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# Conceptual and Procedural Knowledge of Students of Nepal in Algebra: A Mixed Method Study

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**ABSTRACT**

Mathematical knowledge has been defined in several ways in the literature of mathematics education. Procedural knowledge (PK) and conceptual knowledge (CK) or both types of knowledge are the emphasis of knowledge construction. This is a research-based paper extracted from a dissertation of MEd in mathematics education of the first author under the supervision of the remaining two authors. In this context, this explanatory mixed method research study was carried out to find students' level of PK and CK in algebra and explore why students develop such knowledge. In the quantitative part, the survey was conducted among 360 students of grade eight of 9 public schools of Kathmandu Metropolitan City. The study revealed that students have a lower level of CK ($\bar{x}$=8.56) but a higher level of PK ($\bar{y}$=14.05) out of 20 and a moderate positive correlation (r=+0.559, $p<0.05$) between PK and CK. The regression equation was: CK=3.716+0.345(PK). Similarly, PK was dependent, but CK was independent upon the gender of the respondents. In the qualitative part, a two-phase interview was conducted with six participants followed by a group discussion with four mathematics teachers teaching at the same level. This phase concluded that students are weak in reasoning, critical thinking, representational knowledge and comparing algebraic quantities. The reason is because students seemed to be forced/encouraged to develop procedural fluency because of teachers' methods of teaching which oftentimes neglect the progressive pedagogical approaches. The research is useful for everyone who is working on educational reform to emphasize meaningful learning.

**Keywords:** conceptual knowledge, procedural knowledge, algebra learning

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## INTRODUCTION

A debate or a math war (Klein, 2007) between procedural knowledge/skill and conceptual knowledge is not new in mathematics teaching and learning. The relationship between conceptual and procedural knowledge has been an issue of debate among mathematics researchers in education (Zuya, 2017). This debate leads to a question that which knowledge comes first. However, many research studies have done, such as Lenz et al. (2020), Rittle-Johnson and Siegler (1998), Rittle-Johnson et al. (2016), Ross (2010), Zuya (2017), etc., to visualize the relationship between procedural and conceptual knowledge so that we could teach better mathematics and students learn them meaningfully. As a mathematics teacher, most of the time, the researcher heard that for a mathematics knowledge acquisition, the rich concept should be established while teaching and procedures are also important in problem-solving. But the teaching scenario changes when it comes to the classroom. The instructional practices emphasize more on students to memorize formulae, steps or procedures to solve the problems in mathematics than encourage them to be creative, ask questions, think critically, and play with the situation so that they use their highest potentiality to construct knowledge with the underlying concept which is rich in connection with deep meaning, a conceptual knowledge (Lenz et al., 2020; Rittle-Johnson, 2019; Star, 2005).

Due to its abstract nature of symbols and expressions, in algebra, more teaching instructions pursue constructing procedural knowledge by minimizing the number of underlying concepts. For example, problems solving in indices follow the development of procedural knowledge through memorizing rules, steps, and formula of indices. It is difficult to develop the underlying concepts such as the pictorial representation of the basic concept of indices, comparing quantities 1/x and 1/(x^2), etc. So, students may have different abilities to construct the knowledge of algebra at the school level. Some are capable of developing PK, and some CK.

In Nepal, Education Review Office (ERO) found that students in mathematics are weak in reasoning, critical thinking, problem-solving, making pictures and shapes, and representational knowledge (ERO, 2019). It revealed that students of grade eight in mathematics have a lower ability to solve complex problems (higher ability) and that is only 25% of the maximum score in higher ability related questions. Students' performance was found better in lower but poor in higher cognitive skills. Students were much better in recalling types of questions (ERO, 2017). These phenomena show that our students are forced to generate knowledge through memorization, they are evoked to recall and remember the procedures and algorithms without understanding the concept. Moreover, Nepali student has a tendency to take mathematics as a foreign subject (Luitel, 2009).

ERO (2015) has asserted that among the achievement ranges from 28 to 38 percentage in the test of mathematics, students scored only 28% in algebra which was significantly lower than the national mean (35%). Similarly, students are weak in knowledge of algebra in comparison to others. On the other hand, students are disengaged in learning and a huge mass is at the underperforming level (ERO, 2019). They have mathematical anxiety too. At this stage, what is the genuine problem? Is it a problem to emphasize procedures to solve the problems without learning than the underlying concepts? Why are children not enjoying learning concepts in algebra?

With these questions in mind, the main research question of this research were: What is the level of students' procedural and conceptual knowledge in algebra? and Why do students develop conceptual knowledge or/and procedural knowledge?

## REVIEW OF THE LITERATURE

### Conceptual Knowledge of Mathematics

The knowledge based on different connections or concepts is often called conceptual knowledge. In the words of Hiebert and Lefevre (1986), "a knowledge that is rich in relationship. It can be thought as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information which linked to some network" (p. 3-4). Creating the concept of the idea is to connect with other information. For instance, to learn "interest" in mathematics, we connect this idea with other information such as its definition, practical uses in our day-to-day life, relationship with principle, amount, interest rate, time, etc. Similarly, representational knowledge is a conceptual knowledge where connection of a concept is linked to different pictorial representations (Brooks & Freeman, 2018; Rittle-Johnson & Schneider, 2015). The pictorial representation of $x^2$ is a square having length x, for example.

Conceptual knowledge of mathematics is generated through pre-requisite concepts, ideas, and information. We gradually develop mental pictures or schemas of something by interacting with the environment, with different people, and so on. These schemas are very much powerful to create other concepts. Establishing conceptual knowledge of mathematic means to learn about "why" it happens in a particular way (Hiebert & LeFevre, 1986). So, this is not only about what is known but also one way that concepts can be known (Star, 2005). CK gives multiple perspectives to think about a particular problem by developing divergent thinking skills. It means a learner has different methods to tackle with the problem of mathematics (Rittle-Johnson & Schneider, 2015). Having conceptual knowledge means knowing about definition, formulae, and procedures and being able to justify it (Zuya, 2017). So, having conceptual knowledge about something means one can come up with a reasonable answer to it.

