

Conceptual understanding of fractions, decimals, and percentages

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ABSTRACT

This study presents an analysis of a qualitative study that analyzed the conceptual understanding of fractions, decimals, percentages, and their relationships among students in the final grades of primary and secondary schools aged 15 to 19. Based on the knowledge assessment test and interviews conducted, a broader insight into students' conceptual understanding of fractions, decimals, and percentages was obtained. This research was conducted in the Sarajevo Canton in accordance with the mathematics curriculum in Sarajevo. Based on the questions asked and the answers received, information is obtained about their way of thinking about percentages, decimals, and fractions. Also, during the interviews, certain difficulties in understanding fractions, decimals, and percentages were observed in many students. The results of the analysis indicate that greater effort is needed in mastering fractions, decimals, and percentages, both by students and teachers, who need to pay more attention to the conceptual understanding of mathematics in order to improve it and try to raise it to a higher level.

Keywords: conceptual understanding, decimal numbers, fractions, percentages, students

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INTRODUCTION

Mathematics is the foundation of many scientific disciplines and everyday activities, and the development of mathematical abilities plays a key role in education. Learning mathematics completes, explains, proves and analyzes procedures and concepts, and through its study it fosters logical thinking and reasoning and contributes to the intellectual development of students (National Council of Teachers of Mathematics [NCTM], 2000, p. 83). Learning mathematics is based on the fact that mathematics is a subject in which knowledge is continuously built upon, which means that more complex knowledge is built upon the basic knowledge that an individual has already acquired or is expected to acquire (Žakelj & Klančar, 2024).

Knowledge serves to help us to understand certain concepts and facts that can be discussed. According to Davenport and Prusak (1998), knowledge implies an understanding of facts about a certain topic that is acquired through experience and education. Poljak (1984, p. 3) defines knowledge as “a system or logical overview of facts and generalizations about objective reality that a person has adopted and permanently retained in his consciousness.” Knowledge is the key to shaping each individual and society as a whole. Knowledge is acquired through learning and

experience, and experience is the basis of progress and development in general. “Knowledge learned through understanding provides the basis for creating new knowledge and for solving new and unknown problems” (Karega, 2018, p. 23).

Bloom's revised taxonomy contains four categories of knowledge (Krathwohl, 2002, p. 214) shown in **Table 1**.

Clark (2011) considers concepts to be the most useful and powerful cognitive tools available to humans, as concepts have the “ability” to organize and connect. A concept is an idea that is well understood enough to allow other ideas to be connected to it and become part of a network of understanding, and such connections and networks often lead to the formation of conceptual knowledge (Hurrell, 2021). Rittle-Johnson et al. (2016) consider conceptual knowledge to be a network of interconnected ideas that allows students to connect prior knowledge to newly emerging concepts.

However, classroom practice has shown that without knowledge of problem-solving procedures, students cannot have conceptual knowledge. Conceptual and procedural knowledge have a two-way relationship, but they are separate entities that contribute to success with fractions, decimals, and percentages (Rittle-Johnson & Alibali, 1999). Conceptual knowledge is defined as knowledge about

Table 1. Overview of the knowledge dimension structure in the revised taxonomy (the structure of knowledge dimensions in the revised Bloom's taxonomy)

Knowledge	Sub-knowledge
Factual knowledge – Basic facts that the student must know in order to be familiar with the subject or problem to be solved.	1. Knowledge of terminology 2. Knowledge of specific details and elements
Conceptual knowledge – The mutual relationship between fundamental elements within a wider structure that enables joint action.	1. Knowledge of classifications and categories 2. Knowledge of principles and generalization 3. Knowledge of theories, models and structures
Procedural knowledge – How to do something, research methods and criteria for using skills, algorithms, techniques and methods.	1. Knowledge of specific skills and algorithms 2. Knowledge of specific techniques and methods 3. Knowledge of criteria for deciding when to apply appropriate procedures
Metacognitive knowledge – Knowledge about cognition as a whole as well as awareness of knowledge within the framework of one's own cognition.	1. Strategic knowledge 2. Knowledge of cognitive tasks, including appropriate contextual and conditional knowledge. 3. Self-knowledge

concepts and principles that are important in a particular field (Rittle-Johnson & Schneider, 2015) and knowledge about why a mathematical procedure works (Crooks & Alibali, 2014). Hiebert and Lefevre (1986) explain the concept of conceptual knowledge as a form of knowledge that includes understanding concepts and principles, as well as the interrelationships between them. To demonstrate conceptual understanding, a student must be able to explain why a mathematical statement is true or the origin of a mathematical rule (Zuya, 2017). Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM, 2000, p. 11).

From the interviews with the respondents, it was learned that some students understand the concept of part from the whole well both in everyday life and through the application of fractions, decimal numbers and percentages in tasks, while some students cannot visualize these relationships, and this may be a consequence of different learning experiences. While certain students show above-average conceptual understanding, other students, while showing knowledge, do not understand the concepts of certain quantities and show below-average understanding of fractions, decimals, and percentages.

Through the solutions to the interview tasks, insight was gained into how students think about and evaluate the concepts of percentages, fractions, and decimals, and how they explain their actions to the solution. During the interview, insufficient understanding of the relationship between decimals, fractions, and percentages was observed, as well as difficulties in expressing themselves about given quantities.

METHODOLOGY

Research Objective

The aim of this paper is to provide insight into the conceptual understanding of elementary and secondary school students in decimals, fractions and percentages based on an assessment test and interviews. It also aims to explore the difficulties students encounter when solving

problems with decimals, fractions and percentages, and how they think during problem solving.

