Contemporary Mathematics and Science Education
2023, 4(1), ep23004
ISSN 2634-4076 (Online)
https://www.conmaths.com/

Research Article

OPEN ACCESS

Designing a flipped classroom instruction to improve plane geometry learning among pre-service teachers in Ghana

Seth Amoako Atta [1*], Ebenezer Bonyah [1]

[1] Department of Mathematics Education, Akenten Appiah Menka University of Skills Training and Entrepreneurial Development, Kumasi, GHANA
***Corresponding Author:** sethamoako@rocketmail.com

Citation: Atta, S. A., & Bonyah, E. (2023). Designing a flipped classroom instruction to improve plane geometry learning among pre-service teachers in Ghana. *Contemporary Mathematics and Science Education*, 4(1), ep23004. https://doi.org/10.30935/conmaths/12674

ABSTRACT

Teaching mathematics as a STEM subject in this 21st century must be challenging enough to inspire learning, especially in the basic schools. This also depends on the pedagogical content knowledge and skills of the teachers handling the subject. Therefore, there is a need to design modern instructional strategies for training future teachers handling the pupils at the primary schools. Flipped classroom approach is one of the emerging pedagogical strategies that has proven to be very efficacious in teaching almost all subjects. Therefore, this research aimed to investigate the impact of flipped classrooms as a pedagogy in equipping the pre-service teachers with the knowledge and concepts of plane geometry, which happens to be one of the main branches of mathematics. A sample of 101 basic school pre-service teachers was purposively sampled for the study. A pre- and post-test design were used to gather quantitative data for analysis using a quasi-experimental approach. Based on the analysis of the results, the null hypothesis was rejected since there was a statistically significant difference in the pre-service teachers' performance before and after the intervention. The study concluded that there was an improvement in performance due to the intervention. Therefore, the researcher recommends the flipped classroom approach as a formidable pedagogy for teaching pre-service teachers.

Keywords: flipped classroom pedagogy, flipped learning, problem-solving, constructivist learning, geometron

Received: 08 Aug. 2022 ◆ Accepted: 21 Nov. 2022

INTRODUCTION

Euclid defined geometry as "geometron," meaning to measure the earth, becoming a very influential branch of mathematics (Baah-Duodu et al., 2020). It studies the sizes, shapes, position angles, and dimensions. It practically deals with the properties and relations, points, lines, surfaces, solids, and higher analogs (Oladosu, 2014). The application of geometry in daily activities is enormous since it is a potential problem solver. The knowledge and application of geometry date back to the Egyptian civilization, where the concept helped in the architecture of the glorious temples, palaces, dams, and bridges (Uwurukundo et al., 2022). Today, apart from construction and measurements, geometry influences many fields such as engineering, biochemical, modeling, designing, computer graphics, and typography (Hoffer, 1981). Two Dutch educators, Dina Van Hiele-Geldof and Pierre Van Hiele in the 1960s, through their research, tried to uncover why children have difficulty with geometry in order to prescribe a kind of instruction that can help children learn best (Van de Walle et al., 2015). Studies have already proven that around 25-30% of class 9 and 10 mathematics curricula course structure and the assessment blueprints focus on the topic 'geometry' (Charles & Carmel, 2005; Hoffer, 1981; Van de Walle et al., 2015). It was therefore vital for students to have a deeper understanding of the properties of basic geometric shapes and apply the same to understand formal proofs using deductive reasoning and problem-solving. That is the only way to prepare young children to face higher mathematics with some form of flexibility. The duo developed the theory known as the Van Hiele's model (Van de Walle et al., 2015), which prescribes five levels of thinking or understanding in geometry, as detailed below:

Visualization (basic visualization or recognition): Students at the visualization level think about shapes in terms of what they resemble. They can sort shapes into groups that look alike to them in some way. At this stage, their level of understanding is based on looks and attributes such as color, texture, size, and number of sides, among others. The kids are comfortable only working with concrete materials or pictorial representations, mostly two-dimensional shapes (Baffoe & Mereku, 2010).

Level 1-Analysis: Students at this stage can list all of the properties of a given plane figure and distinguish between shapes based on their geometric properties. However, they cannot identify any relationships between the properties and do not realize that some properties imply others (Pusey, 2003).

Level 2-Abstraction (informal deduction or ordering or relational): Students at the informal deduction level not only think about properties but also can notice relationships within and between figures. They create meaningful definitions. They can also give simple

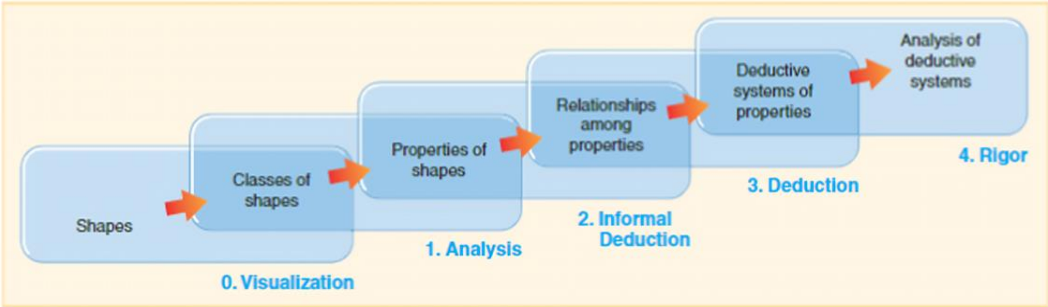

Figure 1. The van Hiele theory of geometric thought (Van de Walle et al., 2015)

arguments to justify their reasoning. They can distinguish between and compare congruent shapes and other patterns (Baah-Duodu et al., 2020).

