

Developing in-service mathematics teachers' pedagogical content knowledge and skills to teach trigonometry: Using cooperative teaching and learning approach

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ABSTRACT

The study is aimed at enhancing pedagogical content knowledge (PCK) and skills of in-service mathematics teachers (IMTs) using cooperative teaching and learning approach (CTLA). A sequential multiple case study was employed to develop eight IMTs knowledge and skills to design, develop, and enact lessons in trigonometry. The lessons were anchored on CTLA in two phases of a professional development arrangement (PDA) initiated in the study. The analysis of classroom observation, interview, CTLA lessons and the independent sample t-test is suggestive of significant improvement in the IMTs PCK. Further, although, the IMTs were rooted in teacher-centered pedagogies and encountered challenges, they effectively incorporated CTLA as a pedagogical tool in the teaching and learning of trigonometry. As a result of which, IMTs exhibited an enhanced content knowledge. Implications and recommendations for effective PDA in incorporating CTLA as an innovative pedagogical strategy for the teaching of mathematics at high school are discussed.

Keywords: cooperative teaching and learning approach, professional development arrangement, in-service mathematics teachers

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INTRODUCTION

The teaching and learning of mathematics have been characterized by a myriad of problems that are often thought of as the remedy to the student's inability to construct their own understanding of the subject matter. In particular, the search for appropriate pedagogical approach within the context of a specific educational cycles has inundated literature in recent times (Asomah et al., 2022b; Vernon et al., 2022). Literature exemplifies pedagogy as the bane of teachers' effort in provoking students' interest and understanding of concepts in the subject matter (Asomah et al., 2018, 2019; Core Practices Consortium, 2020; Jong & Brinkman, 1997). In view of the lack of content specific pedagogical approaches to the teaching of mathematics, many rich opportunities for reasoning and sense-making in the field of mathematics are lost (Ross et al., 2011). To this end, appropriate practical, meaningful and student-centered approach to teaching has been the subject of quite a number of researchers (Addae & Agyei, 2018; Asomah et al., 2018; Blackett & Tall, 1991; Gur, 2009; Johnson et al., 2014).

One of such pedagogies that that allows students to make meaning of the subject matter independently and collectively as a group in the classroom context is the cooperative teaching and learning approach

(CTLA) (Johnson & Johnson, 1998; Slavin, 2013). This is because it provides consciousness on the part of instructors to ensure that learning experiences are tailored in a manner that maximizes learners' engagement during instruction (Johnson et al., 2014).

Further, the used of CTLA as a pedagogical tool in the classroom context, advances a particular a kind of reasoning that is superior, motivates and improves the inter and intra relationships among individuals group members (Edekor & Agbornu, 2020; Slavin, 2013; Vernon et al., 2022). Again, by encouraging learners to work cooperatively, the individuals can appreciate the sense of belongingness to support each one another during the teaching and learning process (Bosson-Amedenu et al., 2021; Gillies, 2003; Gillies & Ashman, 1998). However, its implementation in the classroom context have been researched to be problematic (Gillies & Boyle, 2010; Sharan 2010). Again, the features of cooperative learning seem to be interrelated to each other and interact in varying ways to influence teachers' decisions of its use as an instruction in the classroom (Johnson et al., 2014; Slavin, 2013). Thus, the need to equip teachers with the pre-requisite knowledge and skills to identify and differentiate each of these features of the CTLA to enhance students understanding of the subject matter. Consequently, strengthening the teacher's capacity to design, develop and enact lessons anchored on the CTLA (Hamadi et al., 2022).

In this study, therefore, attempts were made to explore the extent to which these features are distinctly designed to fit into mathematics teachers' pedagogy especially in the area of trigonometry. To this end, we explore the aptitude, instructional as well as content knowledge of the in-service mathematics teachers (IMTs) to gain a deeper understanding of the use of CTLA as a pedagogical tool in the mathematics classroom context.

The Study Context

In the Ghanaian classroom context, the use of CTLA as a pedagogical tool have been characterized by teacher's misconceptions and misrepresentations (Assan-Donkoh et al., 2019; Nurhuda et al., 2018). The teachers limited knowledge and skills on the features of CTLA as instructional strategy in the classroom context, could be a contributory factor for their unwillingness to implement this teaching strategy (Johnson et al., 1991; Shafiuddin, 2010). Although studies in relation to CTLA have been carried out, they fall short in their characterization of cooperative learning as a pedagogy where its features are purposefully designed, iterated, and deployed with the aim of achieving some specific goals in the classroom context (Oloyede et al., 2012; Yousef et al., 2021). In order to achieve this goal, the study engaged IMTs in the senior high school (SHS) to design learning experiences that enhance students understanding of mathematics particularly in the area of trigonometry. This is because many researchers (e.g., Asomah et al., 2022a; Farouq, 2017; Kendal & Stacey, 2003; Miller, 2001) argue that the current teaching practices do not enhance student's learning outcomes in trigonometry. In particular, Farouq (2017) explored students' error in learning trigonometry among final year SHS mathematics students. The findings of the study depicted errors in process skills and transformation in solving trigonometry problems. Perhaps, an effective application of the features CTLA could be the solution to facilitate students understanding of Trigonometric concepts. To this end, cooperative learning was contextualized as a pedagogical tool in enacting lessons in the area of trigonometry by IMTs at the SHSs.

FEATURES OF COOPERATIVE TEACHING AND LEARNING APPROACH

Positive Interdependence

Positive interdependence is the heart of cooperative efforts (Hamadi et al., 2022). The use of positive interdependence requires of students to perceive that:

- (a) they are linked with groupmates in a way so that they cannot succeed unless their groupmates do (and vice versa) and
- (b) groupmates' work benefits them, and their work benefits their groupmates (Johnson et al., 2014).