Sample and Method of Data Collection

This study included 131 participants, students in the final grades of primary and secondary school—comprising 61 (46.56%) females and 70 (63.44%) males. The average age was 17.46 years for female participants and 17.13 years for male participants. The students were tested and interviewed following a random selection process across 15 randomly chosen schools in the Canton of Sarajevo. Data were collected using a test constructed from 6 items involving percentages, fractions, and decimals. The items were taken from or modified from the work *Number sense in school mathematics: Student performance in four countries* by McIntosh et al. (1997). The test was approved by 5 university professors specializing in education—with an average of 19.4 years of experience in mathematics teaching—as well as by the relevant Ministry of Education of the Canton of Sarajevo.

After the test, each student went through a structured interview consisting of the following questions, based on which student's conceptual understanding was analyzed:

- How did you come to the solution? Can you explain your answer?
- How did you think and why did you approach the task in that way?
- Could you do the task in a different way?

Below is the test that was used to recruit students in the final grade of elementary school and the final grade of high school.

Task 1: Round all the given fractions that are greater than $\frac{3}{4}$, but at the same time less than 1: $\frac{2}{3}$, $\frac{5}{8}$, $\frac{4}{5}$, $\frac{7}{10}$, and $\frac{4}{3}$.

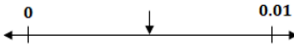
Task 2: How many different decimal numbers are there between 1.34 and 1.35? Circle the letter in front of your answer, then fill in the blank with your answer.

- A. There is none. Why?
- B. Only one. Write which.
- C. Several. Specify how many.
- D. Many. Name at least two: _____ and _____.

Table 2. Percentage presentation of the tasks on the test (number of correctly/incorrectly completed test items from tested subjects [$n = 131$])

	True	False	Blank
Task 1	42 (32.06%)	80 (61.07%)	9 (6.87%)
Task 2	54 (41.22%)	63 (48.09%)	14 (10.69%)
Task 3	73 (55.73%)	48 (36.64%)	10 (7.63%)
Task 4	49 (37.40%)	68 (51.91%)	14 (10.69%)
Task 5	54 (41.22%)	74 (56.49%)	3 (2.29%)
Task 6	54 (41.22%)	72 (54.96%)	5 (3.82%)

Task 3: What decimal number is shown by the arrow on the number line?



Task 4: Arrange the following numbers in order of magnitude: 0.494 , $\frac{4}{5}$, 51% , 0.4 , and 40.5% .

Task 5: The price of the article has been increased from 50 KM to 60 KM. What is the percentage increase? Circle your answer.

- A. 10%
- B. 20%
- C. 25%
- D. 15%

Task 6: Without calculation, answer whether is $\frac{14}{29} \cdot \frac{6}{13}$:

- A. Greater than $\frac{1}{4}$
- B. Less than $\frac{1}{4}$
- C. Equal to $\frac{1}{4}$

RESEARCH RESULTS

The following presents the doability of all tasks among the respondents (Table 2).

The following part of the article presents a sample of correct/incorrect solutions to each of the tasks set and a detailed analysis of the interviews conducted.

By completing task 1 (Figure 1), which refers to comparing fractions by size, students showed an average level of knowledge. Fractions are first mentioned in elementary school in the fourth grade (Ministry of Education of the Sarajevo Canton, 2024, p. 17), and based on the number of years of study, students did not show significant results. Out of 131 tested and interviewed respondents, 42 (32.06%) students correctly completed this task, of which 4 (3.05%) students came to the correct solution during the conversation, leading them to the correct result. The interview shows how students think and connect the relationships between the numerator and denominator. Many students gave the wrong solution

because they view the numerator and denominator separately as a whole number, which is a big mistake, and it is known that the numerator and denominator make up one value. As a result of insufficient understanding of fractions, students have problems when solving tasks with fractions, especially when fractions need to be compared. In a series of incorrect answers, one student gave an interesting explanation to the question of what a fraction is: "A fraction is a number on top, a number on the bottom, with a line between them."

As is known, in primary and secondary schools, teaching is based on rules, or procedures, where fractions can be compared in size and placed on a number line, which are important steps in procedural school learning. However, success in teaching cannot be achieved without strong conceptual understanding (Joutsenlahti & Perkkilä, 2024). This leads to the conclusion that without procedures and actions, there is no understanding of concepts, and therefore no long-term results. Conceptual understanding of fractions includes knowledge of the basic characteristics of fractions: the relationship between the numerator and denominator and different representations of fractions.

Task 1 was done incorrectly by 80 (61.07%) students, while 9 (6.87%) students did not even try to do the task because it was too difficult for them or they did not understand what was being asked at all.

A number of students, 10 (7.63%) of them rounded the fractions $\frac{5}{8}$, $\frac{4}{5}$, and $\frac{7}{10}$ in their answer. When asked why they chose this option, they answered that rounded fractions are less than 1 because the numerators are smaller than the denominator. When asked why these fractions are greater than $\frac{3}{4}$, the explanation was as follows: $\frac{3}{4} < \frac{5}{8}$; $\frac{3}{4} < \frac{4}{5}$; $\frac{3}{4} < \frac{7}{10}$.

In the example of task 1, it is observed that students compare the numerator of one fraction with the numerator of the second fraction and the denominator of the first fraction with the denominator of the second fraction, which represents misunderstanding and ignorance of the relationship between the numerator and the denominator. It has been observed that students rely on the concept of natural numbers when making inferences about fractions. This phenomenon is called natural number bias (Ni & Zhou, 2005).

