any van Hiele level and only 0.5 percent reached level 4, formal deduction (Fitriza et al., 2025). The results show that most Ghanaian students graduating from SHS do not possess the thinking skills required in deductive proof. Performance scores are further evidence of this deficiency in cognition. A correlational study of final-year SHS students in the Central Region of Ghana indicated a moderate mean (M) score of 53.3 percent in geometry (Bashiru & Nyarko, 2019). The investigation indicated that learners' study habits were the most significant factor influencing their performance, with curriculum quality, teaching methods, and assessment practices following in importance (Suleiman et al., 2024). Recent empirical evidence confirms that Euclidean geometry remains one of the most cognitively demanding components of the mathematics curriculum. Current research utilizing the van Hiele model in 2025 indicates a significant developmental gap among graduating students; approximately 42.5% of SHS 3 students have not yet reached any identifiable van Hiele level, with only a negligible 0.5% reaching level 4 (formal deduction). This is particularly concerning as SHS students are the primary recipients of geometry instruction intended to foster deductive reasoning. This study will emphasize the importance of conducting additional research to explore the challenges students face and the teaching methods that are most effective in enhancing their engagement with the proof. This research is important for guiding changes in the curriculum and teaching methods that enhance deductive reasoning and promote a better understanding of proof as a key aspect of mathematical thinking.

Gender Difference and Performance in Geometric Proofs

Students' views and beliefs about mathematics are influenced by their cognitive and emotional experiences, which affect their engagement with geometry. The curriculum in Ghanaian pre-tertiary education promotes discovery learning through exploring real-life scenarios to improve conceptual understanding. However, evidence indicates that there are ongoing gender differences in proof-related tasks among SHS students (Fitriza et al., 2025; Kpotosu et al., 2024; Y. A. Mensah et al., 2022). Again, Else-Quest et al. (2010) studies reported that students in the Central Region achieved an average score of approximately 53 percent, with male students consistently scoring higher than their female counterparts. This means that the various teaching methods a teacher uses in a classroom play a major role in these differences.

In the Ghanaian education system, teacher preparation programs regularly do not focus on geometry enough, which hinders teachers' capacity to establish a strong understanding of the subject concepts. When students lack a solid understanding of geometric concepts, they face greater challenges in proofs, and it appears that female students are disproportionately affected by such situations, particularly in Africa. Some studies have also shown that both male and female students face difficulties, but specific instructional strategies aimed at addressing these issues have been developed to strengthen geometric reasoning and can reduce gender gaps in performance (Boateng et al., 2024; Codjoe et al., 2024). Regarding gender, evidence from 2023 and 2024 suggests that performance gaps in geometry are not biological inevitabilities but are socially constructed. While some regions show male advantages, other recent findings indicate female students performing better or finding no significant difference at all between groups. This highlights the necessity of equitable classroom practices to address persistent gender categorizations in mathematics.

Students' Difficulties in Geometric Proofs

When it comes to students' difficulties in geometric proofs, studies have consistently shown that students face ongoing difficulties in understanding geometric proofs (Haj Yahya & Hershkowitz, 2024; Y. A. Mensah et al., 2022). In this regard, the van Hiele theory of geometric understanding outlines a stage in which a student progresses from visual identification to formal deductive reasoning. It stated that many students, specifically in mathematics, continue to operate at lower levels of geometric reasoning, depending on their intuition instead of logic, which continuously affects students' capacity to create valid proofs (Mensah et al., 2023). Studies conducted in different settings clearly show that students consistently have difficulty in linking geometric diagrams with symbolic reasoning, which can lead to decreased perseverance and confidence in challenging tasks related to proofs. Learners often learn proofs by memorizing examples instead of understanding the foundational logical concepts of theorems. These methods limit their capacity to use knowledge developed in new situations. Moreover, considering classrooms in West Africa, teaching methods that focus on reiterating established proofs tend to promote memorization instead of critical thinking. This leads to a situation in which students do not build a strong understanding of concepts; rather, teachers focus on procedural techniques of teaching because of the demands of exams. Situating these in the Ghanaian education system, the data highlights the seriousness of these problems of geometric Proofs.

Furthermore, contemporary literature identifies the figural concept as a major pedagogical barrier. Learners often treat a geometric drawing as an absolute representation of reality rather than a conceptual illustration, a misconception that allows visual perception to override formal deductive logic, as observed by (Tao & Fu, 2024). Modern studies also suggest that multi-step proofs frequently induce cognitive overload. When the requirements for chained deductions exceed a student's working-memory capacity, they often abandon formal reasoning or commit algebraic collapse by incorrectly simplifying expressions to reduce mental strain.