### Procedural Knowledge of Mathematics

A procedure or process is a systematic way through which something has to be done. Our daily rituals pursue a systematic way so that we can say human life is a kind of routine system, I guess. From the beginning to the end, when we go to bed, in general, most of the people try to make a schedule and follow it accordingly. We need everything in a proper order which should be planned. For acquiring knowledge, knowledge which is generated through a systematic way or procedure is called "procedural knowledge" that is knowledge through process (Hiebert & LeFevre, 1986). Following a strict and predetermined rule or process to get a predetermined answer is known as procedural knowledge (McCormick, 1997). If one tries to grab or behold any idea of mathematics through rote memorization, by using step by step method to get the correct answer or by using too algorithmic problem-solving method to solve the problems of mathematics then he/she tries to develop procedural knowledge (Rittle-Johnson, 2019). Such knowledge of mathematics sounds like a toolbox that includes facts, skills, procedures, algorithms, or methods (Barr et al., 2003).

Procedural knowledge is "knowing how or knowing what", or the knowledge of the steps required to attain the various goals. In the words of Byrnes and Wasik (1991), "procedures have been characterized using such construct as skills, strategies, productions, and interiorized actions" (p. 777). If we investigate this view, we should have the skill to solve a particular mathematical problem. That means, the procedural knowledge cannot be widely generalized (Rittle-Johnson et al., 2001) that promotes convergent thinking skills. Rittle-Johnson and Schneider (2015) have emphasized the following key point regarding procedural knowledge; (1) algorithms–a predetermined sequence of actions that lead to the correct answer when executed correctly and (2) possible action sequences that must be sequenced appropriately to get the solution to a given problem.

There are numerous research studies on procedural and conceptual knowledge. These studies focus on the level of students' CK and PK and their relationship while constructing knowledge. A research study (Rittle-Johnson et al., 2001) done in 2001 on developing conceptual understanding and procedural skill in mathematics: An iterative process through two experiments conducted with fifth and sixth-grade students about learning decimal fraction has concluded that the construction of knowledge should be based on iterative process. That means, one type of knowledge leads to a positive development of the other. After the accomplishment of this study, they have presented a model of the iterative theory of knowledge construction.

As a practitioner-researcher, I believe in this theory of knowledge construction. A learner who can learn mathematics using both methods of procedures and concepts is more intelligent than a learner who learns mathematics through an individual process. Conceptual knowledge and procedural knowledge are interrelated (Hiebert & Lefevre, 1986; Rittle-Johnson & Schneider, 2015; Ross, 2010). Talking about particularly in algebra, Abd Rahman (2006) revealed that the students' conceptual knowledge of algebra is extremely low. Similarly, a research study done by Ross (2010) in his doctoral dissertation among the 8th graders students to measure their conceptual and procedural knowledge in algebra found that there is strong positive correlation between the knowledge. Moreover, a research done by Ghazali and Zakaria (2011) that was carried out among 132 secondary level school students to compare students' procedural and conceptual understanding in algebra found a high level of procedural understanding but a low level of conceptual understanding in algebra. Students have a low level of conceptual knowledge with the average positive correlation ($r=0.512$, $p<0.05$) between students' procedural and conceptual knowledge in algebra. This also supports the interrelationship of CK and PK in the construction of knowledge.

There is a lack of such research in the field of mathematics education in Nepal and most of the research in the world carried out using quantitative methodology. So, this research study has the potential to fill the gap.

## THEORETICAL LENSES OF THE STUDY

Cognitivism and constructivism learning theories were subscribed as theoretical referents to this study. Cognitive learning theory believes that a learner constructs or develops the knowledge of mathematics through his/her active mental process (Yilmaz, 2011). In this process, the previously learned conceptions remain the same in the brain and learner use prior knowledge as a schema to form new knowledge. In this view, construction of procedural knowledge and conceptual knowledge, to some extent, of mathematics is a kind of mental process so that this theory is applicable to justify the meaning and value of PK and CK of mathematics in the process of data collection as well as analyzing both forms of data. This theory helped the researcher examine and explore the meaning of participants' responses more towards procedural knowledge of mathematics. Similarly, this theory helped researcher to develop tools for the survey.

Constructivism keeps the learner at the center of learning. It means learner plays a key role in knowledge development. Constructivists believe in learners' own effort to construct knowledge individually or through social interaction. Constructivists believe that people construct the understanding or knowledge of the world through experiencing things and reflecting on those experiences (Von Glasersfeld, 1995). And the experiences are very rich in developing new knowledge. Conceptual type of knowledge development needs the learner's own experiences. Constructivism theory is a necessary condition for this study to evaluate students' conceptual knowledge of mathematics and justify it theoretically. The exploration of the conceptual kind of mathematical knowledge can be examined through the constructivist eye. The researcher uses this theory to provide more information about the conceptual type of knowledge, develop tools, examine the relationship between conceptual and procedural knowledge of mathematics, and interpret the findings.

## RESEARCH METHODOLOGY

The explanatory mixed method research design (Creswell, 2015) was applied to carry out this study. Therefore, this study meets both post-positivist and constructivist approaches. In this scenario, the research is aligned with the singular and multiple realities. For the first phase, out of N=4,458 students of 61 schools in grade eight, 360 students were randomly selected from 9 public schools of Kathmandu Metropolitan City of Kathmandu district. In the second phase, two-phase interviews, 1st with six students and 2nd with four teachers who were teaching at the same level, were conducted to understand why students develop one or both knowledge. The students were selected purposively based on three categories: high, moderate, and low mark groups (a male and a female from each group) based on their performance in the achievement test for the interview. Similarly, teachers were also selected purposively.