Another explanation for this solution was presented by students who chose these fractions because of the denominator. Some students argue that the larger the denominator, the larger the fraction, while others argue that the larger the denominator, the smaller the fraction. The error in comparing whole numbers occurs when

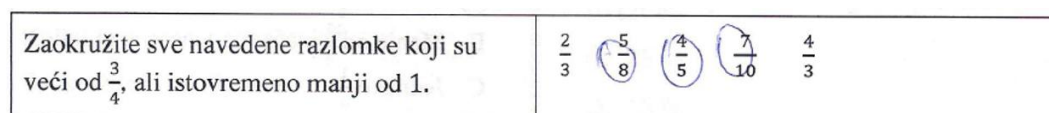


Figure 1. Example of an incorrectly completed task 1 by a research participant (taken from students' test scripts)

students compare fractions based on their numerator or denominator, without clearly considering the relationship between the numerator and denominator (Malone & Fuchs, 2017).

One student circled these three fractions with the following explanation:

$$\left. \begin{array}{l} \frac{3}{4} = 3 \cdot 4 = 12 \\ \frac{5}{8} = 5 \cdot 8 = 40 \\ \frac{4}{5} = 4 \cdot 5 = 20 \\ \frac{7}{10} = 7 \cdot 10 = 70 \end{array} \right\} \text{ So, they are all greater than } \frac{3}{4} \text{ and at the same time less than 1.}$$

Of the answers offered, 11 (8.4%) students circled the second and third fractions $\frac{5}{8}$ and $\frac{4}{5}$ as the solution. Therefore, the explanation for this solution is that “the larger the number, the greater the value.” Some students only looked at the denominators with the explanation “if the denominators are larger, then the fractions are also larger.” The fraction $\frac{7}{10}$ does not belong to the solution because the denominator is not a single-digit number, even though it is larger than 4.

Of the tested students, 3 (2.29%) students chose the first four fractions $\frac{2}{3}$, $\frac{5}{8}$, $\frac{4}{5}$ and $\frac{7}{10}$ as the solution because they are less than 1, i.e., the numerator is less than the denominator. The fifth fraction was not rounded because $\frac{4}{3} = 1\frac{1}{3}$ and it can be seen that it is greater than 1. The students pointed out that they forgot to calculate the exact value of the fractions so they could not determine which of the rounded fractions were greater than $\frac{3}{4}$.

It is interesting that 12 (9.16%) students chose only the first fraction $\frac{2}{3}$ as the correct answer stating that it lack 1 in the numerator by one whole, because the fraction $\frac{3}{4}$ also lacks 1 in the numerator by one whole.

A total of 5 (3.82%) students circled the fifth fraction $\frac{4}{3}$ as a solution, explaining that $\frac{4}{3}$ is the inverse of the fraction $\frac{3}{4}$, and they point out that the fraction $\frac{4}{3}$ is not 1 but close to 1, and 4 is a larger number and cannot be divided by 3.

Also, 5 (3.82%) students circled the second and fourth fractions $\frac{5}{8}$ and $\frac{7}{10}$ as a solution, explaining that they are less than 1 and greater than $\frac{3}{4}$ because the denominator is greater than the numerator by 3, while $\frac{4}{5}$ is not greater than $\frac{3}{4}$ because the numerator and denominator differ by 1.

Two (1.53%) students state that none of the offered fractions meets the set conditions, pointing out: “It can’t be, there are none, because $1 < \frac{3}{4}$.”

A total of 7 (5.34%) students circled the first and fifth fractions $\frac{2}{3}$ and $\frac{4}{3}$ as an answer because both fractions have the number 3 in the denominator because they think they are greater than $\frac{3}{4}$.

A smaller number of students, 2 (1.53%) of them rounded the second, third and fifth fractions $\frac{5}{8}$, $\frac{4}{5}$ and $\frac{4}{3}$ as an answer.

They chose these fractions because they believe they are approximately equal to $\frac{3}{4}$.

Of those tested, 6 (4.58%) students rounded the first and third fractions $\frac{2}{3}$ and $\frac{4}{5}$. These students converted fractions to decimals and rounded as they calculated.

The first, third and fifth fractions $\frac{2}{3}$, $\frac{4}{5}$, $\frac{4}{3}$ were circled by 3 (2.29%) students as an answer with the explanation that each of these fractions contains the number 3 or 4 in the numerator or denominator.

Two (1.53%) students circled the third and fifth fractions $\frac{4}{5}$, $\frac{4}{3}$ as an answer with the explanation that the numerators are equal to 4 and therefore are greater than $\frac{3}{4}$ and less than 1.

A certain number of students, 12 (9.16%) of them randomly rounded fractions without any specific explanations. Mostly, these are their assumptions and opinions that it should have been rounded that way. A total of 2 (1.53%) students rounded the second and fifth fractions $\frac{5}{8}$ and $\frac{4}{3}$, also 3 (2.29%) students rounded the third and fourth fractions $\frac{4}{5}$ and $\frac{7}{10}$ as a solution. One student (0.76%) rounded the fourth fraction $\frac{7}{10}$ for an unknown reason. A total of 2 (1.53%) students rounded the first and second fractions $\frac{2}{3}$, $\frac{5}{8}$ as a solution. One student (0.76%) rounded the first, second and third fractions $\frac{2}{3}$, $\frac{5}{8}$ and $\frac{4}{5}$, while 2 (1.53%) students rounded only the second fraction $\frac{5}{8}$. Also, only 1 (0.76%) student rounded the first, second and fourth fractions $\frac{2}{3}$, $\frac{5}{8}$ and $\frac{7}{10}$.