Level 3-Deduction (formal deduction): Students at the formal deductive level think about relationships between properties of shapes and understand relationships between axioms, definitions, theorems, corollaries, and postulates. They understand how to do a formal proof and know why it is needed (Banson & Arthur-Nyarko, 2021; Hoffer, 1981).

Level 4-Rigor: Students at the rigor level can think in abstract mathematical systems and apply their knowledge to solve complex problems (Agrawal & Morin, 2016; Leinwand et al., 2014; Student, 2017; Van de Walle et al., 2015).

The sequential nature of the van Hiele levels allows students to move from one level to the other until the end. Children can only move to the next level unless they have mastered the current level without skipping. It is interesting to note that there are no age barriers as in Piaget's cognitive development theory. Children, therefore, need to have many experiences in which they are actively involved in exploring and communicating their observations of shapes, properties, and relationships (**Figure 1**).

The Concept of the Flipped Classroom

According to Bishop and Verleger (2013), the flipped classroom is a flip or an inverted classroom that depicts a blended learning experience where the traditional classroom lessons activities take place outside the classroom and vice versa (Abuhmaid & Mohammad, 2020; Bishop & Verleger, 2013). This implies that the teacher records the lesson that should be delivered in class and gives students to watch before the regular class. This model divides learning into two main parts; an interactive learning session where group activities occur inside the classroom and individual computer-based interactive learning outside the classroom (Bishop & Verlager, 2013). A flipped classroom can take the form of a web-based video delivered at home while class time is dedicated to problem-solving, discussions, and the like (Ajmal & Hafeez, 2021). It is essential to remember that a flipped classroom is an expansion of the curriculum but not just a mere rearrangement of activities (Abu-Shanab, 2020).

The approach is mainly based on constructivist teaching and focuses on problem-based learning, group collaborations, research, and creative projects (Bada & Olusegun, 2015). The teacher only acts as a guide or coach, facilitating learning by developing supportive activities and environments and building on what students already know (Atta & Brantuo, 2021; Duit, 2016). Teachers design and present lessons to promote learners' active construction of their knowledge (Kumar, 2018). Therefore, there is a need to modify teaching tasks and strategies, learning tasks and strategies, and the criteria for assessing learning outcomes (Duit, 2016; Gravemeijer, 2020). Teachers provide a means of scaffolding that could help the student extend their zone of proximal development and develop meta-cognitive skills such as reflective thinking and problem-solving techniques to guide learners in assimilation, accommodation, adaptation, and reflection on conflicting ideas (Gómez, 2016).

Flipped Classroom in Ghana

Recent advances in information and communication technologies (ICTs), particularly wireless broadband Internet access and communicating devices, are offering higher education institutions in Ghana a more significant opportunity to implement several innovative learning initiatives. One such technology-enabled learning initiative that has emerged and spreading rapidly is flipped teaching/learning. This approach aims to improve learning outcomes by reversing the traditional pedagogical model: students acquire essential content outside of class (typically video lectures) and then work together in class on application-oriented activities (exercises, projects, discussions, etc.)

University of Education, Winneba, for instance, is making giant steps in the flipped teaching model. As part of the measures to curb the spread of the COVID-19 pandemic, the university has adopted measures to reduce students' contact hours with the lecturers in lecture Halls to only one hour instead of the three contact hours. However, the remaining two hours are taken online. This allows the students to learn at their own pace and still have the opportunity to meet the lecturer in person to discuss or explain complex and incomprehensible concepts. KNUST, LEGON, UENR, and all the tertiary institutions in Ghana have some components of online courses.

Asunka et al. (2018) conducted research into modes of implementing constructivist pedagogy in a flipped classroom in a postgraduate course in a private university in Ghana. Using a sample of 65 MBA students, they flipped the classroom in which the lecturer interacted with the students for two-three hours on all weekdays or eight hours on weekends. The lessons and notes needed for the course were uploaded online for every student to have access to it, and they were assessed after every interaction. It was found that students were highly excited about learning since they had experiences that had never occurred to them before. One of the experiences was, being able to focus on what was taught throughout the interaction. It was so convenience, more students had control of learning experiences, provided richer learning, peer learning, flexibility, and better student-lecturer interaction. The only thing students disliked about the flipped classroom was the cost of data involved in watching videos and downloading materials from the Internet.