After collecting the data in the first phase, the researcher used the Statistical Package for Social Science (SPSS) software to draw out descriptive and inferential analyses. The statistical analysis was used to draw the conclusions. In the second phase, data were coded and generated themes from them and provided a thorough description.

For the survey, an achievement test paper was developed, from algebra section as per the nature of national curriculum, including four questions (20 marks) to measure procedural knowledge and 20 multiple choice questions to measure conceptual knowledge based on the framework developed by Rittle-Johnson and Schneider (2015) by incorporating implicit and explicit measures of CK. These are central to the definition of terms or concepts, representational knowledge, ability to link the core concept of mathematical ideas, ability to compare the quantities, ability to measure students' critical thinking, representation of verbal knowledge into the mathematical structure, ability to measure students' explanation in procedures, etc. For the PK question, the questions were developed so that students could use algorithmic procedures to solve them. The accuracy of the answers and procedures, appropriateness of strategy selection, and solution time used to solve problems are the measures to evaluate the PK (Rittle-Johnson & Schneider, 2015; Schneider & Stern, 2010) of students. For the next phase, the interview checklist was developed to compare the result of the survey. Similarly, an interview checklist and semi-structured questionnaires were developed for the interview and group discussion.

Content, criterion, and construct validity were maintained through various sources. A pilot test was conducted among 30 students and item analysis was done to find the reliability of the items. The internal consisteney (Chronebach alpha) method was used to test the reliability and it was found to be $\alpha$=0.80 representing high level of internal consistency. For the second part, four quality standards: credibility, transferability, dependability, and confirmability were maintained to establish trustworthiness (Shenton, 2004). To maintain the credibility of the qualitative data, the researcher spent a prolonged time collecting data, and qualitative data was examined through the member checking method. For transferability, the researcher used a purposive sampling procedure and a thick description of the data and findings. For conformability, the researcher used the participant's response and there was no potential biasness. The researcher assured that the researcher's bias does not skew the interpretation of what the research participants said. Finally, for dependability researcher tried to maintain the consistency of the data and findings through the audit trail method.

The norms of research were maintained as being ethically sound. Ongoing ethical issues such as participants' voice, rights, informed consent, anonymity, equity and equality, and ethical values were maintained throughout the data collection and interpretation. For administering the achivement test, the researcher physically visited the selected schools, provided the permission letter given by the university to the school principals for seeking thier permission, provided test papers to students, and collected the responses. 40 minutes was provided to attend the test.

## FINDINGS AND DISCUSSIONS

### Quantitative Findings

In the survey of 360 students, 175 were boys and 185 were girls. The following statistical hypothesis were formulated:

**Table 1.** Procedural and conceptual knowledge of students in algebra

| Indicator | N | Minimum | Maximum | Mean | SD |
|---|---|---|---|---|---|
| Procedural knowledge | 360 | 0 | 20 | 14.05 | 6.344 |
| Conceptual knowledge | 360 | 1 | 20 | 8.56 | 3.912 |

**Table 2.** Correlation coefficient between procedural and conceptual knowledge

| | | PK | CK |
|---|---|---|---|
| PK | Pearson Correlation | 1 | .559** |
| | Sig. (2-tailed) | | .000 |
| | N | 360 | 360 |
| CK | Pearson Correlation | .559** | 1 |
| | Sig. (2-tailed) | .000 | |
| | N | 360 | 360 |

Note. **Correlation is significant at the 0.05 level (2-tailed)

**Table 3.** Regression model of conceptual and procedural knowledge

| Model | | Unstandardized coefficients | | Standardized coefficients | R-square | t | Sig. |
|---|---|---|---|---|---|---|---|
| | | B | Std. Error | Beta | | | |
| 1 | (Constant) | 3.716 | .410 | | 0.313 | 9.062 | .000 |
| | Procedural knowledge | .345 | .027 | .559 | | 12.961 | .000 |

Note. Dependent variable: Student's marks in conceptual knowledge

**Table 4.** Procedural knowledge of students based on their gender

| Indicator | Gender | N | Mean | Std. deviation | t - value | Sig. (2-tailed) |
|---|---|---|---|---|---|---|
| Procedural knowledge | Boy | 175 | 14.82 | 6.266 | 2.225 | 0.027* |
| | Girl | 185 | 13.36 | 6.350 | | |

Note. t -value significant at *p>0.05

1. **Hypothesis 1:** There is no relation between procedural and conceptual knowledge of students in algebra.
2. **Hypothesis 2:** There is no significant difference between the mean marks of students in procedural knowledge in algebra based on their gender.
3. **Hypothesis 3:** There is no significant difference between the mean marks of students in conceptual knowledge in algebra based on their gender.

### *Comparision of PK and CK of students in algebra*

The comparison of PK and CK of respondents in algebra addresses the first research question of this study. **Table 1** shows the condition of respondents' procedural and conceptual knowledge in algebra.

**Table 1** shows that the mean mark of respondents in procedural knowledge is 14.05 with a standard deviation 6.344 whereas the mean mark of respondents in conceptual knowledge is 8.56 with a standard deviation 3.912. This shows a huge gap between PK and CK. In conclusion, students are good at procedural knowledge but below average in conceptual knowledge of algebra in grade eight. The standard deviations show that the marks of students in PK are more deviated from the mean mark but more consistent in CK.