In this way, positive interdependence is characterized by assigning students to small groups and assigning problem for them to solve (Smith et al., 2005). The grouping of the students into small teams is to afford facilitators an opportunity to ensure that content that relevant to the subject matter are mastered during instructions by all the individuals who constitute the team (D'Eon & Zhao, 2022). It therefore lends credence to the effect that teachers create an atmosphere in a learning environment such that, the student in the learning environment success is reliant on the ideas, involvement, and accomplishment of the others in the group (Kaymak et al., 2022).

As such, students with a sense of positive interdependence exhibit reliance on the ideas and contributions of group members to promote the learning outcomes of all involved (Johnson et al., 1991; Kyndt et al., 2013; Roseth et al., 2008). In this study, the use of positive interdependence was characterized by a lesson activity that positions the classroom facilitator to put students into groups of specifically assigned responsibilities on a learning task. This is purposed to facilitate the mastery of the subject matter collectively as a group in the learning environment.

Individual Accountability

Each group member is individually accountable to contribute his or her fair share of the group's learning task (Teng, 2022). Thus, it entails carrying out one's part of the task and offering support to other group members to finish off their task in the learning environment (Bores-García et al., 2021). Again, with the incorporation of individual accountability, "students learn together to gain greater individual competency" (Johnson et al., 1991, p. 419). Further, individual accountability is more pronounced among students in diverse backgrounds with academic capabilities. Their assignment to teams of learning is purposed to influence weak students positively (Fernández-González & Franco-Mariscal, 2021; Michaelsen et al., 1982).

Individual accountability is one of the pillars which teachers use to ensure that individual team members in a team are held answerable in their participation of a learning task assigned to the group (Jurkowski & Hänze, 2015). This was purposed to ensure learners master the content of the concept taught. Thus, it ensures that students learn together, but perform alone (Johnson et al., 1991; Kyndt et al., 2013). To this end, Individual accountability was characterized in the current study by a lesson activity that ensures students learn together but performs alone. Thus, the lessons are structured by

- (a) observing students as they work together noting the contributions of the individual members in the group,
- (b) having individual group members explain concepts taught to their colleagues, or
- (c) piecing their individual work output together in order to reach consensus on a solution to a learning task or test.

Promotive Face to Face Interaction

Students promote each other's success by helping, assisting, praising, encouraging, and supporting each other's efforts to learn (Baghcheghi et al., 2011). Doing so results in such cognitive processes as discussing the nature of the concepts being learned, orally explaining to others how to solve problems, teaching one's knowledge to classmates, challenging each other's reasoning and conclusions, and connecting present with past learning (Nam & Zellner, 2011; Van Ryzin & Roseth, 2019). Promotive interaction also includes interpersonal processes such as supporting and encouraging efforts to learn, jointly celebrating the group's success (Slavin, 2013). In this way it affords teachers an opportunity in the classroom context to get students to deliberate, question and offer the needed assistance to one another to complete the work assigned to the group (Johnson et al., 2007). Hence, it is a significant step in establishing CTLA in the learning environment. In this study, therefore, promotive face-to-face interaction was characterized by a lesson activity that positions the classroom facilitator to engage students in groups purposed to share their individual and collective ideas on a learning task with the view to brainstorming on such task to proffer solutions to the learning task.

Social Skills

Contributing to the success of a cooperative effort requires interpersonal and small group co-habituating skills (Choi et al., 2011). In cooperative learning groups, students are expected to use social skills appropriately in order create conducive atmosphere pre-requisite for learning (Tanner et al., 2003). In order to achieve such an atmosphere in the learning environment leadership, trust-building, communication, decision-making, and conflict-management skills have to be taught just as purposefully and precisely as academic skills. (Johnson et al., 2014). Social skill therefore is viewed as one of the proficiencies desirables for an effective learning to take place (Ning, 2011). Thus, if learners cooperate with one another, respect and promote the views of their colleagues and coordinate activities in the group in a hierarchical order, in terms of channeling aggrievances then the feature of social skill is said to have been mastered in such a learning environment (Buchs et al., 2011). Thus, for a fruitful discussion to occur, students need to foster whole class discussions on the need to co-habit with one another in an atmosphere conducive for learning (Opitz, 2008). While the various attributes of social skill as outlined above could not be taught as part of the study owing to standard time acceptable at the SHS in the Ghanaian context, the authors operationalized social skill in the study based on the perspective provided by Opitz (2008). Thus, social skill was characterized by a lesson activity that affords the teacher an opportunity to foster whole class discussions on the guidelines that facilitate successful group activities in the learning environment with the view to creating conditions devoid of mockery and negative dispositions during intra and inter group activities/discussions in the classroom context.

Group Processing

Students need to engage in group processing (Havenga & Swart, 2022). Group processing may be defined as the examination of the effectiveness of the process members use to maximize their own and each other's learning, so that the ways to improve the process may be identified (VanRyzin & Roseth, 2019). Group members need to describe whether member actions are helpful and unhelpful. This is to ensure that all group members

- (a) achieve and maintain effective working relationships,
- (b) decide what behaviors to continue or change, and
- (c) celebrate group members' hard work and success (Johnson et al., 1991; Kyndt et al., 2013; Roseth et al., 2008).