Through the interviews, it was observed that the students more easily recognize the relationship between the part and the whole, e.g., half or third of a whole size, compared to fractions that are larger than one whole. This is probably a consequence of the fact that first in school the parts of the whole are taught through practical examples, e.g., part of an apple.

Students who failed to do this task correctly attributed their ignorance to poor teacher explanations. This problem was investigated by Fuchs et al. (20132) and their study showed that better explanations in class increase conceptual understanding of fractions.

Task 2 requires estimating how many numbers there are between two decimal numbers. Different results and explanations were obtained for the four answers offered. In the fifth grade of elementary school, decimal numbers are introduced into teaching through reading and writing decimal numbers with one decimal place and displaying decimal numbers using number words and/or methods, and through performing arithmetic operations with decimal numbers (Ministry of Education of the Sarajevo Canton, 2024, p. 21). Out of the tested 131 respondents, 54 (41.22%) students did this task correctly with a good understanding and explanation of the solution to the task. One gets the impression that these are students who love mathematics or are gifted in mathematics. Two (1.53%) students got their

Koliko različitih decimalnih brojeva ima između 1.34 i 1.35 ? Zaokružite slovo ispred odgovora, a zatim popunite prazno polje kod svog odgovora.	A) Nema niti jedan. Zašto?
	B) Samo jedan. Napišite koji:
	☒ C) Nekoliko. Navedite koliko:
	☒ D) Mnogo. Navedite barem dva:

~~9~~ (1,341, 1,342, ..., 1,349)
 1,341 i 1,349

Figure 2. Example of a well-done task 2 after an interview with a research participant (taken from students' test scripts)

way to the right answer by leading them to the solution in a reasonable way during the interview. 14 (10.69%) students did not try to do this task, pointing out that they did not understand what was required in the task, and some found it difficult to read the text of the task. Some students give up on tasks with too much text before they even read the task. Task 2 was completed incorrectly by 63 (48.09%) students.

43 (32.82%) students chose the answer that there are no decimal numbers between the numbers 1.34 and 1.35. The explanations for this answer are different:

- There are no numbers in between because 1.34 and 1.35 are close, because they are predecessor and successor.
- 1.34 is immediately followed by 1.35. Number after number, not a single number in between.
- The given numbers go one after the other, 4 then 5 in a row.
- There are no other numbers between these numbers, because they line up like natural numbers.
- If it is 1.341, it is rounded to 1.34, and if it is 1.346, it is rounded to 1.35, so that there is no number between 1.34 and 1.35.
- There are no numbers between 1.34 and 1.35 because the difference of those numbers is 1.
- I solved by intuition that there are no numbers between 1.34 and 1.35. Intuition is a characteristic of deep-minded people. That's what I heard, so I applied it that way.
- There is not a single number between 1.34 and 1.35 or maybe half the number.

These answers indicate a misunderstanding of decimal numbers, that is, the belief that the same rule applies to decimal numbers as to natural numbers where there are immediate predecessors and successors. The answer that there is only one decimal number between the numbers 1.34 and 1.35 was circled by 17 (12.98%) students. The following numbers are listed as potential answers:

- 1.33
- 1.034
- 1.345
- 0.01

- 1.35
- Difference 0.1 so that's 0.01
- 4 and 5
- $1.35 - 1.34 = 0.01$
- 1.346

In these answers, it is observed that the students do not understand decimal numbers, their relationships, the meaning of decimal places and the number of decimals.

The answer that there are several decimal numbers between 1.34 and 1.35 was chosen by 3 (2.29%) students with different explanations.

One student pointed out that there are a total of 1.34 and 1.35 and listed them: 1,34.1, 1,34.2, 1,34.3, 1,34.4, 1,34.5, 1,34.6, 1,34.7, 1,34.8, and 1,34.9.

The student pointed out that the next number after these numbers is 1.35. It remains unclear how a decimal number can contain both a period and a comma, and what the difference is between a period and a comma. When asked why she represents decimal numbers that way, the student stated that she thinks it should be that way.

In the task (Figure 2), the student thought similarly while working on the task, however, during the interview and guidance to the correct answer, the student concluded that the number could be 1.341, 1.3411, and based on that, she came to the solution that there are infinitely many numbers between 1.34 and 1.35.

One student states that there are four numbers between the numbers 1.34 and 1.35, which are the numbers 1.341, 1.342, 1.343, 1.344, and the next number is 1.35 because 1.345 rounds to 1.35.

The third student who opted for the answer that there are several decimal numbers between the numbers 1.34 and 1.35, i.e., there are exactly 10 numbers and he points out that they are the following numbers: 1.341, 1.342, 1.343, 1.344, ..., 1.349, 1.35. The student did not notice that the numbers must be less than 1.35 and considered that number to be the correct answer.

Based on the answers given, it can be seen that students notice that there are several numbers between the given decimal numbers, however, they do not notice that there are infinitely many of them, which suggests that their knowledge of decimal numbers is insufficient.