### *Correlation between PK and CK of respondents*

**Table 2** shows Karl Pearson's correlation coefficient between PK and CK of students in algebra. **Table 2** shows that a moderate positive correlation (r=0.559) between procedural and conceptual knowledge of respondents. This means that the development of one kind of knowledge helps to develop another type of knowledge. For example, if students are familiar with the process of factorization of an algebraic expression; he or she may also be developing the related concept such as defining factorization, relating factors in a diagram or pictures, comparing the expression and understanding the key terms like factors, degree as well as terms, etc. Next, **Table 2** shows that the p-value 0.00 (0% approx.) is less than that of alpha value 0.005 (5%). In this condition, we conclude that there is a relationship between the two types of knowledge.

### *Linear regression between PK and CK of respondents*

Here, the researcher has analyzed the effect of procedural knowledge (independent variable) in conceptual knowledge (dependent variable) to understand how much change occurs in the increase or decrease in the procedural knowledge of respondents. **Table 3** shows the statistical p-value 0.000 which is less than alpha value 0.05. This means that the regression model can predict the outcome variable (CK) with respect to the change in the independent variable (PK). We have moderate R-square value and it is 0.313 meaning that CK can be explained about 31% by PK. The linear regression equation is CK=3.716+0.345(PK). Next, the B coefficient in this equation represents that a 1-point increase on the PK corresponds to 0.35 increase on the CK. This means that when a student gets one mark in PK, they can get a 0.35 mark in CK.

### *Comparison of PK of students based on their gender*

It is widely accepted that gender is one of the factors in the development of mathematical knowledge. **Table 4** shows the phenomenon of gender influence in the procedural knowledge of students in algebra. From **Table 4**, the p-value 0.027 (2.7% approx.)<alpha value 0.05 (5%). In this condition, we can conclude that there is a significant difference between the mean marks in procedural knowledge of respondents according to their gender. If we talk about

**Table 5.** Conceptual knowledge based on gender

| Indicator | Gender | N | Mean | Std. deviation | t - value | Sig. (2-tailed) |
|---|---|---|---|---|---|---|
| Conceptual knowledge | Boy | 175 | 8.63 | 4.087 | 0.316 | 0.752* |
| | Girl | 185 | 8.50 | 3.759 | | |

Note. t -value significant at *p>0.05

the mean value, we can see a slight difference between the mean values of boy (14.82) and girl (13.36) students. This concludes that boys are somewhat better in procedural knowledge than girls.

### *Comparison of CK of students based on their gender*

**Table 5** shows whether gender matters or not in the development of conceptual knowledge of students in algebra. **Table 5** shows that the significant p-value is 0.752 (75.2%) and it is greater than alpha value 0.05 (5%). In this situation, we conclude that there is no significant difference between the average performance of boys and girls in conceptual knowledge. This means students' conceptual knowledge is not affected by the gender of the students as they have approximately the same mean values in CK.

## Qualitative Findings

After quantitative data analysis, the researcher wanted to explain and verify whether students had lower conceptual algebra knowledge. Similarly, to figure out the reason why did such result occur in the first phase. How did it happen? Was the result true? The second phase, qualitative data collection was done to help explain and elaborate on the quantitative finding and results of the first phase in this mixed method study (Creswell, 2015). Through the interview with six students, the following things are explored.

### *Students' ability to provide examples*

The ability to consider and evaluate examples for the related concept shows one level of conceptual knowledge of that concept (Rittle-Johnson & Alibali, 1999). However, the interview with six individuals presents the fact that our students are somewhat weak in considering examples of the concept such as algebraic expression, linear equation, etc. In the question: can you provide the example of algebraic expression? One of the interviewees replied:

> I am quite good at mathematics because I normally score above 80 in the examinations of mathematics. I practice the questions given in the textbooks and practice books time and again. I sometimes read definitions. However, I usually skip examples because they are not important and have not been asked in the examinations. Also, the teachers say the same thing and tell us to do the problems. So, I cannot provide the examples of algebraic expressions.

In the same line, another participant also said, "I am not good in mathematics, and I find difficulties understanding mathematical concepts when shared by the teacher. But I learn to solve problems with the help of examples provided in the textbook."

### *Students' representational knowledge in algebra*

Another key term to measure conceptual knowledge of the learner is to see whether s/he can represent mathematical knowledge with pictures such as representing the symbolic number with pictures (Hecht, 1998). Here, in this study, questions were asked to measure students' pictorial representational knowledge in factorization and linear equations. Interviewees were asked how and why they chose the correct or the wrong option. In addition, they were asked to represent some other concept in the diagram such as the factorization of $x^2 + 7x + 12=(x + 3)\times(x + 4)$.

Among six, three selected the correct option. The first respondent used the formula to find the area of the rectangle, but he did not have the concept to represent all the factorization problems into the diagram. On the other hand, the second respondent used prior knowledge and already established knowledge such as the diagram of $(a+b)^2=(a+b)\times(a+b)$ to choose the correct option. When asked, the student expressed, "We have used this diagram in grade seven while establishing the relation $(a–b)^2=a^2-2ab+b^2$. So, I quickly remembered it and chose the correct option. But I do not know how to represent all other expressions in pictures or diagrams." In this context, the student somewhat had the conceptual knowledge as Hiebert & Lefevre (1986) asserted that CK is a knowledge that is rich in relationship and connections. Here, prior knowledge can be taken as a concept. However, the student also did not have the proper concept of factorization of the second-degree polynomial that can be represented pictorially as the area of the rectangle is equal to the product of length and breadth.

Another question was selecting the diagram of $x – y=0$. Among the interviewees (who chose the correct option), one replied that putting the numeric values of x and y on a graph gives such a picture. However, the student was unknown about other diagrams of the equations. Others were unknown about representations of these equation to the pictorial forms. It seems that students do not have a proper understanding about liner equation and its pictorial representation.