In this way, an opportunity is afforded the group members to explain their collective work output to their peers based on the learning task assigned to them (Bores-García et al., 2021). In assessing the success or otherwise of the work undertaken by a group in a cooperative learning environment, group processing affords the individuals who make up the team a platform in the learning environment to do a retrospection of the quality of work executed by them (Johnson et al., 2014). To this end, group processing was characterized in this study, as a lesson activity that maximize students understanding of the subject matter. Thus, an atmosphere in the learning environment where students present/explain their solution to a task to the class. It further, positions the classroom facilitator to solicit an alternative (superior) process used in solving same task from other students in the class. Following which the teacher provides clarity to all questions from the students.

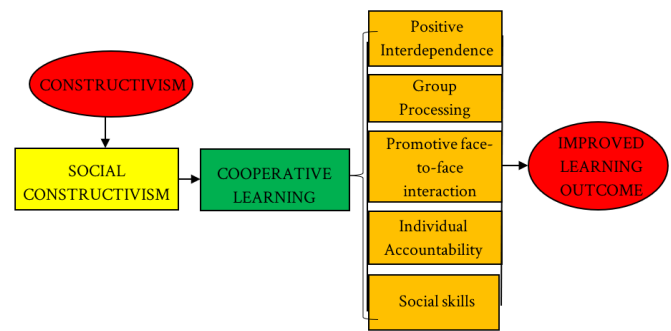


Figure 1. The conceptual framework of the study
(Authors' own source 2022- from Asomah, 2022)

In a nutshell, these five basic features as discussed served as the items that informed the preparations of lessons by the IMTs in the current study. This was purposed to afford IMTs to:

- (a) structure their lessons to be reflective of the feature of CTLA,
- (b) fine-tune and adapt CTLA to the specific needs and circumstances of their students, and
- (c) intervene in malfunctioning groups to improve their effectiveness.

Thus, these five features position teachers to organize lessons where students are actively engaged in the learning task with the aim of making meaning of the materials individually and collectively as a group (Choi et al., 2011; Fernández-González & Franco-Mariscal, 2021; Johnson et al., 1991).

TOWARDS THE CONCEPTUALIZATION OF THE STUDY

The CTLA as a pedagogical tool was employed in the current study to facilitate the teaching and learning of mathematics. This is because it subsumes student working in teams to accomplish a common goal (Johnson et al., 2014; Slavin, 2013). The CTLA is largely based on the theory of constructivism that projects a particular type of learning environment where learners make meaning of the content themselves (Dalingier et al., 2020). In this way learners are the fulcrum around which the teaching and learning process revolves (Bonk & Cunningham, 1998), and teacher's teaching strategies focus on student-centered pedagogical strategies and cognitive learning (Darling-Hammond, 2017; Ormrod, 2004). Specifics is made on the social constructivism, which according to Prawatt and Floden (1994) maintain that the social nature of knowledge, the belief that knowledge is the result of social interaction and language usage, and thus is a shared, rather than an individual experience. As result, a meaningful learning occurs when individuals engage in social activities and consequently unite into a "dynamic whole"; in this way innovations from individual learners could be modified to reflect innovations of learners working together in groups (Kyndt et al., 2013; Roseth et al., 2008; Vernon et al., 2022). The theory therefore positions the facilitator to drive the individual learners' intrinsic motivation towards the completion of a task (Hamadi et al., 2022).

Figure 1 presents the conceptual framework for the study. It highlights the study's overarching theory of constructivism specifically, its social aspect of which cooperative learning is a tenant to yield an increased learning outcome in mathematics.

Thus, the framework illustrates that, an improvement in learning outcomes would be actualized from mathematics lessons where teachers design lessons based on CTLA in a manner that allows students to make meaning of the materials independently and collectively as a group. CTLA has therefore been operationalized in this study as a teaching method that involves student working in teams to accomplish a common goal, conditioned on inclusion of positive interdependence, individual accountability, promotive face-to-face interaction, social skills, and group processing (Johnson et al., 1991, 2014).

Professional Development Arrangement

The study is aimed at enhancing the pedagogical strategies of IMTs through the effective utilization of CTLA-intervention as a pedagogical tool in the classroom context. To this end, IMTs were introduced to a professional development arrangement (PDA) where series of teaching try outs anchored on the features of CTLA were conducted. The overall goal was to provide an arrangement to develop the IMTs pedagogical knowledge and skill in developing and enacting cooperative-based lessons. The PDA was organized in two phases categorized in the current study as phases one and two. Each of these phases was characterized by three stage activities, as follows:

1. an introductory workshop for the IMTs,
2. design of CTLA- lessons by IMTs in groups of two, and
3. implementation of lessons designed by the IMTs.

The essence of which was to equip the IMTs with the pre-requisite knowledge and skills as well as the competencies needed to ensure an effective application of the features of cooperative learning as a teaching method in the classroom context. At the introductory workshop for phase one of the study, the first cohort of four IMTs were placed in teams of two; team 1 (T1T1; T1T2) being team one teacher one (T1T1) and team one teacher two (T1T2) as well as team 2 (T2T1; T2T2) also designated as team two teacher one (T2T1) and team two teacher two (T2T2). The IMTs were tasked with designing their own lessons (lesson plans and students' worksheets) that incorporate CTLA as a pedagogical strategy to teach mathematics (the concept of trigonometry). Each team had a week to complete this task focusing on the topics, angles of elevation and angles of depression. A teacher from each of the teams enacted CTLA lessons with their colleagues (other members of the team) and some mathematics teachers who served as students (student teachers) in the learning environment in a peer-teaching session.