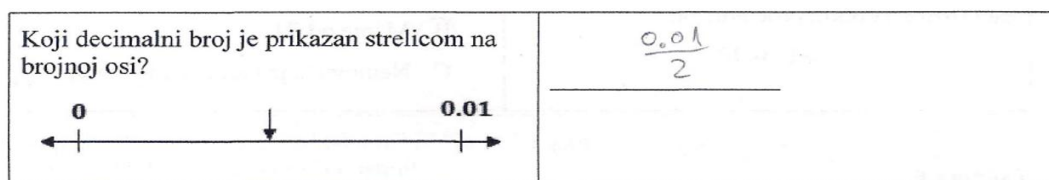


Figure 3. Example of an incorrectly completed task 3 by a research participant (taken from students' test scripts)

In task 3, which requires the representation of decimal numbers on a number line, the students were somewhat more successful. 73 (55.73%) students had the correct solution and explanation, of which 1 (0.76%) student was led to the correct solution during the conversation. There are many examples where the approach to this task is incomplete and incorrect, and in some cases there is a complete lack of explanation of the solution. 48 (36.64%) students have the wrong approach, way of working and incorrect solution. In task 3, it is noted that the reasoning is incorrect, the students have difficulties with representing numbers on the number line and cannot transfer knowledge to the number line without a good understanding of the concept of decimal number. Some of the students have an understanding without calculation, but not well enough, i.e., they have a superficial understanding and fail to reach the correct solution. In this task (**Figure 3**), one student did not know how to write a fraction in a form other than $\frac{0.01}{2}$, and she believes that this type of writing is correct.

One student presented in response that 0.5 is in the middle of a number line, explaining that no matter how many zeros we put in front of 5, it will still be the same number. This student did not adopt the rule that a decimal number does not change its value if one or more zeros are added or omitted from the right side.

A total of 5 (3.82%) respondents stated that in the middle of the numerous lines between 0 and 0.01 is the number 0.1 with the explanation that it is the number before 0.01. Another explanation for this solution is that it is the number $\frac{0.01}{10}$. The student pointed out that 0.01 is shifted by 10, which gives the number 0.1, which represents the middle between 0 and 0.01.

On the other hand, 18 (13.84%) respondents made a mistake in this task and opted for the answer 0.001, where they gave the following answers as an explanation:

- The number 0.001 seems smaller than 0.01.
- It may not have a number in the middle, but I think 0.001 is less than 0.01.
- It is the number 0.001 because there is no other number between 0 and 0.01.
- Maybe it's $\frac{1}{2}$ because it's between 0 and 0.01.
- I consider 0.001 to be less than 0.01, but not halfway. I don't think there's a number on half of it.
- Since the number is halfway between 0 and 0.01, I'd add another zero so it's 0.001.
- That number is 0.001 because it is greater than 0 and less than 0.01.

A total of 6 (4.58%) students think that it is in the middle of 0.05, while half of those students think that it is in the middle of 0.050, which clearly shows the misunderstanding of the number of zeros after the decimal point.

One (0.76%) of the interviewed students believes that the middle number is 0.5, while 3 (2.29%) students believe that the middle number is 0.50 because it is half of 0.01.

Interestingly, 4 (3.05%) students point out that the middle is 0.00. The reason for this answer is that in the number 0.01, in the place where the digit 1 is, they put the number that comes before 1, which is 0. This is an indicator that students do not understand decimal numbers, i.e., they view decimal numbers as whole numbers.

One student (0.76%) pointed out that the number 1 is in the middle of a number of lines between 0 and 0.01 because after 0 comes 1, while when asked how the number 0.01 comes after 1, he remains confused and without comment.

A total of 3 (2.29%) students think that the number 0 is in the middle between 0 and 0.01 because between those two numbers all the numbers are equal to zero.

The explanation for the answer was that in the middle of numerous lines between 0 and 0.01 the number 0.025 was missing because the student with that answer just shrugged his shoulders and looked at the task in confusion.

The student who wrote that in the middle of a number of lines between 0 and 0.01 the number $\frac{1}{20}$ states that it is exactly that number because she believes in it.

One student divided the line from 0 to the middle of a number of lines into four equal parts and wrote that the required number is 0.4 with the explanation that this is how numbers go.

One student wrote in her answer to task 3 that the number 5 is in the middle between 0 and 0.01. The explanation for this answer is that the numbers go 0, 1, 2, 3, then 5, and then decrease. So, the numbers increase to 5, then decrease to 0.01.

Out of the 131 students tested and interviewed, 10 (7.63%) students did not try to solve this task because they did not understand what was required in the task.

Based on the answers given, it is observed that some students have difficulty with representing decimal numbers on a number line or generally have a poor understanding of the size of decimal numbers. Bell et al. (1981) also investigated students' difficulties with decimal numbers and observed that students view decimal numbers as negative numbers.

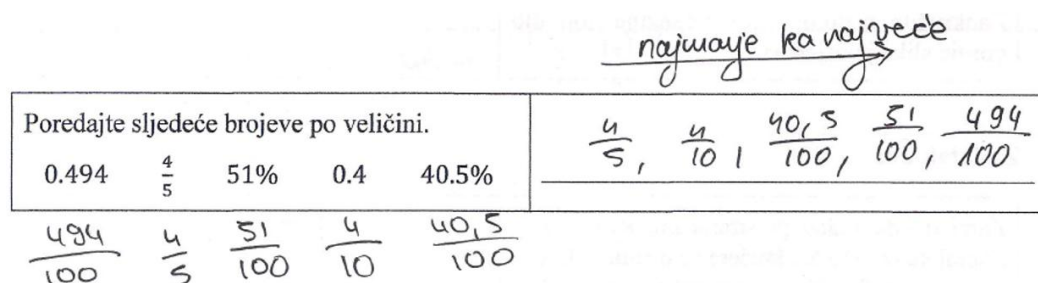


Figure 4. Example of incorrectly completed task 4 by one research participant (taken from students' test scripts)