METHODOLOGY

Research Approach and Design

This current investigation followed a mixed-method approach. Within the mixed-methods research approach, an explanatory sequential research design was used (Creswell & Creswell, 2018) In an explanatory sequential design, priority is given to quantitative data. This implies that the findings of the quantitative study inform the collection of qualitative data to provide a deeper explanation of the initial results. Integrating numerical and narrative data provides a more nuanced and comprehensive understanding of the research problem. This design improves the interpretation of the results, as the qualitative phase is explicitly informed by and builds upon the quantitative findings

Population and Sampling Technique

The target population consisted of 240 final-year SHS students within the Atwima Nwabiagya Municipal in the Ashanti Region of Ghana. A purposive sampling technique was employed to select 62 students (32 males and 30 females) from three specific classrooms. These students were selected because they had completed the Euclidean geometry module as stipulated in the national mathematics curriculum and were in the final stages of preparation for their exit examinations.

Research Instrument

The study used a teacher-made test to collect data. The test comprised 10 open-ended items on Euclidean geometric proof questions. The items were constructed based on the researcher's past teaching experience of students' understanding and difficulties with Euclidean geometric proofs and other insights obtained from a review of related studies on the study objectives. The open-ended items were chosen to enable the participants to compose their responses in proving Euclidean geometry so that their procedures can be analyzed in detail in terms of being appropriate or inappropriate. Some participants' thinking processes underlying the problem-solving skills provided were sought through interviews. The interview was conducted one-on-one with some selected participants in the investigation.

Validity and Reliability

The essential components for forming accurate conclusions and making a reasonable decision based on research results are validity and reliability. William (2024) proposed that the reliability of measuring instruments is based on their accuracy in measuring what they are intended to measure. The open-ended items were given to SHS mathematics teachers and two university professors to assess the quality of the items constructed in line with the research objectives of the study. The instrument was further subjected to pilot testing on students of the same characteristics as those used in the study. By using the KR-21 formula $\alpha = \frac{k}{k-1}\left(1 - \frac{M(k-M)}{k\sigma^2}\right)$, where the number of test items $k = 10$, the M is 7.02 and the variance $\sigma^2 = 10.70$. The KR-21 reliability coefficient of test items was found to be 0.89 and was considered good for use.

Trustworthiness

The trustworthiness in qualitative study, which is similar to validity and reliability in quantitative research, was established using peer debriefing, where two mathematics facilitators from the school reviewed the data by challenging the researchers' underlying assumptions and posing questions related to interpretations and methodological decisions (Lotey et al., 2025). Additionally, an audit trail involving detailed documentation of the research process, including data collection, analysis, and interpretation, was maintained throughout the study to ensure transparency. Moreover, participant validation (member checking) was conducted by returning the final interpretations to participants for feedback, validation, or correction. Member checking was done to ensure that participants agreed with how their opinions were reported and also assisted in identifying any inaccuracies, misinterpretations, or gaps in the researchers' understanding of participants' perspectives (Lotey et al., 2025).

Data Collection and Analysis

Before data collection, the researchers visited the schools to seek permission from school authorities, after explaining the rationale of the study. They were informed that they were voluntary participants in the study and that anyone could decide not to participate at any time. The teacher-made tests were given to the participant, and they were allowed adequate time to provide their written responses to the tasks, after which their answer scripts were collected. The test results were analyzed based on the difficulties experienced by students in reading and constructing a geometric proof. Learners' results were reported utilizing descriptive statistics (M and standard deviation [SD]) and inferential statistics, specifically the independent t-test. To ensure proper representation of participants' performance, those who answered fewer than 4 questions were eliminated from the study. In order to minimize gender-related bias in the study, the sample comprised 62 students, including 30 females and 32 males. This near-equal representation was deliberately adopted to ensure that findings were not disproportionately influenced by a single gender group. Earlier studies have indicated that gender imbalance in selecting the participants is most likely to create bias in results, particularly in technology-enabled learning and mathematics education contexts, where variations in participation and perceptions between male and female students have been found (Fitriza et al., 2025)

Balanced gender representation thus makes the outcome more valid as well as generalizable.

Furthermore, qualitative data were collected through a one-on-one interview with purposively selected participants. Participants were selected based on their willingness to share their experiences and, where feasible, to reflect on the range of performance levels on the

Table 1. Students' background information

Categories	Frequency (N)	Percentage (%)
Age	62	100
13-15	22	35.5
16-17	25	40.3
18+	15	24.2
Gender	62	100
Male	32	51.6
Female	30	48.4

Euclidean proof test. The interview lasted for about thirty minutes and was conducted in the participants' classroom, providing a neutral and comfortable environment that encouraged open and honest interaction. The discussion was facilitated by the first author. The discussion was audio-recorded, and supplementary field notes were taken. Following data collection, the audio recordings were transcribed verbatim, and the data were analyzed using a thematic analysis approach (Braun & Clarke, 2006). An initial coding process was conducted to identify recurring patterns and ideas, after which the emerging themes were reviewed and refined to generate meaningful insights aligned with the study's objectives.