An evaluation of the CTLA lessons and the pedagogical approach used as the medium of instruction were reviewed at the end of each round of the peer-teaching sessions. This was to enable the teams to improve upon their lessons and enact second round of peer-teaching try-out lessons following from the feedback received. The outcome of the first phase of the PDA for the first cohorts of IMTs was used to inform the second phase of the PDA. In the second phase of the PDA, the second cohorts of four IMTs were also grouped in teams of two; teams 3 (T3T1; T3T2) and teams 4 (T4T1; T4T2). They were assigned to design CTLA lessons after an introductory workshop. A member each from the teams (T3T1; T4T1) enacted their designed lessons first among themselves and colleague mathematics teachers who served as students in a designed classroom situation (peer-teaching). The last teachers (T3T2; T4T2) used the feedback obtained from their peers to modify their lessons. Lastly, T3T2 and T4T2 implemented their modified lessons in a real classroom learning environment at the SHS.

Table 1. Gender distribution of IMTs involved in the study

Gender	n	%
Male	7	87.5
Female	1	12.5
Total	8	100.0

Participants

The study purposively engaged two cohorts of four IMTs (thus, professional mathematics teachers who teach mathematics at the SHS) in four out of the ten SHS in the Cape Coast metropolis in Ghana. Thus, professional mathematic teachers who were credited with bachelor's education degree in mathematics from recognized tertiary awarding institutions (university level) participated in the study. This is because, they could be said to possess the pre-requisite content knowledge of the subject matter and can bring to bear critical and reflective thinking in their analysis of educational issues in general as well as curricular and pedagogical issues in the study. The gender distribution of the IMTs in the study is shown in **Table 1**.

From **Table 1**, majority of the IMTs involved in this study were males with only one of IMTs being a female. The average age of IMTs in this study was 37 years (SD=6.33). The minimum age was observed to be 27 years while the maximum age was 42 years. It was found that the IMTs in the study had an average teaching experience of approximately 10 years (SD=3.85). This depicts the ample experience of IMTs in the teaching of mathematics at the SHS level.

Research Question and Research Design

The main research question that guided the conduct of the study was, to what extent did the IMTs' pedagogical content knowledge and skills in the design and implementation of CTLA lessons develop and impact on their experiences? Data was collected at phases one and two of the PDA initiated in the study. The study which engaged eight IMTs at the two phases of the PDA employed a sequential multiple case study design as the appropriate research design. This design was considered suitable as it afforded the researcher an opportunity to provide a detailed description of the phenomenon under study (Gerring, 2007). Further, data for the second cohort (case) of IMTs were informed by the outcome of the first cohort (case). Thus, grounding the appropriateness of the design used. Purposive sampling technique was used in the selection of the eight IMTs who participated in the study. This is because teachers with bachelor's education degree in mathematics from were used in the study. As they were positioned to possess the pre-requisite content knowledge of the subject matter and brought to bear critical and reflective thinking in their analysis of phenomenon as well as curricular and pedagogical issues in the study. Thus, the participants were best suited to respond appropriately to the research questions, which guided the study.

Instrument

Both quantitative and qualitative methods were employed as result of which different research instruments were used in the data collection. A pre- and post-IMTs CTLA self-assessment questionnaire and an essay type question on the concept of trigonometry were administered in the study. A five-point Likert scale format (from 5-strongly agree to 1-strongly disagree) questionnaire grouped under the features of CTLA was used. This was employed to track the knowledge of the IMTs about the five main features that underpinned the implementation of CTLA lessons. Cronbach's alpha reliability estimates

Table 2. IMTs pre- and post-PDT on CTLA in the first phase of the study

CTLA subscale	Pre-test		Post-test		Gains
	Mean	Standard deviation	Mean	Standard deviation	
Positive interdependence	2.13	.29	5.00	.00	2.87
Promotive face-to-face interaction	2.58	.32	4.63	.24	2.05
Social skills	2.54	.37	4.88	.16	2.34
Group processing	2.35	.33	4.44	.17	2.05
Individual accountability	2.74	.32	4.14	.17	1.40

of 0.75-0.88 was considered reliable in the conduct of this study. Further, essay type questions in the area of trigonometry were administered and aimed at determining improvement in the IMTs content knowledge prior to and post-organization of the PDA. In addition, in each of the peer-teaching try-outs as well as the real classroom implementation of the CTLA lessons, the IMTs were interviewed. The interview guide was premised on IMTs experiences and opinions in the implementation of the CTLA lesson. In ensuring the content validity of the questions on the interview guide, the first draft of the guide was given to two experts in mathematics and ICT Education Department at the University of Cape Coast. The quality of the items in the interview guide were improved upon by the recommendations from the experts. Again, the lesson documents (lesson plans and student's worksheet) designed by the IMTs were collected at the end of each teaching session and analyzed.

Finally, an observation protocol was employed to collect data particularly during the implementation of the CTLA lessons. The adapted instrument has inter-rater reliability agreement of $\kappa=0.67$. According to Landis and Koch (1997), this is "substantial agreement," which is only second to "almost perfect agreement." The instruments used in the collection of the were analyzed. In particular, in relation to the interview guide, the dialogues were transcribed from audio to text and were given to respondents to cross-check for the purposes of confirming if their responses to the questions asked during the interview sessions were reflective of their views, and that, they were the true representations as captured by the researchers. In analyzing the interview data, pattern coding techniques (Miles et al., 2014) were used. The information recorded in the researcher's cooperative learning observation protocol was analyzed using data reduction techniques (Miles et al., 2014). Also, with regards to the CTLA lessons, a document analysis was conducted to determine the extent of the IMTs developed knowledge and skills in the use of the features of the CTLA. Thus, whether the lessons as prepared (in the form written lesson notes including the student's worksheet collected at the end of the lesson) and taught were reflective on the features of the CLTA. Descriptive statistics specifically, means and standard deviations was used in the analysis of quantitative instrument administered to the IMTs prior to and post-introduction of PDA.