Task 4 represents the ordering of numbers in the form of fractions, decimals and percentages and proved to be one of the relatively difficult tasks in the test. Percentages are introduced in the sixth grade of primary school through the connection and creation of connections between positive fractions, decimals and percentages (Ministry of Education of the Sarajevo Canton, 2024, p. 28). Through interviews with students, difficulties were observed in representing numbers in different forms according to size (Figure 4). Greater ambiguities were observed in placing the decimal point, representing a fraction in the form of a decimal number and calculating percentages, i.e., in reducing all given numbers to the same form so that the numbers could be ordered by size. When completing task 4, only 49 (37.40%) students were successful, of which 6 (4.58%) came up with the correct answer by leading them to the solution through discussion. To evaluate the solution of a problem that contains fractions, decimals and percentages, a good knowledge of the relationships between these quantities and a good knowledge of their concepts is required. Since many answers were incomplete and incorrect, it is assumed that this area of mathematics is not well mastered. Students used different methods to create the problem, and only some students managed to solve this problem correctly and give a correct explanation. Of all the respondents, a total of 14 (10.68%) students did not attempt to solve the problem, assuming that they did not understand how fractions, decimals and percentages work together to be ordered by size.

A total of 68 (61.91%) students did this task incorrectly, with very different answers:

The most common wrong answer was that the students ordered decimal numbers first, then fractions and finally percentages. This procedure was used by 30 (22.9%) students. In this task, students consider the smallest decimal numbers because they are less than 1 in appearance. According to their thinking, fractions are greater than decimal numbers because they consider the numerator and denominator as whole numbers, and do not understand their meaning and mutual relationship. After fractions, percentages are the largest in size because they are specifically large numbers. Here, a lack of understanding of decimal numbers, percentages and fractions is observed. One student's interesting explanation is that he thinks that $40.5 = \frac{4}{5}$, that is, he assumes that "maybe it's the same, it's just written differently," and for that reason he wrote these

numbers next to each other. This student explains that $\frac{4}{5} > 0.494$ is valid, because if $\frac{4}{5} > 0.4$, then it is "logically greater than 0.494." The explanation for $0.4 > 0.494$ of his assumption that decimal numbers with more decimal places are smaller than decimal numbers with fewer decimal places. Most students point out that they do not know percentages because they were not in school when they did percentages. One of the explanations for the order where $\frac{4}{5} < 40.5\%$ is the opinion that $\frac{4}{5} = 0.5$ and based on that, 40.5 is a large number compared to $\frac{4}{5}$.

The next incorrect solution that students presented in this task is that decimals are the smallest, then percentages, and fractions are the largest. 9 (6.87%) students chose this order. They think that $0.494 < 0.4$ because more decimals in a decimal number means a smaller number. These students think that fractions are always the largest, then percentages, and that decimals are the smallest because "decimals are small, percentages are large, and fractions are the largest."

The next incorrect solution is that the smallest fractions are followed by decimals followed by the largest percentages. This opinion is shared by 7 (5.34%) students. The reason for this order is an error in calculating the decimal value of the fraction where the students think that $\frac{4}{5} = 0.25$. Decimal numbers seem larger to students, and percentages seem larger.

A common mistake made by students is to write the fraction between the decimal numbers provided. As many as 5 (3.82%) students have the order $0.494 < \frac{4}{5} < 0.4$ in their solutions, while the percentages are the largest. In this task, students think and explain the solution that numbers are smaller if they have more decimals after the decimal point, while most students do not understand how to compare $\frac{4}{5}$ and 0.4, and write them next to each other. It was observed that students cannot divide the number 4 by the number 5, and therefore do not know how much $\frac{4}{5}$ is, and are very confused when comparing numbers. On the other hand, a total of 2 (1.53%) students wrote the order $0.4 < \frac{4}{5} < 0.494$, where they state that $\frac{4}{5} = \frac{1}{2} = 0.5$, and that $0.5 < 0.494$ is because $5 < 494$. As with the other incorrect solutions, the largest percentages are.

Cijena artikla je povećana sa 50 KM na 60 KM. Koliki je procenat povećanja? Zaokružite svoj odgovor.	☒ A. 10 % ☐ B. 20 % ☐ C. 25 % ☐ D. 15 %
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Figure 5. Example of an incorrectly completed task 5 by a research participant (taken from students' test scripts)

Only 1 (0.76%) student had the order $40.5\% < \frac{4}{5} < 51\%$ in their solution, while decimal numbers were presented as smaller. It seems that the fraction $\frac{4}{5}$ caused the most confusion among students, which most students could not convert correctly into a percentage or decimal number.

A total of 4 (3.05%) students attempted to solve this task by converting all numbers into fractions. It was observed that students did not know how to correctly convert all numerical values into fractions, and when comparing these fractions, they compared numerator with numerator and denominator with denominator as whole numbers. One of the incorrect solutions is the following: $0.494 = \frac{494}{100}$, $\frac{4}{5} = \frac{8}{10}$; $51\% = \frac{51}{100}$; $0.4 = \frac{4}{10}$; $40.5\% = \frac{4,050}{100}$.

Based on this conversion of decimal numbers and percentages into fractions, the order is as follows: $0.4 < \frac{4}{5} < 40.5\% < 0.494 < 51\%$.

Here it is observed that the fractions $\frac{4}{10}$ and $\frac{8}{10}$ are ordered by the numerator from smaller to larger, followed by fractions with a larger denominator $\frac{494}{100}$, $\frac{51}{100}$, and $\frac{4,050}{100}$ which are ordered by the numerator from larger to smaller.