The population statistics of the participants in the study are provided in **Table 1**. They were broken down to 51.6% males and 48.4% females, and together they comprised the total sample. Out of their age, 22(35.5 %) were between 13-15 years old, 25(40.3 %) were between 16-17 and 15(24.2%) were 18 years and above.

RESULTS AND DISCUSSION

Students' Performance in Euclidean Proofs

A total of 62 students completed the Euclidean geometry proof assessment with scores ranging from 0 to 20 points. The results show that students generally performed poorly in constructing Euclidean proofs. The highest number of students (n = 15, 24.2%) scored only 6-7 points out of 20, while 13 students (21.0%) scored 8-9 points. The researchers grouped the scores into three categories, and the poor performance became clear, with nearly a quarter of students (n = 14, 22.6%) getting a low performance with scores of 5 points or less. The majority of students (n = 42, 67.7%) showed moderate performance between 6-11 points, which is still below expectations for a 20-point assessment. Only 6 students (9.7%) achieved what can be considered a good performance with scores of 12 points or higher. The data reveal significant weaknesses in students' ability to construct Euclidean proofs. Taking all scores into consideration, 90% of students scored below 12 points, and the average performance clustered to the lower-middle range. These results indicate that students struggle considerably with geometric reasoning and formal proof construction. The low performance suggests that current teaching methods may be inadequate for developing strong proof-writing skills in Euclidean geometry (**Figure 1**).

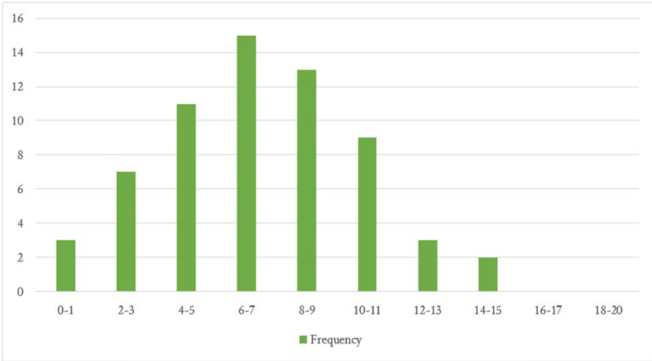

Figure 1. Students' performance in Euclidean proofs (Source: Authors' creation, 2025)

Gender Difference in Students' Performance in Euclidean Proofs

The results of **Table 2** show a significant difference in student performance on Euclidean proof tasks based on gender. The data indicate that a higher percentage of female students are in the lower performance quartiles. Nineteen of the thirty female participants (63.3%) scored between 0 and 7, while sixteen of the thirty-two male participants (50.0%) achieved scores in that same range. The 13.3% difference suggests that there may be differences in geometric reasoning skills or proof-building methods between genders. Male students had higher median achievement levels, with 13 participants (40.6%) scoring between 8 and 11 points, while 9 female participants (30.0%) achieved the same range of scores. This trend indicates that male students show better consistent performance in moderately challenging test situations. The 10.6 percentage point difference in this range represents the largest gender gap observed across all scoring intervals.

Both groups performed similarly, with scores in the 6-7-point range. A slightly higher percentage of females scored in this range, at 26.7%, compared to 21.9% of males. Male students had a small advantage in the higher performance levels, with 3 participants (9.4%) scoring between 12 and 15 points, while 2 female participants (6.7%) reached the same scores. No participants from either gender achieved a score higher than 15 points, which suggests that Euclidean geometry proofs are difficult for students. The distribution of the results suggests that both genders achieved similar peak frequencies in the lower-middle ranges. However, males showed a slight advantage in higher-level performance on Euclidean proof tasks.

Tests of Normality

A normality test was conducted using SPSS (v27) by the researchers to decide whether to do a parametric test or a non-parametric test for assessing differences between independent groups. **Table 3** displays the results of the normality tests for both male and female groups.

The study's results of the Kolmogorov-Smirnov test (D [32] = 0.106, p = .200) and the Shapiro-Wilk test (W [32] = 0.988, p = .965) for the male students were not significant, suggesting that their scores

Table 2. Gender difference in students' performance in Euclidean proofs

Scores	0-1	2-3	4-5	6-7	8-9	10-11	12-13	14-15	16-17	18-20	Total
Male	1	3	5	7	7	6	2	1	0	0	32
Female	2	3	6	8	6	3	1	1	0	0	30
Total	3	6	11	15	13	9	3	2	0	0	62

Table 3. Tests of normality

Gender	Kolmogorov-Smirnov			Shapiro-Wilk		
	Statistic	df	Significance	Statistic	df	Significance
Male	.106	32	.200*	.988	32	.965
Female	.100	30	.200*	.978	30	.781

Table 4. Group statistics

Gender	N	M	SD	Standard error mean
Male	32	7.5625	3.30139	.58361
Female	30	6.7333	3.18329	.58119