### *Students' knowledge in comparison of the expressions in algebra*

We can take an issue that most of the students in upper primary and middle school grades feel difficult to compare two distinct fractions. Many research studies have been done to claim that students are very poor in comparison of fractions such as determining the bigger among ½ and ¼ (Heemsoth & Heinze, 2014; Pantziara & Philippou, 2012; Tian & Siegler, 2017); also in comparison of decimal expression such as determining the large one among 0.25 and 0.5 (here is misconception of students that 25>5 implies that 0.25>0.5). And coming to algebra, it is more difficult as it constitutes abstract symbols and representations (variables and constants).

In comparing two algebraic expressions, the question was to compare $1/x$ and $1/(x^2)$ for any natural number x. Here, I was astonished by their responses because no one of these six respondents was able to compare these two quantities. One of the students said, "I think the expression having more exponent is always greater than the expression having less. Here, $(1/x^2)=(1/x)^2$ and $(1/x)=(1/x)^1$. Here, the power of $(1/x^2)$ is 2 and $(1/x)$ is only 1. So, $(1/x^2)>(1/x)$". It is one of the challenging situations for teachers and students to grab the concept of comparing two or more quantities. For example, in fractions, perhaps, students feel difficult to compare fractions like 1/5 and 1/25. They probably say that (1/25) is greater than (1/5) because they think that 25 is greater than that of 5. Even, students are not able to compare numerical quantities which are given into fractions. In this case, they have poor conceptual knowledge of fraction as well as fraction in algebraic expressions.

### *Students' explicit CK in algebra*

Now, in explicit measure of conceptual knowledge, I have evaluated respondents' ability to define cconceptsor terms, for instance, defining the equal sign (Knuth et al., 2006), ability to explain why the procedures work, an ability to explain why is it ok to borrow when subtracting (Fuson & Kown, 1992), and some of the major focus areas on the ability of critical thinking. First, I asked the questions about defining the factorization and then the linear equation. Most of them replied that they do not emphasize on reading definitions, do not spend time in conceptualizing the definition of core ideas, terms, etc. Two of the participants replied that factorization is the process of conveying the expression into the product of its two or more factors. They created this definition with the help of examples done in the procedural knowledge questions. On the other hand, students do not know what a linear equation is. They understand what the figure of a line is, but they do not know about the equation of a line. This implies that they are not more familiar with the definition and core concept of linear equations. So, they have somewhat weak conceptual knowledge of the linear equation.

I had asked another question to evaluate their ability about why a procedure work. Four of them were able to explain the process. This means that students are quite familiar with steps and they somewhat know how to perform these action sequences while solving a problem. This quite seems that the ability to action sequences to solve problems comes under procedural knowledge (Rittle-Johnson & Alibali, 1999).

However, most of the students are quite familiar with how to do them and their explanations. The other participants who selected the wrong option did not understand how those procedures work. At last, I had asked a verbal problem to evaluate respondents' ability to think critically and change the verbal problem into mathematical sentences. For this, question was asked to select the correct linear equation when Jiya is five years older than Rina. In this situation also, students felt difficulty when they were asked verbal problems to express in a mathematical sentence. Another group of students who chose the wrong option expressed that older means multiply and younger means divide. I requested them to give one example from their daily life, but they were unable to create a verbal problem and felt it difficult to relate to their living life. One of them asked,

> Sir, can we relate these mathematics concepts to our day-to-day life?

I then replied,

> Yes, we can. For instance, your dad is 25 years older than you. That is, suppose your dad is 39 (asking with her) now and your age is 14, then tell me how we can do with 25 to your age to reach to your dad's age?

She certainly replied,

> Sir, we should add up 25 years to reach my dad's age.

I asked.

> Did you get that how we can relate mathematics to our living life?

She then replied,

> Yes, sir a little bit.

All these explanations indicate that students of grade eight are weak to the most extent in the conceptual knowledge of algebra. They are weak in defining the core ideas, giving the definition of the concept, generating examples, expressing abstract knowledge through pictures and diagrams, comparing the quantities of algebraic expressions. These findings force us to conclude the fact that students have a very lower level of understanding of core ideas in comparison to the procedural knowledge of algebra. Similarly, our middle schools students are weak in critical thinking and meaning-making.

### *Findings and discussion from second phase interview*

The second phase interview revealed and tried to articulate why students are weak in conceptual knowledge. The interview with six participant students (three-higher marks achiever, three-lower marks achiever) had explored some interesting findings.

One of the students, Rashmi, from the high marks achiever group, expressed her focus while learning algebra:

> Sir, most of the contents in algebra are abstract in nature, such as long division, indices, linear equations, and verbal problems. So, I do learn these concepts with the help of formulas or rules and sometimes reading definitions. But I know how to perform procedures when solving problems. I regularly get help from my teachers in/outside the classroom, and I look for solutions in the guidebooks or on the net. This is how I learn algebra.

However, students from low marks achiever group had to say different things:

> I am not good in mathematics. Last year, I hardly passed the exam but failed in mathematics. However, I would like to learn mathematics by using step by step approach more and concepts very little. I try to learn solutions. I do one or two steps and I forget again. I forget to do the steps all the time. The teacher said that I needed to memorize the formula and steps to make it error-free. But what to do I don't enjoy memorizing things. Mostly, teachers emphasize so-called talented students. These students ask questions with teachers, but we do not get chance or are afraid of asking questions because we have fears of teachers.

From the above two responses, students, to the most extent, have a similar approach to learning algebra. The focus is on performing technical steps, using a static formulas, and memorizing them. They, however, understand some of the conceptual portions, but not properly. Getting support from teachers, students who are good at mathematics usually get support from teachers. Nevertheless, it is a bitter truth that most students who are considered to be do not get sufficient support or are usually neglected. This scenario is a vast inequality among two classes (high and low performing) of students in our education system.