Another example of a wrong solution is $\frac{2}{5} < \frac{4}{5} < \frac{51}{100} < \frac{405}{1,000} < \frac{494}{1,000}$, where the student pointed out that the order of the fractions goes from smaller to larger according to the denominator. After a few moments, she changed her mind and pointed out that she ordered from largest to smallest because $\frac{2}{5}$ is the largest because it is equal to 0.4, while the other fractions are smaller than 0.009.

In the next attempt to solve the problem $\frac{4}{5} < \frac{4}{10} < \frac{40.5}{100} < \frac{51}{100} < \frac{494}{1,000}$, it is observed that the fractions are ordered by size in both the numerator and denominator.

In the case of 10 (7.63%) students, the wrong solution of this task had no explanation. These students point out that they "arranged the numbers randomly, hoping that something would work out."

This task, which involves arranging numbers in different forms of fractions, decimals, and percentages, proved to be challenging for students.

Task 5 represents the students' understanding of percentages and their level of conceptual knowledge about percentages. Understanding conceptual knowledge is determined based on the students' ability to estimate a part of a whole of a quantity, i.e., the ability to estimate percentage values of certain quantities. Out of the 131 tested

subjects, 54 (41.22%) students answered task 5 correctly, of which 2 (1.53%) students arrived at the correct answer with the help of a solution. Students with the correct answer generally had a good explanation of how they arrived at the solution. They noticed that 10% of 50 KM is 5 KM, and that 10 KM represents 20% of 50 KM. 74 (56.49%) students gave the wrong answer to this task. A total of 3 (2.29%) subjects did not even attempt to answer the task. The explanation for the uncompleted task is that they do not know percentages and/or were not in the class when they learned percentages. In the interview, students demonstrated a lack of understanding of the concept of percentages, which probably has deeper roots, perhaps they do not have a good learning style or they lack a better strategy for learning percentages to achieve a higher level of understanding.

The problems that some students face when solving this task are presented in a very interesting and interesting way. A total of 57 (43.51%) respondents point out that the increase in the price of the item in task 5 (Figure 5) is equal to 10% because the difference between the prices is 10 KM, so $10\% = 10 \text{ KM}$. One student rounded this assignment result randomly, explaining that she did not understand increasing percentages. One student presented his solution as follows:

1% = 51 KM
 2% = 52 KM
 :
 10% = 60 KM.

This confirms the fact that students perceive percentages as whole numbers.

A total of 4 (3.05%) students circled the answer 25% with the explanation that they followed their own logic and that 10 KM represents 25%. One student pointed out that he did not find the sum of 50+60 in the answers offered, and he rounded 25% as a closer value. One student had a calculation error even though the procedure was correct

$$\frac{\text{New price} - \text{old price}}{\text{Old price}} \times 100\% = \frac{60 - 50}{50} \times 100\% = 25\%.$$

A total of 13 (9.92%) students gave the answer 15% with the recognition that they did the task randomly, by guessing, with some logic of their own according to appearance. One student confidently emphasized his explanation: "If it was 10%, then it would be 5 KM, if it were 25%, it would be 75 KM, and if it was 20%, it would be 70 KM, which means that the correct answer is 15%."

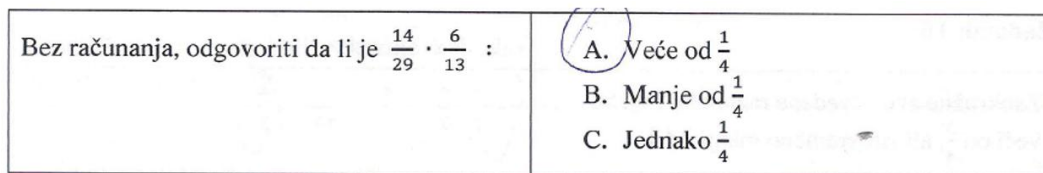


Figure 6. Example of an incorrectly completed task 6 by a research participant (taken from students' test scripts)

Some students find it very challenging to introduce the concept of percentages and explanations end up being trial and error-like.

Bağdat and Bağdat (2023) in their study concluded that students have difficulty interpreting real-life situations that involve percentages. Also, Erdem et al. (2018) through their study conclude that students misunderstand the concept of percentage, which they view as a whole number.

In task 6, it is necessary to estimate the solution of the product of two fractions and compare it with $\frac{1}{4}$. The ability of students to explain the procedure of multiplying two fractions and estimate its size is very diverse. Out of 131 respondents, 54 (41.22%) answered task 6 correctly, while 72 (54.96%) students did the task incorrectly (Figure 6). A total of 5 (3.82%) students did not complete this task, emphasizing that it was difficult.

60 (45.8%) respondents chose the answer that the product $\frac{14}{29} \times \frac{6}{13}$ is greater than $\frac{1}{4}$. The explanations for this answer were different:

- It is logical that when these two fractions are multiplied, the result will be greater than $\frac{1}{4}$.
- Adequately. That's what it looks like.
- We have two-digit numbers and when we multiply them the result will be greater than $\frac{1}{4}$.
- Because the larger numbers are in the denominators.
- A total of 3 (2.29%) students gave the explanation that: $\frac{14}{29} \cdot \frac{6}{13} > 1 \Rightarrow \frac{14}{29} \cdot \frac{6}{13}$ is greater than $\frac{1}{4}$.
- One student, when asked what a fraction represents for her, answered that for her, the fraction is what annoys her on the test.
- If we look at 13 parts of something and 29 parts of something, it's certainly greater than $\frac{1}{4}$.
- By appearance, when these two fractions are multiplied, a fraction greater than $\frac{1}{4}$ must be obtained.
- I count in my head, draw in the air, and conclude that the product $\frac{14}{29} \cdot \frac{6}{13}$ is greater than $\frac{1}{4}$.
- Two students pointed out that they are multiplying large numbers, so they concluded that the result is greater than $\frac{1}{4}$.
- Four students answered the questions by random guessing or, as they themselves stated, they were "just guessing."