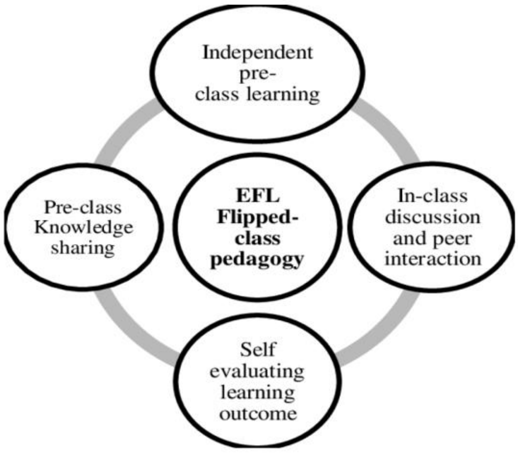

Figure 2. How student learns geometry using flipped classroom approach (Zainuddin et al., 2019)

Mensah et al. (2017) also using WhatsApp, flipped the form 2 leatherwork students' classroom in the Serwaa Nyarko Girls Senior High School. Teachers had initially had difficulty covering one-third of what was supposed to be taught for a term aside from other schedules like taking them to leather studios to go and look at what was to be done there. Through the help of flipped classrooms, teachers took four of the topics perceived to be difficult and treated them. Students were happy since they understood them better and had additional time to visit studios to see how things were done since most of the things taught were too theoretical.

Atta and Brantuo (2021) also conducted a similar study using Remedial students in Ghana. The study was conducted using the flipped-classroom approach to teach one of the problematic concepts in the senior high school curriculum under geometry, called circle theorems. The study documented the flipped classroom because the results were overwhelming. Since internet connectivity is limited in many parts of the country, the best form of the flipped classroom, for now, is giving out the recorded video to students to watch. Several other studies have been conducted, especially in COVID-19 and post-COVID-19 era (Akuffo et al., 2019).

Geometry and the Flipped Classroom Approach

Several studies have proven the effectiveness of technology in teaching geometry (Ajmal & Hafeez, 2021; Atta & Brantuo, 2021; Erbil, 2020). Especially the use of computer software like multi-media courseware, GeoGebra, among others. In most of these cases, the studies use classroom students, forgetting the pre-service teachers who are also equally important (Adu et al., 2017; Arbain & Shukor, 2015; Benning, 2021; Muñante-Toledo et al., 2021; Uwurukundo et al., 2022). As technology advances, school curricula worldwide are being restructured to integrate STEM approaches into teaching and learning (Belbase, 2019; Diego-Mantecón et al., 2022). A flipped classroom is one of the technology-based teaching instructional techniques whereby teachers record lessons for students to learn on their own and use the class hours for discussions and problem-solving (Atta & Asiedu-Addo, 2021; Arfiana & Wijaya, 2018). Studies have demonstrated that Flipped classroom offers students the opportunity to learn actively and at their own pace (Abeysekera & Dawson, 2015; Aksu & Colak, 2021; Akuffo et al., 2019; Atta & Brantuo, 2021). As pre-service teachers, there is a need for them to appreciate the concept of plane geometry well to impart it to the future leaders of the country, hence the need for this study.

Conceptual Framework

Figure 2 shows how the student learns geometry using the flipped classroom approach.

Statement of the Problem

Ghana's performance in international ranked tests in mathematics has been very abysmal, which was why it pulled out of the teams after two unsuccessful attempts (Abdul Gafoor & Kurukkan, 2015; Abreh et al., 2018; Mereku, 2010). One of the areas of mathematics that remains a challenge in primary schools is geometry. Most of the studies in this area have also focused on students, especially in secondary school (Aksu & Colak, 2021; Uwurukundo et al., 2022). However, not many studies have been done focusing on pre-service teachers being trained for the primary schools; the present study, therefore, seeks to bring to light the need to enhance the content knowledge of the pre-service teachers for effective delivery.

Researchers have shown that the traditional methods of teaching mathematics in Ghana are not helping the students achieve the subject's objectives and goals as stipulated by the mathematics syllabus (Arthur et al., 2017; Baah-Duodu et al., 2020). This is because much emphasis is laid on computational algorithms in the classroom, which encourages the memorization of facts and procedures.

Consequently, students are not motivated to learn the subject and continue to perform poorly. The person needs to be creative, innovative, and critical thinking. These are the very skills that mathematics seeks to develop. Therefore, mathematics must be taught using hands-on and mind-on approaches to motivate students to learn mathematics quickly (NaCCA, MOE, 2019).

In addition to critical thinking and problem solving, the mathematics curriculum observes personal development and leadership as an integral part of mathematics education (MOE, NaCCA, 2019). Upon this background, the students will be able to acquire cognitive and reasoning abilities to enhance their analytical thinking and problem-solving abilities. Critical thinking and problem-solving skills embedded in the curriculum will enable the learners to draw their own experiences to analyze situations and choose the most appropriate out of several possible solutions (MOE, NaCCA, 2019).

The rationale is to equip the students to identify and develop talents and fulfilling dreams and aspirations. Self-awareness and self-esteem will enable students to learn from their mistakes and failures of the past (MOE, NaCCA, 2019). To fulfill this mandate, the teachers must also be abreast with the knowledge and the cross-cutting skills needed to inspire learning, hence, the study.

Objectives of the Study

1. To investigate the impact of the flipped classroom approach on the interest of pre-service teachers in learning geometry concepts.
2. To determine the extent to which the use of flipped classroom improves the pre-service teachers' performance.