Table 5. Independent sample t-test

		Levene's test for equality of variance					95% confidence interval	
		F	Significance	t	df	Significance	Lower	Upper
Scores	Variance assumed	.178	.675	1.006	60	.319	-.82032	2.47865

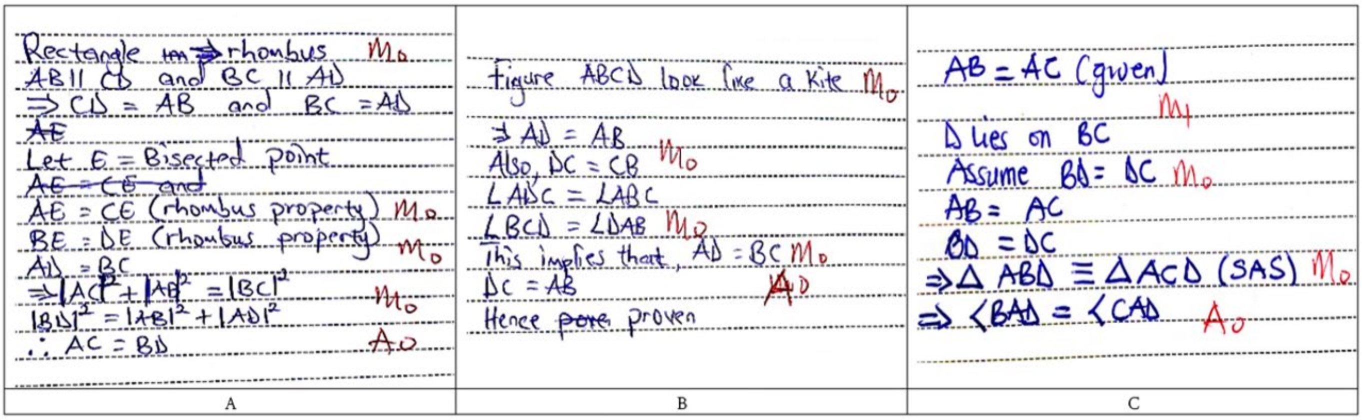

Figure 2. Interpreting diagrams and translating into formal statements (Source: Students' responses, 2025)

were normally distributed. A similar trend was seen among the female students, as both the Kolmogorov-Smirnov test (D [30] = 0.100, p = .200) and the Shapiro-Wilk test (W [30] = 0.978, p = .781) indicated that there were no violations of normality. These results suggest that the data for both groups follow a normal distribution, which permits the use of parametric tests in subsequent analyses without concern for bias from distribution-related issues.

A t-test for independent samples was performed to determine if there were significant differences in student performance based on gender. The findings indicated that male students had a slightly higher average score (M = 7.56, SD = 3.30) compared to female students (M = 6.73, SD = 3.18). However, this difference was not statistically significant, with t (60) = 1.00, p = .319, and a confidence interval of 95% (-0.82, 2.48) (**Table 4**).

The effect size was small (Cohen's d = 0.26), which indicates that gender accounted for minimal variation in the scores. In practical terms, the findings suggest that male and female students performed comparably in the task, and that factors such as instructional methods or prior preparation may have had a stronger influence on outcomes than gender (**Table 5**).

Students' Difficulties in Euclidean Geometry Proofs

Interpreting diagrams and translating into formal statements

From the analysis of the results, students frequently relied on the visual appearance of diagrams rather than the formal properties required in Euclidean geometry proofs. For instance, in Q1 of **Appendix A**, in step three of part C in **Figure 2**, a learner assumed BD = DC without any given or derived justification simply because point D was said to be on side BC. Again, that extra hypothesis is necessary for the SSS congruence claim in step five, because it is not established from the problem data, the congruence and the derived angle quality are invalid (SAS). For example, when P#20 was interviewed, he said that "*I thought D was the midpoint because in the drawing it looks like the two parts of BC will be the same if D lies on BC*"**.** Similarly, in Q6 of **Appendix A**, P#30 classified the quadrilateral by visual similarity (Kite) rather than using labels that identify a parallelogram. The properties of a kite differ from those of a parallelogram; applying kite properties here is therefore invalid and leads to an incorrect chain of reasoning.

Again, in part B in **Figure 2**, for Q2 in **Appendix A**, the equal subdivision of diagonals was inferred directly from the apparent symmetry of the drawing. However, P#15 misclassifies a rectangle as a rhombus; that is the student swaps properties of figure classes. Because rhombus properties do not necessarily hold in a generic rectangle, any subsequent inference based on rhombus properties is invalid. These examples illustrate the misconception that a sketch is an exact representation of geometrical reality, a belief that allows learners to substitute visual perception for deductive reasoning. Such misconceptions are consistent with Goswami's (2010) notion of the "figural concept," where the drawing is perceived simultaneously as a figure and as a conceptual object, often blurring the boundaries between illustration and mathematical structure.