- One girl explained her solution by swapping the numerator and denominator in the second fraction so she could shorten the numerators and denominators. Observing the newly obtained expression $\frac{14}{29} \cdot \frac{13}{6}$, she concluded that she could shorten the numerator of the first fraction and the denominator of the second fraction by 2, which resulted in $\frac{7}{29} \cdot \frac{13}{3}$. After that, she multiplied 29 by 3 and got 87. Then she multiplied 7 and 13 in the following way: she added 13 and 13, then she added 26 and 26, after that she added 52 and 52, and finally she added 104 and 13, with which she got that the product of the numbers 7 and 13 is equal to 117. Here it can be seen that the student does not know how to multiply two whole number so it cannot be expected to evaluate a fraction. Also the method he uses when multiplying two numbers is applied incorrectly. It has generally been observed that a greater number of students multiply two integers by adding the larger factor to itself as many times as the other smaller factor. After multiplication, the fraction $\frac{117}{87}$ was obtained. Then the student subtracted the denominator from the numerator and got $117 - 87 = 30$. The final score is $1 \frac{30}{84}$. The student probably wrote the denominator incorrectly, and based on the result, she concluded that the correct answer is under A. Greater than $\frac{1}{4}$.
- The product $\frac{14}{29} \cdot \frac{6}{13}$ is greater than $\frac{1}{4}$ because both fractions are greater than $\frac{1}{4}$, and therefore their product must also be greater than $\frac{1}{4}$. The explanation for the statement that the fractions are greater than $\frac{1}{4}$ is as follows: for the first fraction $14 > 1$ and $29 > 4$, and for the second fraction $6 > 1$ and $13 > 4$.
- One student pointed out that he multiplied the numerators and denominators and got $14 \times 6 = 84$ and $29 \times 13 = 377$. After that, he emphasized that it is greater than $\frac{1}{4}$ because it is 0.25.
- The product $\frac{14}{29} \cdot \frac{6}{13}$ is greater than $\frac{1}{4}$ because the denominators are greater than 4 and the numerators are greater than 1. Therefore, larger numbers and the result is greater than $\frac{1}{4}$.
- The product $\frac{14}{29} \cdot \frac{6}{13}$ is greater than $\frac{1}{4}$ because $\frac{1}{4} = 1 \times 4$ and 1×4 cannot possibly be greater than $\frac{14}{29} \cdot \frac{6}{13}$. He views the fractions $\frac{14}{29}$ and $\frac{6}{13}$ as the product of the

numerator and denominator 14×29 and 6×13 . Furthermore, this student points out that he would calculate these two products, get their results, and then multiply these results to get the final result. The student states that this is a large number that is greater than 4.

The answer that the product $\frac{14}{29} \cdot \frac{6}{13}$ is equal to $\frac{1}{4}$ was rounded by 12 (9.16%) students, explaining that the fraction $\frac{14}{29}$ is approximately equal to $\frac{1}{2}$ and that the fraction $\frac{6}{13}$ is approximately equal to $\frac{1}{2}$, so based on this it follows that $\frac{14}{29} \times \frac{6}{13} \times \frac{1}{4}$. Some students gave this solution by random rounding.

CONCLUSION

Practice in education has shown that the teaching of mathematics in primary and secondary schools takes place in a procedural manner (Jacobs et al., 2006; Stovner & Klette, 2022). Through long-term research of mathematical concepts, teachers try to enable students to build knowledge by connecting procedures and concepts, that is, learning with conceptual understanding. An indicator of the natural combination of conceptual and procedural knowledge is the ability to present mathematical situations in different ways and the knowledge of how different approaches can be used for different purposes (Ministry of Education of the Sarajevo Canton, 2024, p. 125).

The results of the research show that the way to achieve correct mathematical thinking in students is to connect concepts, that is, to acquire quality conceptual knowledge through different ways and approaches to mathematical problems. The results also show that estimating quantities and providing appropriate explanations can influence students' conceptual understanding of quantities.

From the solutions to the given tasks and the interviews conducted, it can be concluded that many students think well and have an excellent understanding of concepts. It has been established that students who are gifted in mathematics achieve better results and have a better conceptual understanding of certain quantities. On the other hand, some students are confused and do not understand comparisons of differently presented quantities, e.g., percentages and decimals. When solving test tasks, some students show their mathematical thinking very superficially, and many are left without an explanation of the task. Considering that percentages, fractions and decimals are introduced in the second triad of primary school, by the end of primary school, students should be able to apply their understanding of percentages, decimals and fractions in different situations.

Teachers should pay more attention to encouraging students to think in mathematics lessons, which would greatly contribute to a better understanding of mathematical concepts. In order to encourage students to work and stimulate them to concentrate more in class,

additional practical explanations are needed to improve understanding of the material, especially when it comes to percentages.

Recommendations for future research on the development of concepts and conceptual understanding in students include introducing new learning methods and working on understanding the relationship between the magnitudes of different representations of numbers from the lower grades of primary school, and then continuously working and building on them. This primarily refers to a change in the rote learning system, as there are certainly other learning methods, such as learning through practical connections between certain areas and application in everyday life.

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