Moreover, the essay type questions (in the area of trigonometry) were scored out of 100 as result of which scores below and above 50 were interpreted as below and above average respectively. To this end, independent sample t-test was used to determine if there is a significant difference between the means of two cohorts of IMTs who responded to the test. The test (IMTs pre- and post-test in relation to questions from trigonometry) was independent of each other. As there was no relationship between the observable groups. The data for each group were found to be approximately normally distributed. As a result of which no significant outliers and homogeneity of variance were observed in the data (Kim & Park, 2019). In conclusion, owing to the

qualitative nature of the current study, trustworthiness of the study was ensured based on criteria of "credibility", "transferability", "dependability" and "confirmability" (Lincoln & Guba, 1985).

RESULTS

The main research question was "to what extent did the IMTs pedagogical content knowledge and skills in the design and implementation of CTLA lessons develop and impact on their experiences?" Thus, the question sought to find out how the improvements in teaching using CTLA as a medium of instruction could be explained in relation to the improvements in the IMTs pedagogical knowledge and skills as well as their content knowledge. Based on the data obtained from the study, the improvements in IMTs pedagogical knowledge and skills and content knowledge in the design and implementation of CTLA lessons in mathematics as well as its impact on their experiences can be explained from two main perspectives, as follows:

1. IMTs developed knowledge and skills in enacting CTLA lessons (thus, the extent to which the lessons were reflective of the five features of CTLA) and
2. IMTs improved content knowledge in the subject matter (trigonometry) as a result of the application of CTLA lessons.

In-Service Mathematics Teachers' Developed Knowledge and Skills in Enacting CTLA Lessons

In responding to the research question, the IMTs developed knowledge and skills in enacting CTLA lessons, data was measured quantitatively in the light of the administration of the 5-point Likert scale questionnaire prior to and post-introduction of the PDA to the two cohorts of four IMTs who were involved in phases one and two of the study. The purpose of the CTLA self-assessment questionnaire (pre- and post-) was to track the IMTs knowledge of the features of the CTLA intervention before and after the PDA to ascertain the extent of their developed knowledge and skills in enacting CTLA lesson. The results of the pre- and post-CTLA self-assessment questionnaire of eight IMTs are presented. Thus, the results of first batch of four IMTs from School X and last batch of four IMTs from School Y are presented in **Table 2** and **Table 3**, respectively.

Generally, the results depicted an improvement in all the CTLA subscales. This is because the mean scores for the IMTs' CTLA self-assessment before the professional development training are lower than the mean scores after the PDA. For example, for the first cohort of IMTs, it can be observed from **Table 2** that positive interdependence was rated as the highest with mean gain of (gain=2.87) followed by social skills (gain=2.34), promotive face to face interaction (gain=2.05), group processing and Individual accountability recording equal gains (gain=2.05), and (gain=1.40), respectively. This shows the improvement in the pedagogical knowledge and skills of the IMTs prior

Table 3. IMTs pre- and post-PDT on CTLA in the second phase of the study

CTLA subscale	Pre-test		Post-test		Gains
	Mean	Standard deviation	Mean	Standard deviation	
Positive interdependence	2.34	.29	4.92	.67	2.58
Promotive face-to-face interaction	2.48	.32	4.73	.55	2.25
Social skills	2.35	.37	4.85	.72	2.50
Group processing	2.33	.33	4.15	.54	1.82
Individual accountability	2.31	.32	3.81	.67	1.50

to and post implementation of the PDA. A further indication that the intervention (CTLA) achieved its intended purpose and the proof of materialization of the rationale behind the introduction of the PDA in the study.

Table 3 presents the mean scores for pre and post CTLA self-assessment for the last cohort of IMTs who were involved in second study. Similarly, analysis from **Table 3** reiterates an improvement in all the CTLA self-assessment for the second cohort of IMTs. For example, before the PDT, the IMTs indicated that they had less or no knowledge in relation to any form of activity where during the teaching and learning process deliberate effort ought to be made for the group's mastery of the subject matter to reflect on the individual student mastery of that subject matter. Again, the IMTs indicated that they had inadequate knowledge in relation to any form of activity during lessons where students project their work (solution to a task) on the chalkboard purposed to elicit alternate ideas (different approach to the resolution of the question) from other students in the learning environment.

The lowest recorded mean values of individual accountability ($M=2.31$ and $SD=0.32$) and group processing ($M=2.33$ and $SD=0.33$) affirm these assertions. However, the post-PDA revealed that IMTs seem to have improved not only on the subscales of individual accountability ($M=3.81$ and $SD=0.67$) and group processing ($M=4.15$ and $SD=0.54$) with gains of 1.50 and 1.82, respectively over their pre-PDA but in all domains relating to CTLA. For instance, promotive face-to-face interaction ($M=4.73$ and $SD=0.55$) an increment of 2.25 main gain from the pre-PDT mean with a higher standard deviation. Also, positive interdependence and social skills recorded mean values of 4.92 ($SD=0.67$) and 4.85 ($SD=0.71$), respectively.

It is obvious from **Table 2** and **Table 3** that both first and second cohorts of IMTs had little knowledge about the use of CTLA in teaching mathematics prior to the PDA conducted in the study. However, the results seem suggestive of IMTs being knowledgeable about the grouping of students with some assigned roles (positive interdependence) and organizing the learning environment to be accommodative of the views of other student during the teaching and learning process (social skills).