On the discussion of the usage of discussion, project, collaborative activities, practical works, and materials while learning algebra and other contents of mathematics, both groups of students expressed the same thing. One of them expressed:

> Project work, practical tasks, and collaborative discussion in the class? No, sir. I did not experience such tasks while learning. In terms of project work, teachers sometimes tell us to bring chart papers by writing formulae on itthemIs that a project work, sir? Except these, we did not get a chance to go outside and explore mathematics. We sit inside the classroom, the teacher writes on the board, and we copy them.

This is a sad story of our pedagogical context. The progressive and emerging pedagogical approaches have not been explored and implemented. These approaches such as project-based learning, activity-based learning, inquiry-based learning, story-telling approach, etc., are not in the access. However, these days, schools and teachers are started implementing these practices. These approaches are efficient in developing a conceptual understanding of mathematics. In implementing these approaches, students get direct experience of mathematical ideas from others, especially from the context (the world they live in).

### *Findings and discussion from group discussion with teachers*

Next, an extensive discussion with four teachers who are teaching at the same level came to outstanding findings. The teacher expressed that they want their students to develop PK as well as CK. However, the knowledge construction process depends on the nature of the contents. They said that the abstract nature of algebra makes learning difficult; it is better we use rules, formulae, and steps to solve problems. For instance, solving problems of indices, polynomial equations, and long divisions. They also expressed that teachers' structional approaches can be a key factor in developing CK of algebra. They agreed that the usage of progressive methodologies in teaching mathematics help students construct knowledge conceptually. When implementing them, there are some constraints. Regarding this, one of them expressed:

> The abstract nature of contents is often difficult to use the progressive methodologies inside and outside the classroom. These need a great amount of time, extensive resources, and skills. However, the limited available resources and insufficient skills in developing the tasks, activities, and materials are complex. Next, the content-loaded curriculum is another problem that is limited in providing space for these practices. Moreover, our interest matters. Usually, we are not ready to implement them. We think, 'who cares!' Also, the large number of students inside the classroom is an issue to use emerging methodologies in teaching and learning.

The usual problems in pedagogical contexts make things challenging to implement innovative methods in teaching. Also, the traditional mindset of teachers that prevents the innovative changes in developing conceptual knowledge of mathematics. Teachers seem comfortable with the one-size-fits-all (Luitel, 2009) method of teaching. The conventional frame of references such as teachers are the source of knowledge, mathematics needs practice and memorization to learn, answers are important, etc., are promoting procedural ways of knowing. Therefore, teachers are not able to face the innovative changes in education.

Next, the participant teachers agreed that real-life examples could enhance a deeper understanding of mathematics. However, they felt that creating examples was a challenging task. One of the teachers expressed:

> We have heterogeneous and diverse students in the classroom. Creating real-world examples to represent each student's cultures and their living world is a humongous challenge. I sometimes try to use real-life examples when I teach some basic concepts about variables and constants. For example, the amount of water consumption in a daily basis (a variable) and time in hours (constant). Students try to construct and relate them to their practices. These examples help them construct conceptual knowing in mathematics. However, the job is tough.

The construction of real-world examples and implementation in teaching promotes a culturally contextualized nature in mathematics (Pant et al., 2020). These examples can enhance students' procedural and conceptual knowledge in mathematics because they can relate their learning of mathematics to their own living life.

In addition, the use of concrete materials in algebra teaching at the basic school level is a great way of developing CK. One of the materials I use is algebra tiles. These 3D materials can be used to construct conceptual knowledge of algebra. Algebra tiles to develop the fundamental concepts through a representation of objects.

Similarly, the concept of addition, subtraction, division, multiplication of algebraic expressions up to degree two, factorization of algebraic expression having degree two, solving the linear and quadratic equations can be taught and learned through these awesome materials.

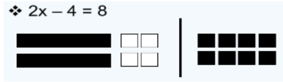
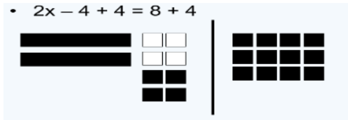
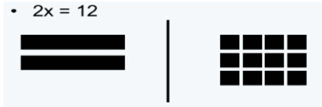
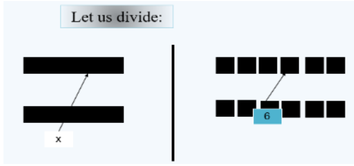
Here is one example shown in **Table 6** for solving a linear equation, let us denote white color as negative (-) and black color as positive (+). The area of the rectangle having length x and breadth 1 is x (black) and –x (white), and unit square 1 (black) and -1 (white).

The tiles should be used in the foundation phase, and later, students should be presented with problems. This way, algebra tiles can be one of the effective materials to learn algebra conceptually. Similarly, other materials such as cardboard papers, etc. can be used in algebra teaching and learning.

The use of project work and other practical tasks are essential in developing conceptual knowledge in algebra. Participant teachers agreed that involving students in exploring the outer world and learning algebra is effective and essential to learning mathematics effectively. These methods help in building creative reasoning, critical thinking, communication, creativity, collaboration, and cooperation (Goodman & Stivers, 2010; Viro et al., 2020). Including these, students have a plethora of opportunities to develop other 21st century skills. However, developing tasks, managing resources, and implementing them are the usual problems teachers are facing.

In the thorough discussion with teachers, we found that teachers want their students to develop both CK and PK in mathematics using the emerging and innovative pedagogical approaches in teaching and learning because the instructional practices matter in knowledge creation, the real-life examples used by teachers and created by students can enhance a deeper understanding of contents presented, however, the job is challenging, and the use of teaching-learning materials are the great resources to develop CK.