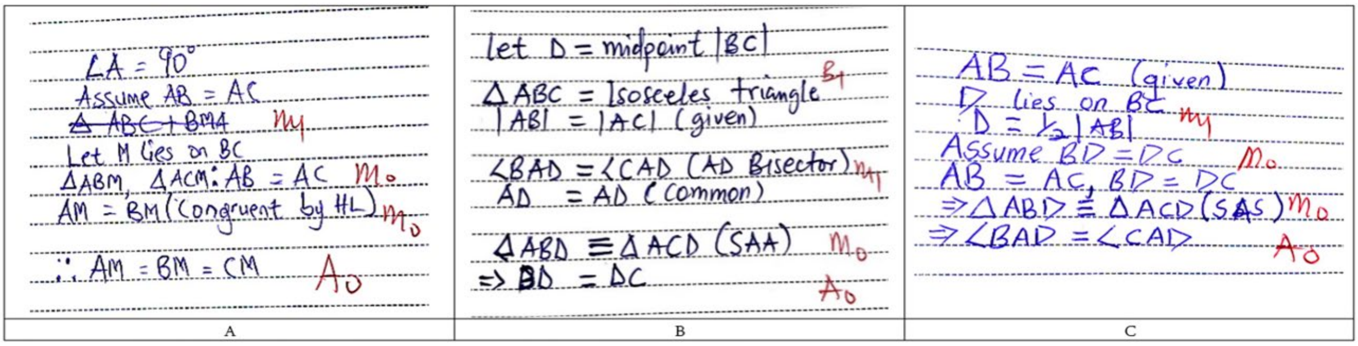

Figure 3. Recalling and applying theorems incorrectly (Source: Students' responses, 2025)

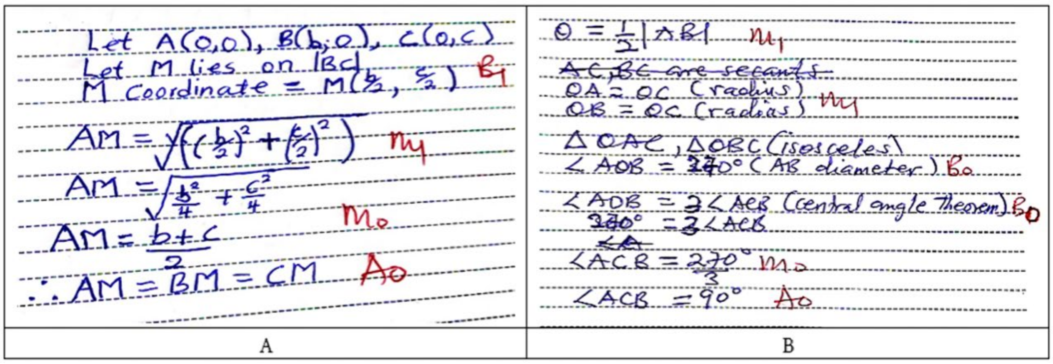

Figure 4. Perseverance and cognitive overload in multi-step proofs (Source: Students' responses, 2025)

Recalling and applying theorems incorrectly

Students' difficulties were also observed in recalling and applying theorems to prove Euclidean geometry. For example, Q7 of **Appendix A** for part B in **Figure 3**, a student uses the non-standard congruence label "SAA". The classical congruence tests are SAS, ASA, SSS, AAS, and HL (for right triangles). The student may have intended to state ASA or SAS but has misnamed or misapplied the pattern. Because the congruence pattern is not correctly formulated, the resulting inference ($BD = DC$) is not properly justified. When P#8 was interviewed, he disclosed that, *"I used SAA because I thought equal sides and one angle were enough for congruence."*

Similarly, in Q4 of **Appendix A**, the student P#36 inserts an extra assumption (AB = AC) that is not given; subsequently, they try to use HL (hypotenuse leg), but the HL criterion requires specific identification of the hypotenuse and a corresponding leg. The conditions are not met as presented. This is a double error, a wrong assumption and a misapplied congruence rule. This reflects problem-schema drift: the student attempts to map the problem onto a familiar congruence pattern (HL) but forces the givens to fit by inventing AB = AC. This indicates that students frequently force data to fit preferred proof templates. When P#36 was asked, she said, *"Hmmm, Sir, I just assumed AB = AC so that I could use HL congruence."*