Research Questions

1. What is the impact of the flipped classroom method on the interest of pre-service teachers in learning geometry concepts?
2. To what extent will the use of flipped classroom improve the performance of the pre-service teachers in geometry.

METHODOLOGY

The quantitative research approach is the second form of study whose primary focus is to test or measure reality by examining the relationship among variables (Apuke, 2017). It employs descriptive and inferential statistics in analyzing data collected to answer the research questions. These tools are most mean, standard deviation, regression, and collection analysis (Bloomfield & Fisher, 2019). Those researchers who favor quantitative studies are deep-rooted in positivism which beliefs in doing everything scientifically. In so doing, theories are tested deductively, bias is to a greater extent avoided, and measures are put in place to control other variables just to generalize and replicate the findings. Quantitative research provides a researcher with quantifiable data and can therefore take extensive samples. This is because numbers are assigned to answers so researchers can objectively measure and compare. It is beneficial for collecting data when the sample size is large (Bloomfield & Fisher, 2019).

101 second-year basic school pre-service teachers offering Bachelor of Education (basic education) in distance mode were selected in the Ashanti Region of Ghana sampled for this study (**Table 1**). The aim was to help the basic school pre-service teachers to get a conceptual understanding of plane geometry by using the flipped classroom approach as an instructional pedagogy (Baah-Duodu et al., 2020). The study was practically based on a quasi-experimental design since intact classes were used. The selection of the participants was purposive because the focus was on the pre-service teacher education offering B.Ed. JHS education and in distance mode. A pre- and post-test were the main instruments used for data collection. The pre-test consisted of twenty items to test the pre-service teacher's basic conceptual understanding of plane geometry and was administered to the participants before the intervention was rolled out. Immediately after the implementation of the intervention, a post-test was also administered. The post-test also had 20 items mainly to ascertain: their level of understanding and the impact of the intervention.

The intervention was implemented within three weeks of three hours of face-to-face interaction each week. The lessons were partitioned into three separate videos, each given to the pre-service teachers five days before the face-to-face. Learners were supposed to watch the videos and follow up with the assigned tasks within the five days at home. Face-to-face was on Sundays from 8:30 am to 11:30 am each week for class A and 12: 00 noon to 3: 00 pm each week for class B. The sub-topics covered under plane geometry were angles at a point, properties of parallel lines, and properties of triangles. Each video lasted not more than 45 minutes. Several practical examples were given after each concept had been established. Students had the opportunity to pause, rewind or watch the video repeatedly until all the activities provided were answered thoroughly.

During the face-to-face, participants were supposed to do a presentation. The presentation tasks were part of the activities provided in the video, and the groupings had been done earlier. Each group consisted of five students randomly assigned from each class. The presentations made by each group were based on the assigned tasks in the video. Each group was assigned random numbers. So, the group members were supposed to work on the tasks corresponding to their group numbers and present the solution in class for discussion. Each group was given a total of 15 minutes; 10 minutes for presentation and five minutes for critique by colleagues. The last 30 minutes of each session were devoted to general class discussion.

The pre- and post-test results were coded, keyed, and analyzed using the descriptive statistics and the one-sample t-tests embedded in the SPSS software package version 22. The analysis of the results helped the researcher answer the research questions that guided the study and test the hypothesis generated.

Table 1. Characteristics of the participants

		Frequency	%	Valid %	Cumulative %
Valid	Male	72	71.3	71.3	71.3
	Female	29	28.7	28.7	100.0
Total		**101**	**100.0**	**100.0**	

RESULTS

Table 2 shows the pre-test results. **Table 3** depicts the post-test results.

Table 2. Pre-test results

		Frequency	%	Valid %	Cumulative %
Valid	29	7	6.9	6.9	6.9
	34	7	6.9	6.9	13.9
	35	2	2.0	2.0	15.8
	36	5	5.0	5.0	20.8
	37	5	5.0	5.0	25.7
	38	7	6.9	6.9	32.7
	41	1	1.0	1.0	33.7
	43	1	1.0	1.0	34.7
	44	5	5.0	5.0	39.6
	45	10	9.9	9.9	49.5
	46	5	5.0	5.0	54.5
	47	6	5.9	5.9	60.4
	53	1	1.0	1.0	61.4
	54	17	16.8	16.8	78.2
	55	1	1.0	1.0	79.2
	56	6	5.9	5.9	85.1
	60	1	1.0	1.0	86.1
	62	1	1.0	1.0	87.1
	64	5	5.0	5.0	92.1
	65	6	5.9	5.9	98.0
	67	2	2.0	2.0	100.0
Total		**101**	**100.0**	**100.0**	

Table 3. Post-tests scores

		Frequency	%	Valid %	Cumulative %
Valid	50	1	1.0	1.0	1.0
	56	10	9.9	9.9	10.9
	64	3	3.0	3.0	13.9
	65	4	4.0	4.0	17.8
	66	5	5.0	5.0	22.8
	67	10	9.9	9.9	32.7
	68	5	5.0	5.0	37.6
	69	1	1.0	1.0	38.6
	72	6	5.9	5.9	44.6
	73	6	5.9	5.9	50.5
	74	6	5.9	5.9	56.4
	75	1	1.0	1.0	57.4
	76	1	1.0	1.0	58.4
	77	3	3.0	3.0	61.4
	78	12	11.9	11.9	73.3
	80	1	1.0	1.0	74.3