**Table 6.** Example for solving a linear equation

| Step | Operation |
|---|---|
| First, set the algebra tiles model as shown in the figure. |  |
| Next, to make 2x alone, add 4 positive unit tiles to both sides. |  |
| Now, take away 4 negative and positive tiles from the left side. Then, we get. |  |
| Let us divide, at last we get 6 equal units for 2 same x. In the diagram, the equal each x has equal 6 units in the right-hand side. Hence, x = 6. |  |

## CONCLUSIONS

The quantitative findings show the lower level of CK as compared to PK with the moderate positive correlation and positive dependency of CK on PK as per the regression model. This means that students are good at procedural knowledge, but CK is dependent upon PK. Thus, our practices should focus on conceptual knowledge development.

The qualitative findings show that students were influenced and encouraged by textbooks, teachers, and learning activities to develop procedural skills, memorize formulae, and solve problems. Most of them felt mathematics as a difficult subject in comparison to others. They did not see the practical use of mathematics in their living life. They spend more time learning procedures rather than understanding the underlying concepts. They did not have a critical discussion related to mathematics inside and outside the classroom. Most of them did not get the equal opportunity of learning inside the classroom, and psychologically, they feel inferior when teachers discriminate against them and support talented students. In this context, students seem to study just to pass the examination. As a result, most of them are trying to learn and memorize steps to leave the conceptual part of learning.

On the other hand, it was found that teachers wanted their students to learn both concepts and procedures of mathematics learning. So, emphasis has been given to both types of knowledge construction. Because of lack of well-trained teachers, poor management of teacher training and professional development programs, a heavy number of contents, large number of students, etc., affect the implementation of students-cantered approaches. The abstract nature of Algebra forces teachers to teach to construct PK with the minimum focus on the underlying concepts. Teachers are forced to implement lectures and other conventional methods. In this situation, teachers are in a rush to finish the course on time. Project-based learning, fieldwork, and practical works are considered the weapons/mediums of constructing an authentic understanding of mathematics. It was found that students can learn mathematics concepts when they play with problems related to their day-to-day life using more contextual examples. However, in the context of Nepal, it is very challenging to create issues and examples representing each community practice of students. Another factor affecting CK in algebra is the appropriate use of teaching materials and manipulatives, as it needs a greater number of concrete materials. The use of technology seems effective for CK, but teachers take it a difficult task to construct manipulatives for each concept and use it inside the classroom. Therefore, teachers are forced to encourage students to learn steps and procedures to solve problems in algebra.

Procedural and conceptual knowledge is the central focus in teaching and learning mathematics throughout the world. This is a debate among the people from the early 80's to till date about which is essential, which we should give more emphasis, and the discussion of the relationship between these two types of knowledge and so do in the context of Nepal. Algebra is a part of our mathematics curriculum from the primary grades to university level. In the teaching and learning context, the curriculum also emphasizes learning algebra in all grades. Middle grades in schooling are considered the backbone of knowledge construction in Nepal. Students get a chance to reshape their mathematical knowledge and skills in these grades. The curriculum has emphasized both types of knowledge construction in mathematics. Finally, the knowledge construction process differs from person to person. Some people can learn mathematics with the help of step-by-step algorithmic procedures, and some can generate mathematical knowledge with a deep understanding of the concepts. However, each type of knowledge depends upon how an individual constructs knowledge, past experiences as prior knowledge, the environment of learning mathematics, etc. However, they are dependent on each other. So, learning is an iterative process of developing procedural and conceptual knowledge by emphasizing CK.

### Limitations and Recommendations

This research was limited to measure the level of CK and PK of students of grade eight in algebra and sample was drawn from only Kathmandu Metropolitan City of Nepal. Thus, the further studies can be down by broadening the research area and including the other grades or entire school level and other chapters of mathematics. The study emphasized only a few areas of PK and CK (see methodology and other sections). The future study can be conducted to include various and emerging dimensions related to PK and CK. The explanatory mixed method design was one of the limitations of this study because the voice of students, teachers, and other stakeholders was not included. So, the future researcher can conduct studies by using more qualitative research methods.

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## REFERENCES

Abd Rahman, Z. (2006). *Mastery of algebraic concepts and attitudes towards algebra among secondary school students* [Unpublished master of education project report]. Universiti Kebangsaan Malaysia.

Barr, C., Doyle, M., Clifford, J., De Leo, T., & Dubeau, C. (2003). There is more to math: A framework for learning and math instruction. *Waterloo Catholic District School Board*.

Brooks, J. A., & Freeman, J. B. (2018). Conceptual knowledge predicts the representational structure of facial emotion perception. *Nature Human Behaviour, 2*(8), 581-591. https://doi.org/10.1038/s41562-018-0376-6

Byrnes, J. P., & Wasik, B. A. (1991). Role of conceptual knowledge in mathematical procedural learning. *Developmental Psychology, 27*(5), 777-786. https://doi.org//10.1037/0012-1649.27.5.777

Creswell, J. W. (2015). *Research design: Qualitative, quantitative, and mixed method approaches*. Pearson.

Education Review Office (ERO). (2015). *Report on national assessment of student achievement (NASA) 2013*. The Author.

Education Review Office (ERO). (2017). *Report on national assessment of student achievement (NASA) 2017*. The Author.

Education Review Office (ERO). (2019). *Report on national assessment of student achievement (NASA) 2018*. The Author.

Fuson, K. C., & Kwon, Y. (1992). Korean children's understanding of multi-digit addition and subtraction. *Child Development, 63*(2), 491-506. https://doi.org/10.1111/j.1467-8624.1992.tb01642.x

Ghazali, N. H., & Zakaria, E. (2011). Students' procedural and conceptual understanding of mathematics. *Australian Journal of Basic and Applied Science, 5(7)*, 684-691.