Perseverance and cognitive overload in multi-step proofs

Errors in this section occur when students abandon formal reasoning midway or resort to shortcuts to manage the cognitive load of multi-step proofs. For example, in Q3 of **Appendix A** for part B in **Figure 4**, the student begins correctly by identifying symmetric radii (OA = OC and OB = OC) in step 2 and step 3 but then abandons the required angle chase that connects these equalities to the right-angle conclusion. The final line is an unsupported leap; recognition of structure occurred, but the operational deductions were not carried out. This is a persistence or working-memory failure. This means that complex proofs require keeping interim relations active while performing chained deductions. Cognitive load theory explains that students may prematurely stop when the chaining exceeds their working-memory capacity. Again, Q4 for part A in **Figure 4**, the coordinate method was applied correctly at first but broke down when the student simplified $\sqrt{((b/2)^2 + (c/2)^2)}$ as $(b + c)/2$, a clear instance of overload leading to algebraic collapse. Similar patterns were observed where learners skipped intermediate congruence arguments and jumped to conclusions. These attempts suggest that the misconception is not merely factual but procedural. Students appear to believe that once a result "looks obvious," it is legitimate to stop. When P#51 was asked why he skipped steps, he said, *"Sir, the calculation for AM was too long, so I simplified $\sqrt{((b/2)^2 + (c/2)^2)}$ as $(b + c)/2$."*

Language, notation, and logical structure errors

From the analysis, the half-length conclusion is true for Q9 of **Appendix A** for part A in **Figure 5**, but the student jumps directly from the midpoint relations for AB and AC to $DE = ½BC$ without providing the similarity or proportionality chain that justifies this equality. The logical gap weakens the proof. This is an omission of justificatory steps. The student knows the right numerical relation but lacks the linking argument. This pattern appears often in novice proofs where students have partial 'stored results' but not the structural reasoning. Stylianides notes this tendency to state results without operational justification. Similarly, in Q8 of part C in **Figure 5**, the idea is correct, but the student's arc labelling is ambiguous, and the notation is disordered. Such disorder increases cognitive load and makes the reasoning hard to follow or verify. Notation confusion and overloaded short-term memory can cause otherwise-correct conceptual chains to fail in practice. Sweller's cognitive load and Duval's representation arguments both suggest that clarity of representation reduces errors.

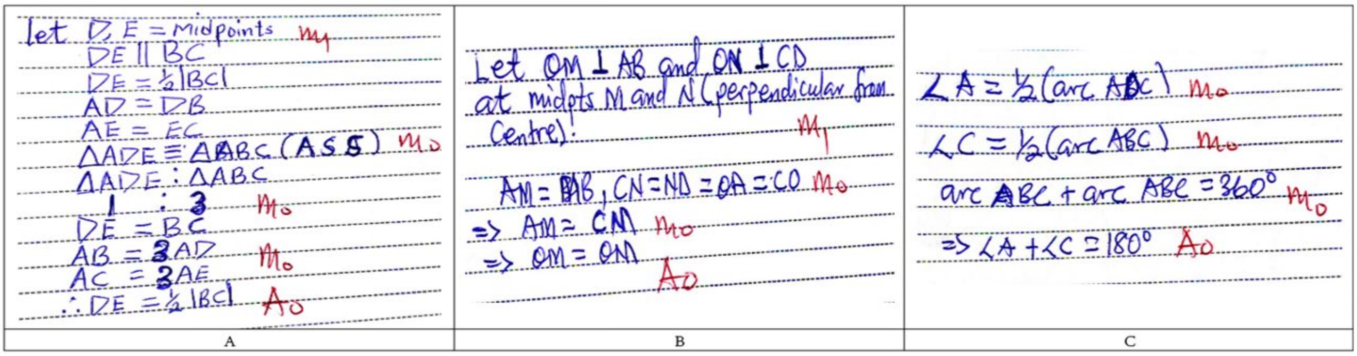

Figure 5. Language, notation, and logical structure errors (Source: Students' responses, 2025)

Again, in the final step of Q10, the learner omits an explicit congruence argument. To deduce OM = ON, the student must show right triangles OMA and ONC are congruent (by hypotenuse and leg). The student's leap compresses multiple steps into one and leaves the chain unverifiable. Omission of the congruence step is a logical compression error that indicates limited proof-scripting skill. Students know what should be true but cannot articulate the triangle-congruence warrant. One student said, *"Sir, since AB = CD, I just wrote $AB \perp OO$. I thought that was the correct way to show perpendicular."*

These errors reveal the misconception that proof writing tolerates informal or approximate language and that logical rigor is secondary to intuitive plausibility. Duval's (2008) theory of semiotic registers helps explain this difficulty. Learners struggle to coordinate between the natural language register, symbolic register, and diagram register, leading to notational ambiguity and logical incoherence.

DISCUSSION OF RESULTS

Students' Performance in Geometry Proofs

The results of this study offer significant information about how SHS students in Ghana work with Euclidean geometry proofs. The most notable finding is that students consistently perform poorly in this area. Geometry proof is an essential part of the mathematics curriculum and serves as a key method for fostering deductive reasoning and logical thinking skills. The poor students' performance identified in the study is not a unique problem but rather a sign of an ongoing challenge in mathematics education in Ghana. Kpotosu et al. (2024) reported that SHS students struggle with basic geometry topics. Previous national-level studies conducted by Mensah et al. (2022) identified geometry as one of the weakest areas of mathematical performance. This indicates that the issue has continued for many years, even with changes in curriculum and assessment.