Table 3 (Continued). Post-tests scores

		Frequency	%	Valid %	Cumulative %
Valid	83	1	1.0	1.0	75.2
	85	7	6.9	6.9	82.2
	86	6	5.9	5.9	88.1
	87	12	11.9	11.9	100.0
Total		**101**	**100.0**	**100.0**	

Table 4. Descriptive statistics

		Pre-test	Post-test
n	Valid	101	101
	Missing	0	0
Mean		46.89	73.48
Median		46.00	73.00
Mode		54	78[a]
Standard deviation		10.749	9.766

Note. [a]Multiple modes exist & the smallest value is shown

Table 4 compares the pre- and post-test from a statistical perspective. The pre-test recorded a mean (X) of 46.89 and a standard deviation (SD) of 10.749 as against a mean of 73.48 and SD=9.766 in the post-intervention test. There is every indication that the intervention has dramatically enhanced the pre-service teachers' performance. Therefore, it is not a mistake to say that the intervention has positively impacted the pre-service teachers.

A pair sampled test was computed, shown in **Table 5**, to determine if there was a statistically significant difference in the pre-service teacher's performance in both tests. The p-value from **Table 5** is less than 0.05, indicating that the difference in the mean scores of the tests was statistically significant. Based on this information, the null hypothesis stands rejected.

Table 6 tried to find if there was a significant difference in terms of performance of male and female pre-service teachers. Across the tests only the independent-sample Mann-Whitney U test for the pre-test gave a sig=0.044<0.05. the rest of the test had sig>0.05, which is an indication that there was no significant difference in performance in terms gender. This confirms the work by Armah et al. (2020), which also declared that there was no significant difference in male and female students' performance in mathematics.

DISCUSSION

The results from the post-test were highly commendable, and the analysis has proven the flipped classroom as a robust and formidable pedagogy used as a tool for enhancing conceptual understanding. It is no wonder that modern researchers (Abuhmaid & Mohammad, 2020; Akuffo et al., 2019; Atta & Brantuo, 2021) have recommended it. This study is also in line with research on flipped learning conducted by Abah et al. (2017) in applied mathematics classrooms. The study was designed to determine university students' experience and perceptions of using flipped learning. A survey research design was employed to measure students' responses after a successful seven-week deployment of the flipped instructional delivery model in a university in North Central Nigeria. (Abah et al., 2017) explained that mathematics as a subject area is highly amenable to the flipped model of instruction delivery because of its loaded contents and the need for in-depth practice. Since students have surpassed simple understanding and memorizing levels, having been exposed to provided lecture videos, a higher level of studying can be done in the class. The study's findings indicated that the flipped classroom stimulates curiosity, self-paced study, and behavioral engagement in the students. Participating students in the study also recommend the approach to handle other courses in the school.

Educational Implications

ICTs have become the most fundamental building block of modern industrial society (Agyei & Agyei, 2021; Agyei & Voogt, 2011). Mastering and understanding basic skills and concepts of ICTs are now highly recognized and adopted by many African countries in teaching STEM. Flipped classroom instruction is one of the instructional strategies that make use of it. Flipped classroom approach is highly based on the constructivist theory of learning since constructivists believe that learning is an active social process in which learners use existing knowledge and prior experiences to build a unique understanding of new material (Brown et al., 1989, as cited in Shimamoto, 2012). As also observed in this Van Hiele's model, the students can progress to the next stage only after they have mastered the previous stage. Therefore, they act as facilitators, guiding students through the learning process while allowing them to shape their understanding of the instruction (Rhodes & Bellamy, 1999, as cited in Shimamoto, 2012). With the use of the flipped classroom, the teacher's role shifts from knowledge provider to learning facilitator, and the student's role shifts from information collector to the active practitioner (Ameyaw et al., 2019; Kenney & Kastberg, 2013; Kholid et al., 2021)

In terms of Bloom's taxonomy of learning, the flipped classroom has students perform the lower levels of cognitive work outside of class, and the higher levels of cognitive work in class, alongside their fellow students and instructor (Abeysekera & Dawson, 2015). The teacher serving as a facilitator and curator in a constructivist sense (Altaftazani et al., 2020; Bada & Olusegun, 2015) provides an opportunity for

Table 5. Paired samples test

	Paired differences					t	df	Sig. (2-tailed)
	Mean difference	Standard deviation	Standard error mean	95% confidence interval of difference				
				Lower	Upper			
Post-/pre-test	26.584	14.355	1.428	23.750	29.418	18.611	100	.000

Table 6. Hypothesis test summary

	Null hypothesis	Test	Sig.	Decision
1	The distribution of post-test is the same across categories of gender	Independent-sample Mann-Whitney U test	.593	Retain the null hypothesis
2	The distribution of post-test is the same across categories of gender	Independent-sample Kolmogorov-Smirnov test	.833	Retain the null hypothesis
3	The distribution of pre-test is the same across categories of gender	Independent-sample Mann-Whitney U test	.044	Reject the null hypothesis
4	The distribution of pre-test is the same across categories of gender	Independent-sample Kolmogorov-Smirnov test	.431	Retain the null hypothesis

Note. Asymptotic significances are displayed & the significance level is .050

students to gain first exposure to the course material prior to the lesson in the form of videos or audios. The students can follow the lesson at their convenience and practice or replicate the activities involved or embedded in the lesson.