Goodman, B., & Stivers, J. (2010). Project-based learning. *Educational Psychology*. https://www.fsmilitary.org/pdf/Project_Based_Learning.pdf

Hecht, S. A. (1998). Toward an information-processing account of individual differences in fraction skills. *Journal of Educational Psychology, 90*(3), 545-559. https://doi.org/10.1037/0022-0663.90.3.545

Heemsoth, T., & Heinze, A. (2014). The impact of incorrect examples on learning fractions: A field experiment with 6th grade students. *Instructional Science, 42*(4), 639-657. https://doi.org/10.1007/s11251-013-9302-5

Hiebert, J., & LeFevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 1-27). Lawrence Erlbaum Associates, Inc.

Klein, D. (2007). A quarter century of US 'math wars' and political partisanship. *Journal of the British Society for the History of Mathematics, 22*(1), 22-33. https://doi.org/10.1080/17498430601148762

Knuth, E. J., Stephens, A. C., McNeil, N. M., & Alibali, M. W. (2006). Does understanding the equal sign matter? Evidence from solving equations. *Journal for Research in Mathematics Education, 37*(4), 297-312.

Lenz, K., Dreher, A., Holzäpfel, L., & Wittmann, G. (2020). Are conceptual knowledge and procedural knowledge empirically separable? The case of fractions. *British Journal of Educational Psychology, 90*(3), 809-829. https://doi.org/10.1111/bjep.12333

Luitel, B. C. (2009). *Culture, worldview and transformative philosophy of mathematics education in Nepal: A cultural-philosophical inquiry* [Unpublished doctoral dissertation]. Curtin University of Technology.

McCormick, R. (1997). Conceptual and procedural knowledge. *International Journal of Technology and Design Education, 7*(1), 141-159. https://doi.org/10.1023/A:1008819912213

Pant, B. P., Luitel, B. C., & Shrestha, I. M. (2020). Incorporating STEAM pedagogy in mathematics education. In *Proceedings of Episteme 8 International Conference to Review Research in in Science, Technology and Mathematics Education, January 3-6*, 2020 (pp. 319-326). Homi Bhabha Centre for Science Education, Tata Institute of Fundamental Research, Mumbai, India.

Pantziara, M., & Philippou, G. (2012). Levels of students' "conception" of fractions. *Educational Studies in Mathematics, 79*(1), 61-83. https://doi.org/10.1007/s10649-011-9338-x

Rittle-Hohnson, B., & Siegler, R. S. (1998). The relations between conceptual and procedural knowledge in learning mathematics: A review. In C. Donlan (Ed.), *The development of mathematical skill* (pp. 75-110). Phychology Press. https://doi.org/10.4324/9781315784755-6

Rittle-Johnson, B. (2019). Iterative development of conceptual and procedural knowledge in mathematics learning and instruction. In J. Dunlosky, & K. Rawson (Eds.), *The Cambridge handbook of cognition and education* (pp. 124-147). Cambridge University Press. https://doi.org/10.1017/9781108235631.007

Rittle-Johnson, B., & Alibali, M. W. (1999). Conceptual and procedural knowledge of mathematics: Does one lead to the other? *Journal of Educational Psychology, 91*(1), 175-189. https://doi.org/10.1037/0022-0663.91.1.175

Rittle-Johnson, B., & Schneider, M. (2015). Developing conceptual and procedural knowledge in mathematics. In R. Cohen Kadosh, & A. Dowker (Eds.), *Oxford handbook of numerical cognition* (pp. 1102-1118). Oxford University Press. https://doi.org/10.1093/oxfordhb/9780199642342.013.014

Rittle-Johnson, B., Fyfe, E. R., & Loehr, A. M. (2016). Improving conceptual and procedural knowledge: The impact of instructional content within a mathematics lesson. *British Journal of Educational Psychology*, *86*(4), 576-591. https://doi.org/10.1111/bjep.12124

Rittle-Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology, 93*, 346-–362. https://doi.org/10.1037//0022-0663.93.2.346

Ross, A. A. (2010). *The effects of constructivist teaching approaches on middle school students' algebraic understanding* [Doctoral thesis]. Texas A & M University. https://core.ac.uk/download/pdf/147133472.pdf

Shenton, A. K. (2004). Strategies for ensuring trustworthiness in qualitative research projects. *Education for Information*, *22*(2), 63-75. https://doi.org/10.3233/EFI-2004-22201

Star, J. R. (2005). Re-conceptualizing procedural knowledge. *Journal for Research in Mathematics Education*, *36*(5), 404-411. https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.544.4487&rep=rep1&type=pdf

Tian, J., & Siegler, R. S. (2017). Fractions learning in children with mathematics difficulties. *Journal of Learning Disabilities, 50*(6), 614-620. https://doi.org/10.1177/0022219416662032

Viro, E., Lehtonen, D., Joutsenlahti, J., & Tahvanainen, V. (2020). Teachers' perspectives on project-based learning in mathematics and science. *European Journal of Science and Mathematics Education, 8*(1), 12-31. https://doi.org/10.30935/scimath/9544

Von Glasersfeld, E. (1995). *Radical constructivism: A way of knowing and learning.* Falmer.

Yilmaz, K. (2011). The cognitive perspective on learning: Its theoretical underpinnings and implications for classroom practices. *The Clearing House: A Journal of Educational Strategies, Issues and Ideas, 84*(5), 204-212. https://doi.org/10.1080/00098655.2011.568989

Zuya, Z. H. (2017). Prospective teachers' conceptual and procedural knowledge in mathematics: The case of algebra. *American Journal of Educational Research, 5*(3), 310-315. https://doi.org/10.12691/education-5-3-12