This is not surprising since it lends credence to the misconceived definition (as observed on the observation guide and recorded during interview sessions and focal group discussions between the researchers and MTs) attributed to the use of CTLA as just the organization of students into groups with some assigned tasks to the detriment of the other features, which corroborate to ensure a cooperative-based learning environment (with all of the five features at play in the classroom).

The responses given by the IMTs (first cohort and second cohort) suggest that as a result of their developed knowledge and skills in designing and enacting CTLA-lessons, they gained deeper understanding and appreciation of CTLA as a teaching method,

Lesson Plan		
	Teacher Activity	Student Activity
Main Lesson (30minutes)	<p>In order to prepare the students for the main concept in the lesson, the teacher</p> <ul style="list-style-type: none"> puts students into groups of five (5). assigns students into roles of responsibilities. Asks students to mention the trigonometrical ratios and define them base on the diagrams given. 	<ul style="list-style-type: none"> students are randomly assigned into groups of five students elect themselves into the following roles Leader- leads all discussions Groups Recorder- records all proceedings and presents solutions on behalf of group Timer- keeps the group checks of time during group discussion.
P. I		
ACTIVITY I		<ul style="list-style-type: none"> Groups Recorder- records all proceedings and presents solutions on behalf of group Timer- keeps the group checks of time during group discussion.
S. S	<ul style="list-style-type: none"> admonishes students to respect each other's opinion build consensus and cooperate with one another before a particular solution to a task is reached during group activities 	<ul style="list-style-type: none"> Students cooperate with one another in group activities.
Conclusion stage (20minutes)	<ul style="list-style-type: none"> The lesson ends by providing students with example and non-examples for the students to try <p>Given that $\sin x = \frac{\sqrt{3}}{2}$ for $90^\circ < x < 180^\circ$ and $\cos y = \frac{\sqrt{2}}{2}$ for $0^\circ < y < 90^\circ$</p> <p>Evaluate</p> <ul style="list-style-type: none"> $\cos(y - x)$ $\sin(x + y)$ 	<p>Sample questions:</p> <ul style="list-style-type: none"> Find, without using tables, the values of $\cos(\beta - \alpha)$ if $\tan \alpha = -\frac{3}{4}$, $\cos \beta = \frac{1}{2}$ where α is in quadrant II and β is in quadrant I.
G. P		

Figure 2. Excerpt of IMTs developed lesson plan on compound angles by team 4 (G.P: Group processing; S.S: Social skill; & P.I: Promotive interdependence)

(Authors' own source 2022- from Asomah, 2022)

identified weaknesses in this teaching method for mathematics lessons (as observed under focus group discussions) and in addition, became more informed about the features of the CTLA to adopt in order to maximize learners understanding and conceptualization of mathematical concept. Further, the IMTs were interviewed, and their lessons observed to provide the qualitative data to explain improvements in the knowledge and skills of teaching with the CTLA. Thus, data was collected from the IMTs reported lessons, observed data, focus group discussions and interview.

The IMTs lessons (lesson plans and activity sheets) were analyzed through document analysis. Also, data from the interview and focus group discussions were analyzed by the identification of themes as well as patterns in relation to the IMTs developed knowledge and skills in enacting CTLA lessons. For instance, the lesson plan designed by team 4 (T4T1 and T4T2) and team 3 (T3T1 and T3T2) for teaching of the topics: Compound Angles and three ratios of trigonometry respectively, showed evidence of IMTs improvements in their competencies in teaching SHS trigonometry using CTLA through a series of teaching try outs with an improved lesson plan document. This is because the first lesson plan document designed by team 4 (T4T1 and T4T2) and team 3 (T3T1 and T3T2) depicted an incomplete reflection on all the features of the CTLA (thus, there was an omission of the features of face-to-face promotive interaction and individual accountability) of the lesson plan initially developed by team 4.

Figure 2 shows excerpt of illustrations of IMTs initially developed lesson plan on compound angles team 4.

Lesson Plan	
Teacher Activity	Student Activity
<p>Main lesson stage (50 minutes)</p> <p>In order for students to acquire their own practical understanding of the concept of Sine, Tangent and Cosine of an angle in a right-angle triangle, the teacher</p> <ul style="list-style-type: none"> puts students into groups of four assigns students into roles of responsibilities <p>POSITIVE INTERDEPENDANCE attends to the various groupings in the class for the purposes of ensuring successful group activities</p> <p>INDIVIDUAL ACCOUNTABILITY</p>	<ul style="list-style-type: none"> students are randomly assigned into groups of three students elect themselves into the following roles Leader- leads all discussions in Groups Recorder- records all proceedings and present solutions on behalf of the group Timer- keeps the group in checks of time group discussion. <p>Some discussion points expected from students:</p> $f(x) = \text{Sin}x = \frac{\text{opposite}}{\text{hypotenuse}}$ $f(x) = \text{Cos}x = \frac{\text{hypotenuse}}{\text{adjacent}}$ $f(x) = \text{Tan}x = \frac{\text{opposite}}{\text{adjacent}}$ <p>students' pieces together the most appropriate solution to the problem as a collective solution reflective of the groups</p>
<p>Conclusion stage (15 min)</p> <p>Finally, sums up the whole lesson by highlighting the key concepts discussed as follows:</p> $f(x) = \text{Sin}x = \frac{\text{opposite}}{\text{hypotenuse}}$ $f(x) = \text{Cos}x = \frac{\text{adjacent}}{\text{hypotenuse}}$ $f(x) = \text{Tan}x = \frac{\text{opposite}}{\text{adjacent}}$	<ul style="list-style-type: none"> Group leaders lead discussions on the solutions to the problems after the group's recorder presentation of the solution on the board GROUP PROCESSING Students listen and take notes of the core points as the teacher provides clarity to some questions.