Finally, the in-class activities allow the teacher and the students to engage in discussions (Minarni & Napitupulu, 2020). This kind of language activity develops the students, self-confidence, creative thinking, problem-solving skills, and cooperation among students. This is because the in-class discussion allows the students to engage their colleagues at peer-to-peer, small group, and whole-group levels (Bautista et al., 2020). This is based on the social constructivism theory, where students learn through interaction with peers and more experienced adults (Knapp, 2019).

CONCLUSION

As observed above, the flipped classroom tool helped the pre-service teachers grasp the concept of plane geometry, making the designed intervention very successful. Since the performance of the pre-service teachers improved significantly, it is an indication that the pre-service teachers have acquired the perquisite knowledge to impart to their learners. And this conceptual understanding would enable them to teach the topic effectively and efficiently. It therefore prudent to recommend the flipped classroom pedagogy as an effective tool to teach the geometry to pre-service teachers. Notwithstanding, the study can be replicated at different locations to ascertain the true potency of the approach with respect to prevailing conditions.

Author contributions: All authors were involved in concept, design, collection of data, interpretation, writing, and critically revising the article. All authors approve final version of the article.

Funding: The authors received no financial support for the research and/or authorship of this article.

Ethics declaration: Authors declared that they are faculty members of the mathematics Department of AAMUSTED and are part of the tutors who teach the Basic Education Preservice teachers (Distance Learners). Ethics committee approval was not required by the institution since it is a common practice in the department for lectures to modify their instructions to find out which learning strategies are more efficacious and formidable. Authors further declared that the issues of voluntary participation, privacy and confidentiality were explained to participants, and that their informed consents were obtained before their participation.

Declaration of interest: Authors declare no competing interest.

Data availability: Data generated or analysed during this study are available from the authors on request.

REFERENCES

Abah, J., Anyagh, P., & Age, T. (2017). A flipped applied mathematics classroom: Nigerian university students' experience and perceptions. *Abacus, 42*(1), 78-87.

Abdul Gafoor, K., & Kurukkan, A. (2015). *Why high school students feel mathematics difficult. An exploration of affective beliefs* [Paper presentation]. National Seminar on Pedagogy of Teacher Education-Trends and Challenges, Farook Training College, Kozhikode, Kerala, India.

Abeysekera, L., & Dawson, P. (2015). Motivation and cognitive load in the flipped classroom: Definition, rationale, and a call for research. *Higher Education Research & Development, 34*(1), 1-14. https://doi.org/10.1080/07294360.2014.934336

Abreh, M. K., Owusu, K. A., & Amedahe, F. K. (2018). Trends in performance of WASSCE candidates in the science and mathematics in Ghana: Perceived contributing factors and the way forward. *Journal of Education, 198*(1), 113-123. https://doi.org/10.1177/0022057418800950

Abuhmaid, A., & Mohammad, A. (2020). The impact of flipped learning on procrastination and students' attitudes toward it. *Universal Journal of Educational Research, 8*(3), 566-573. https://doi.org/10.13189/ujer.2020.080228

Abu-Shanab, E. A. (2020). Students' perceptions of flipped classrooms, gender, and country difference. *International Journal of Web-Based Learning and Teaching Technologies, 15*(4), 36-56. https://doi.org/10.4018/IJWLTT.2020100103

Adu, E., Mereku, D. K., Assuah, C. K., & Okpoti, C. A. (2017). Effect of multimedia courseware with cooperative learning on senior high school students' proficiency in solving linear equation word problems. *African Journal of Educational Studies in Mathematics and Sciences, 13*, 1-11.

Agrawal, J., & Morin, L. L. (2016). Evidence-based practices: Applications of concrete representational abstract framework across math concepts for students with mathematics disabilities. *Learning Disabilities Research & Practice, 31*(1), 34-44. https://doi.org/10.1111/ldrp.12093

Agyei, D. D., & Voogt, J. (2011). ICT use in the teaching of mathematics: Implications for professional development of pre-service teachers in Ghana. *Education and Information Technologies, 16*(4), 423-439. https://doi.org/10.1007/s10639-010-9141-9

Agyei, E. D., & Agyei, D. D. (2021). Promoting interactive teaching with ICT: Features of intervention for the realities in the Ghanaian physics senior high school classroom. *International Journal of Interactive Mobile Technologies, 15*(19), 93-117. https://doi.org/10.3991/ijim.v15i19.22735

Ajmal, S. F., & Hafeez, M. (2021). Critical review on flipped classroom model versus traditional lecture method. *International Journal of Education and Practice, 9*(1), 128-140. https://doi.org/10.18488/journal.61.2021.91.128.140

Aksu, H. H., & Colak, S. O. (2021). The effect of realistic mathematics education on student achievement in 8th grades geometric objects teaching. *African Educational Research Journal, 9*(1), 20-31. https://doi.org/10.30918/AERJ.91.20.205