This finding is significant because the difficulty with geometry proofs is not limited to just students in Ghana. A study conducted on future mathematics teachers in Ghana indicates that many demonstrate limited geometric reasoning skills, especially in the areas of deduction and proof as outlined by the van Hiele model (Okeke et al., 2024). This indicates that deficiencies at the secondary level influence teacher education, which continues to sustain a cycle of inadequate skills in evidence-based reasoning. The study supports the idea that enhancing performance in geometry necessitates a comprehensive approach. This should involve focused curriculum development, specialized training for teachers, and the use of more interactive and visual teaching methods to help clarify the process of constructing proofs.

Gender Difference in Proving Euclidean Geometry

The second important finding from this study is also fascinating. The study data showed that there was no significant difference in performance between male and female students in geometry proofs, despite ongoing issues about gender differences in mathematics. This result questions the belief that gender differences in mathematics education are inevitable. It again indicates that when provided with supportive and fair learning environments, both boys and girls can achieve similar performance levels. This aligns with the results of Kpotosu et al. (2024), who reported that gender did not play a significant role in students' comprehension of geometry. It is important to recognize that findings related to gender and mathematics vary across contexts. Armah (2024) found that female students performed better than male students in core mathematics in the Cape Coast Metropolis. On the other hand, Codjoe et al. (2024) revealed a modest but significant advantage for male students in the Assin North District. These differences indicate that gender disparities are not facts of nature but are produced by classroom contexts, cultural expectations, and opportunities given to the students. World evidence also attests in the same direction. Else-Quest et al. (2010) report that in some regions, boys perform better than girls in math, but in other regions, girls perform better than boys. In others, there is no sizeable difference between the two groups. The variation in outcome highlights the need to think of gender differences as variable and socially constructed, not fixed.

Difficulties in Euclidean Geometry Proofs

The challenges found in this study align with van Hiele's geometric reasoning model (Stiff & Curcio, 1999), which suggests that most Ghanaian SHS students are in the lower to middle levels of development. Misunderstanding of diagrams is aligned with the visualization level since Tao and Fu (2024) observed that students have ideas based on appearances rather than properties. Misuses of theorems point to the condition of analysis, just like Heinesen and Mariotti (2024) found that learners memorized but could not use theorems properly in proof contexts. Inability to carry out multi-step reasoning points towards learners being at the informal deduction level, just as Okeke et al. (2024) found that the majority of secondary school students can deal with single-step and not long proofs. Finally, difficulties with notation and formal expression are a mirror of difficulties in the transition to formal deduction, an issue further highlighted by Heinesen and Mariotti (2024). These patterns are consistent with global research, showing that most students are between van Hiele level 1 and van Hiele level 3, with little progress to extensive deductive reasoning (Forgasz & Leder, 2017; Haj-Yahya & Olsher, 2025; Suárez & González, 2025).

CONCLUSION

The study revealed three main contributions based on these findings. It highlights the urgent need to enhance students' abilities in deductive reasoning and the development of mathematical proofs, particularly in geometry, based on new evidence. To address this issue, it is essential to modify the curriculum, offer continuous professional development for teachers, and adopt classroom practices that engage students in evidence-based problem-solving. It provides strong evidence that the differences in math achievement between genders are not inevitable. The absence of gender differences in the sample indicates that when learning opportunities are equitable, both boys and girls can achieve comparable success. The results of the study are significant for educators and policymakers in Ghana, as they revealed that equitable classroom practices can improve academic achievement and address persistent gender categorizations associated with mathematics.

Implication and Practice

The results of this study indicate that mathematics teachers should enhance student involvement with proofs, as they are essential for understanding geometry. Classroom teaching should focus on logical reasoning, problem-solving, and student-led explanations rather than memorization, since many students struggle with Euclidean proofs. Utilizing visual aids, structured exploration, and clearly outlined opportunities to describe proof steps can enhance the comprehension of abstract concepts. The lack of performance differences between genders suggests that male and female students can attain comparable outcomes when provided with the same educational opportunities. Teachers should design activities that purposefully lead students through the van Hiele levels by emphasizing the analysis of properties, supporting the application of theorems, and involving students in organized proof exercises. It is important to use reasoning that considers the features of visual tasks to reduce reliance on visual signals. Furthermore, activities that require multiple steps and a structured approach can improve students' informal deductive reasoning abilities.

Suggestion for Future Studies

This study has identified evidence of consistent difficulties of SHS students in the development of Euclidean geometry; however, there is no gender difference in performance where equal opportunity prevails. On this ground, it is advised that subsequent work should focus on broader and more varied populations of students and factor in intervention-based strategies involving innovative methods such as inquiry methods and dynamic geometry software. Future research could then explore teacher-specific factors, as well as cognitive and affective factors, to further understand proof learning.