Figure 3. Excerpt of IMTs developed lesson plan on trigonometric ratios by team 3 (Authors' own source 2022- from Asomah, 2022)

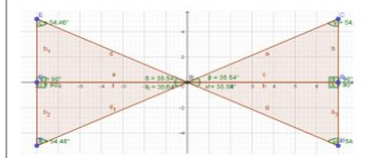
Lesson Plan	
Teacher Activity	Student Activity
<p>Main Lesson (30minutes)</p> <p>Activity II: (30minutes)</p> <p>Students resolve the length of hypotenuse side individually = INDIVIDUAL ACCOUNTABILITY</p> <p>Student divides in groups the processes of solving the trigonometric ratios in the diagram = FACE-TO-FACE PROMOTIVE INTERACTION</p>  <p>Presents the task to the various groups as Activity III = POSITIVE INTERDEPENDANCE</p>	<ul style="list-style-type: none"> Students follow the guided activities of the student's worksheet on activity II Students' reminiscent of their pair share model finds solution to the task on activity sheet II. SOCIAL SKILLS students' pieces together the most appropriate solution to the problem as a collective solution reflective of the groups
<p>Conclusion stage (20 minutes)</p> <p>The lesson is ended by providing students with example and non-examples for the students to try</p> <p>Given that $\sin x = \frac{\sqrt{2}}{2}$ for $90^\circ < x < 180^\circ$ and $\cos y = \frac{\sqrt{2}}{2}$ for $0^\circ < y < 90^\circ$ Evaluate</p> <ul style="list-style-type: none"> $\cos(y - x)$ $\sin(Ce + 9)$ <p>Teacher guides the students to solve question 11 and asks students to solve question 13 in groups = GROUP PROCESSING</p>	<p>In groups students try question 'b' and present their solution nicely on board and shared it with class.</p> <p>Sample questions:</p> <ul style="list-style-type: none"> Find, without using tables, the values of $\cos(\beta - \theta)$, if $\tan \theta = \frac{1}{2}$, $\cos \beta = \frac{1}{2}$ where θ is in quadrant I.

Figure 5. Excerpt of the IMTs (in team 4) developed CTLA lesson on compound angles (Authors' own source 2022- from Asomah, 2022)

Lesson Plan	
Teacher Activity	Student Activity
<p>Main lesson stage (60 minutes)</p> <p>Activity 1 Definition of Sine, Tangent and Cosine of an Acute angle in a right-angle triangle (20 minutes)</p> <p>In order for students to acquire their own practical understanding of the concept of Sine, Tangent and Cosine of an angle in a right-angle triangle, the teacher</p> <ul style="list-style-type: none"> puts students into groups of four with assigned students into roles. <p>SOCIAL SKILLS</p> <p>solomonides students to respect each other's opinion both consensus and cooperate with one another before a particular solution to a task is reached during group activities</p> <p>Asks students to explore the comparison of the ratios of length in a right-angle triangle individually using the activity sheet</p>	<p>POSITIVE INTERDEPENDANCE</p> <ul style="list-style-type: none"> students are randomly assigned into groups of three students cooperate with one another in group activities <p>Some discussion points expected from students:</p> $\text{Sin}A = \frac{\text{opposite side to angle } A}{\text{hypotenuse side to angle } A} \quad A = 30^\circ$ $\text{Cos}A = \frac{\text{adjacent length to angle } A}{\text{hypotenuse length to angle } A} \quad A = 30^\circ$ $\text{Tan}A = \frac{\text{opposite length to angle } A}{\text{adjacent length to angle } A} \quad A = 30^\circ$ <p>students' pieces together individual solutions to the problem as a collective solution reflective of the group</p>
<p>Conclusion stage (40 min)</p> <p>Students transition on the concept in groups to find solutions to the task</p> <p>calls to the board, the group's recorder to present solutions on behalf of the group</p> <p>FACE TO FACE PROMOTIVE INTERACTION</p> <p>GROUP PROCESSING</p>	<ul style="list-style-type: none"> Group leaders lead discussions on the solutions to the problems after the group's recorder presentation of the solution on the board

Figure 4. Excerpt of the IMTs (in team 3) developed CTLA lesson on trigonometric ratios (Authors' own source 2022- from Asomah, 2022)

Similarly, the design of the initial lesson plan on the concept of trigonometric ratios by team 3 showed evidence of absence of face-to-face promotive interaction and social skills features of the CTLA although they were implied during the teaching and learning process. The features present in the lesson plan have been color shaded for the purposes of clarity and identification in the excerpt.

Figure 3 shows excerpts of the first lesson designed by team 3.

However, after a series of teaching try outs, focal group discussions and cooperation among team members, teams 3 and 4 improved in their competencies in the design of lessons reflective on all the features of the CTLA on the concepts of trigonometric ratios and compound angles

respectively. In particular, IMTs in team 4 and team 3 used their knowledge of the affordances of the features of CTLA to

- guide learners in the acquisition of knowledge through the use of the students' worksheet of activities to conceptualize compound angles and the three trigonometric ratios of sine, cosine and tangent of angles taught during instruction,
- stimulate learners' thinking abilities and enhance their understanding of concepts in trigonometry, and
- enhance their teaching method in student-centered manner (i.e., making use of an activity-oriented based lesson reflective on the features of CTLA).