Akuffo, B., Okae-Adjei, S., & Dzisi, S. (2019). Flipped learning as an alternative for effective and efficient learning pathway in technical and vocational education and training (TVET): Evidence from Koforidua Technical University-Ghana. *Commonwealth of Learning.* https://oasis.col.org/handle/11599/3372

Altaftazani, D. H., Rahayu, G. D. S., Kelana, J. B., Firdaus, A. R., & Wardani, D. S. (2020). Application of the constructivism approach to improve students' understanding of multiplication material. *Journal of Physics: Conference Series, 1657*(1), 012007. https://doi.org/10.1088/1742-6596/1657/1/012007

Ameyaw, J., Turnhout, E., Arts, B., & Wals, A. (2019). Creating a responsive curriculum for postgraduates: Lessons from a case in Ghana. *Journal of Further and Higher Education, 43*(4), 573-588. https://doi.org/10.1080/0309877X.2017.1386285

Apuke, O. D. (2017). Quantitative research methods: A synopsis approach. *Kuwait Chapter of Arabian Journal of Business and Management Review, 6*(11), 40-47. https://doi.org/10.12816/0040336

Arbain, N., & Shukor, N. A. (2015). The effects of GeoGebra on students achievement. *Procedia-Social and Behavioral Sciences, 172*, 208-214. https://doi.org/10.1016/j.sbspro.2015.01.356

Arfiana, A., & Wijaya, A. (2018). Problem solving skill of students of senior high schools and Islamic high schools in Tegal Regency in solving the problem of PISA based on Polya's stage. *Jurnal Riset Pendidikan Matematika* [*Journal of Mathematics Education Research*], *5*(2), 211-222. https://doi.org/10.21831/jrpm.v5i2.15783

Armah, S. E., Akayuure, P., & Armah, R. B. (2020). A comparative study of male and female distance learners' mathematics achievement. *Contemporary Mathematics and Science Education, 2*(1), ep21001. https://doi.org/10.30935/conmaths/9288

Arthur, Y. D., Asiedu-Addo, S., & Assuah, C. (2017). Students' perception and its impact on Ghanaian students' interest in mathematics: Multivariate statistical analytical approach. *Asian Research Journal of Mathematics, 4*(2), 1-12. https://doi.org/10.9734/ARJOM/2017/33023

Asunka, S., Freeman, E., & Sheeta Arthur, L. (2018). Implementing constructivist pedagogy in a flipped mode in a postgraduate course. *ICERI2018 Proceedings, 1*, 3301-3309. https://doi.org/10.21125/iceri.2018.1733

Atta, S. A., & Asiedu-Addo, S. (2021). The use of problem-solving approach linking classroom mathematics to real life activities at Bekwai SDA SHS. *Global Scientific Jpurnals, 9*(3), 1174-1195.

Atta, S. A., & Brantuo, W. A. (2021). Digitalizing the teaching and learning of mathematics at the senior high schools in Ghana: The case of flipped classroom approach. *American Journal of Education and Practice, 5*(3), 29-37. https://doi.org/10.47672/ajep.869

Baah-Duodu, S., Osei-Buabeng, V., Cornelius, E. F., Hegan, J. E., & Nabie, M. J. (2020). Review of literature on teaching and learning geometry and measurement: A case of Ghanaian standards based mathematics curriculum. *International Journal of Advances in Scientific Research and Engineering, 6*(3), 103-124. https://doi.org/10.31695/IJASRE.2020.33766

Bada, S. O., & Olusegun, S. (2015). Constructivism learning theory: A paradigm for teaching and learning. *Journal of Research & Method in Education, 5*(6), 66-70.

Baffoe, E., & Mereku, D. K. (2010). The van Hiele levels of understanding of students entering senior high school in Ghana. *African Journal of Educational Studies in Mathematics and Sciences, 8*, 51-62. https://doi.org/10.4314/ajesms.v8i1.69103

Banson, J., & Arthur-Nyarko, E. (2021). Interactive courseware and academic performance in geometry in junior high schools. *British Journal of Education, 9*(9), 31-54.

Bautista, J., Samonte, I., Improgo, C. M., & Gutierrez, M. R. (2020). Mother tongue versus English as a second language in mathematical word problems: Implications to language policy development in the Philippines. *International Journal of Language and Literary Studies, 2*(2), 18-29. https://doi.org/10.36892/ijlls.v2i2.283

Belbase, S. (2019). STEAM education initiatives in Nepal. *The STEAM Journal, 4*, 7. https://doi.org/10.5642/steam.20190401.07

Benning, I. (2021). Enacting core practices of effective mathematics pedagogy with GeoGebra. *Mathematics Teacher Education and Development, 23*(2), 102-127.

Bishop, J., & Verleger, M. (2013). The flipped classroom: A survey of the research. In *Proceedings of the 2013 ASEE Annual Conference & Exposition* (pp. 1-18). https://doi.org/10.18260/1-2--22585

Bloomfield, J., & Fisher, M. J. (2019). Quantitative research design. *Journal of the Australasian Rehabilitation Nurses Association, 22*(2), 27-30. https://doi.org/10.33235/jarna.22.2.27-30

Charles, R. I., & Carmel, C. A. (2005). Big ideas and understandings as the foundation for elementary and middle school mathematics. *Journal of Mathematics Education, 7*(3), 9-24.