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AI statement: No generative AI tools were used in writing this manuscript. Conventional tools, including spell-check, grammar software, and Mendeley for reference management, were used for editing and formatting only. All analysis, interpretation, and writing were completed solely by the authors.

Declaration of interest: Authors declared no competing interest.

Data availability: Data generated or analyzed during this study are available from the authors on request.

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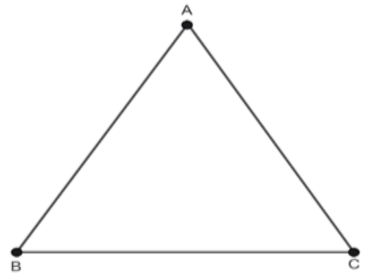
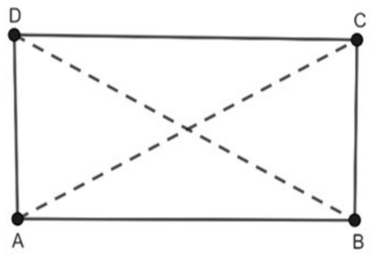
APPENDIX A: SENIOR HIGH SCHOOL MATHEMATICS - GEOMETRY PROOF TEST

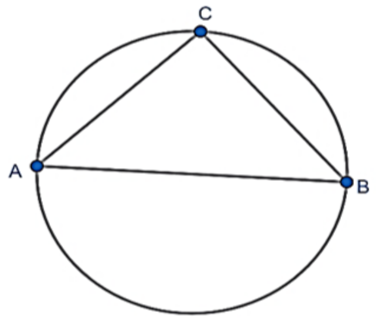
1. In △ABC, AB = AC. Point D lies on BC. Prove that ∠BAD = ∠CAD.



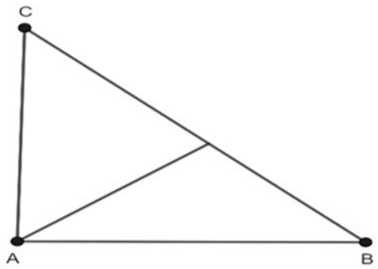
2. ABCD is a rectangle. Prove that the diagonals AC and BD are equal and bisect each other.



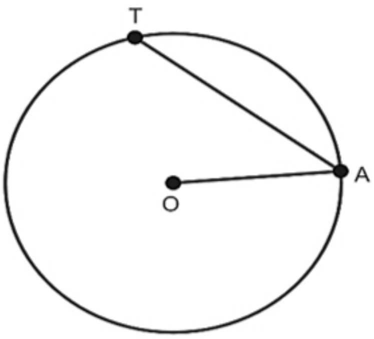
3. In a circle with center O, AB is the diameter and C is a point on the circle. Prove that ∠ACB = 90°.



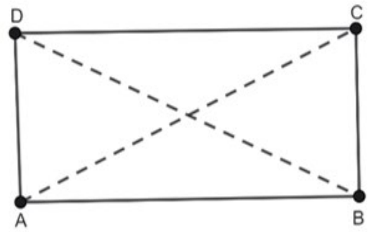
4. In △ABC, right-angled at A, M is the midpoint of BC. Prove that AM = BM = CM.



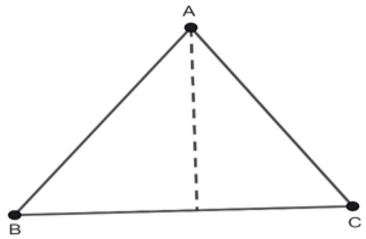
5. A tangent is drawn to a circle with center O at point A. Prove that OA is tangent to AT.



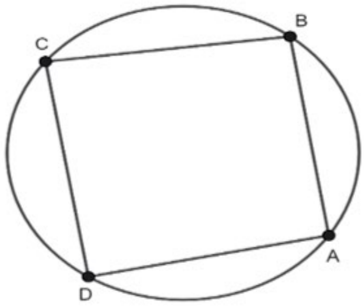
6. ABCD is a parallelogram. Prove that opposite sides are equal and opposite angles are equal.



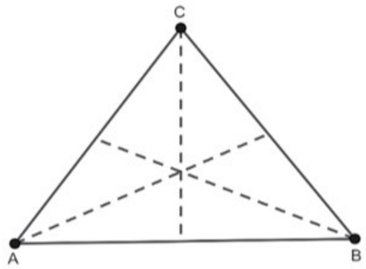
7. In isosceles △ABC with AB = AC, prove that the bisector of ∠A also bisects BC.



8. In a circle, ABCD is a cyclic quadrilateral. Prove that opposite angles are supplementary.



9. In △ABC, the midpoints of two sides are joined. Prove that the line joining them is parallel to the third side and half its length.



10. In a circle with center O, prove that equal chords are equidistant from the center.