Diego-Mantecón, J.-M., Ortiz-Laso, Z., & Blanco, T. F. (2022). Implementing STEM projects through the EDP to learn mathematics: The importance of teachers' specialization. In P. R. Richard, M. P. Vélez, & S. Van Vaerenbergh (Eds.), *Mathematics education in the age of artificial intelligence* (pp. 399-415). Springer. https://doi.org/10.1007/978-3-030-86909-0_17

Duit, R. (2016). The constructivist view in science education–what it has to offer and what should not be expected from it. *Investigações Em Ensino de Ciências* [*Investigations in Science Teaching*], *1*(1), 40-75.

Erbil, D. G. (2020). A review of flipped classroom and cooperative learning method within the context of Vygotsky theory. *Frontiers in Psychology, 11*, 1157. https://doi.org/10.3389/fpsyg.2020.01157

Gómez, P. J. S. (2016). Students' ideas and radical constructivism. *Science & Education, 25*(5), 629-650. https://doi.org/10.1007/s11191-016-9829-3

Gravemeijer, K. (2020). A socio-constructivist elaboration of realistic mathematics education. In M. Van den Heuvel-Panhuizen (Ed.), *National reflections on the Netherlands didactics of mathematics* (pp. 217-233). Springer. https://doi.org/10.1007/978-3-030-33824-4_12

Hoffer, A. (1981). Geometry is more than proof. *The Mathematics Teacher, 74*(1), 11-18. https://doi.org/10.5951/MT.74.1.0011

Kenney, R., & Kastberg, S. (2013). Links in learning and transferable skills. *Australian Senior Mathematics Journal, 27*(1), 12-20.

Kholid, M. N., Imawati, A., Swastika, A., Maharani, S., & Pradana, L. N. (2021). How are students' conceptual understanding for solving mathematical problem? *Journal of Physics: Conference Series, 1776*(1), 012018. https://doi.org/10.1088/1742-6596/1776/1/012018

Knapp, N. F. (2019). The shape activity: Social constructivism in the psychology classroom. *Teaching of Psychology, 46*(1), 87-91. https://doi.org/10.1177/0098628318816181

Kumar, R. (2018). *Research methodology: A step-by-step guide for beginners*. SAGE.

Leinwand, S., Huinker, D., & Brahier, D. (2014). Principles to actions: Mathematics programs as the core for student learning. *Mathematics Teacher, 19*(9), 516-519. https://doi.org/10.5951/mathteacmiddscho.19.9.0516

Mensah, C. P., Yeboah, A., & Adom, D. (2017). Flipped classroom model as an instructional tool for effective teaching and learning of leatherwork. *American Scientific Research Journal for Engineering, Technology, and Sciences, 30*(1), 195-212.

Mereku, D. K. (2010). Five decades of school mathematics in Ghana. *Mathematics Connection, 9*(8), 73-86. https://doi.org/10.4314/mc.v9i1.61558

Minarni, A., & Napitupulu, E. E. (2020). The role of constructivism-based learning in improving mathematical high order thinking skills of Indonesian students. *Infinity Journal, 9*(1), 111-132. https://doi.org/10.22460/infinity.v9i1.p111-132

MOE, NaCCA. (2019). Ministry of Education Ghana new curriculum. *National Council for Curriculum Assessment*. https://nacca.gov.gh/

Muñante-Toledo, M. F., Salazar, G. D. C., Rojas-Plasencia, K. M., & Flores, J. M. V. E. (2021). Geogebra software in mathematical skills of high school students: Systematic teview. *Turkish Journal of Computer and Mathematics Education, 12*(6), 4164-4172.

NaCCA, MOE. (2019). Ministry of Education English syllabus for primary 4-6. *National Council for Curriculum Assessment*. https://nacca.gov.gh/wp-content/uploads/2019/06/ENGLISH-B4-B6.pdf

Oladosu, L. O. (2014). *Secondary school students' meaning and learning of circle geometry* [Unpublished doctoral thesis] [University of Calgary]. https://doi.org/10.11575/PRISM/27723

Pusey, E. L. (2003). *The van Hiele model of reasoning in geometry: A literature review* [Master's thesis, North Carolina State University].

Shimamoto, D. (2012). *Implementing a flipped classroom: An instructional module* [Powerpoint presentation]. The Technology, Colleges, and Community Worldwide Online Conference.

Student, A. (2017). *Does the use of manipulatives to move pupils from a concrete to an abstract idea help or hinder their problem solving skills?* [PhD thesis, University of Chichester].

Uwurukundo, M. S., Maniraho, J. F., & Tusiime Rwibasira, M. (2022). Effect of GeoGebra software on secondary school students' achievement in 3-D geometry. *Education and Information Technologies, 27*, 5749-5765. https://doi.org/10.1007/s10639-021-10852-1

Van de Walle, J., Karp, K. S., Bay-Williams, J. M. (2015). *Elementary and middle school mathematics: teaching developmentally*. Pearson.