Figure 4 and Figure 5 show evidence of an improvement in the design of lessons reflective on the five features of CTLA over their first developed lesson. Thus, an illustration of how team 3 improved in their lesson plan document to be reflective on all the five features of the CTLA as seen in their re-designed lesson plan informed their teaching of trigonometric ratios in the SHS (schools A). Similarly, the IMTs in team 4 made use of their improved lesson plan document reflective on the features of the CLTA in a way that enhanced their teaching strategies as evidenced in Figure 5. Figure 5 illustrates how the IMTs in team 4 developed CTLA as seen in their lesson plan informed their teaching of compound angles in the SHS.

The lesson plan documents of teams one and two were used to inform and improve the teaching methods (CTLA lessons) of teams three and four. As such, the two lesson plan documents that were respectively developed by teams three and four and consequently implemented in the real classroom context were an improvement on the initially developed lessons of teams one and two and reflected on the five features of the CTLA. Hence, the CTLA lessons enacted in the study were a total deviation from the conventional lesson plan documents (see Figure 4 and Figure 5) used by the teachers in the Ghanaian SHSs. Again, the IMTs corroborated the improvement in their teaching methods during an interview after CTLA-based lessons. For example, the three ratios of sine, cosine, and tangent of acute angles

CORE MATHEMATICS (SHS 2)

Student Worksheet on Trigonometric Identities

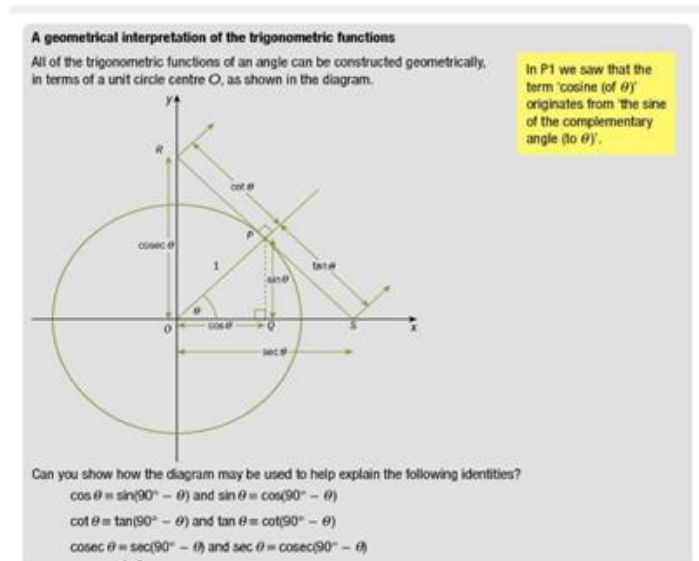
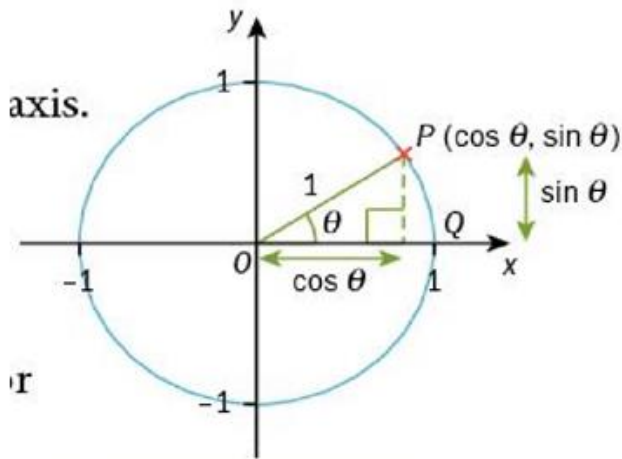


Figure 6. Excerpt of activity sheet used by team 1 (T1T1 and T1T2) for the lesson on trigonometric identities (Authors' own source 2022- from Asomah, 2022)

in a right-angle triangle, the compound angles, the angles of elevation and depression as well as the trigonometric identities, which ordinarily would have been taught in abstract was implemented with ease through the affordances of the features of the CTLA.

Some of the responses given by the IMTs to the question: “Was the integration of the activity-oriented worksheets helpful in teaching trigonometry with the CTLA in this lesson?” were reported, as follows:

T1T1: It ensured an orderly and systematic presentation of the concept during the teaching and learning process.

T1T2: It provided students with hands on activities needed to mobilize the knowledge base to understand the concept.

T2T1: It was a source of motivation to the students in developing a good understanding of the concept taught.

T2T2: Students made sense of the materials themselves as result of their participation in the activities in the lesson.

T3T1: Yes, it helped me to engage the students and sustain their attention throughout the lesson.

The responses given by the IMTs (first cohort and second cohort) suggest that as a result of their developed knowledge and skills in the design and enactment of CTLA-lessons, they gained deeper understanding of CTLA as an instructional approach in the learning environment. They became more informed about the features of the CTLA to adopt in order to maximize learners understanding and conceptualization of trigonometric concept.

In-Service Mathematics Teachers' Improved Content Knowledge in the Subject Matter

In measuring IMTs improved content knowledge in the subject matter, three main criteria were employed. The first criterion was the quality of the information provided in the lesson plan documents, the second criterion was the quality of delivery (i.e., clarity in explanation of concepts). Thus, the quality of the content of trigonometric concept presented in the lesson plan (notes) and the ease with which the IMTs articulated and responded to the students' questions. The last criterion employed was the pre-test and post-test on trigonometric questions conducted prior to and post implementation of the PDA in the study. In particular reference to team 1 (T1T1 and T1T2) who instructed their peers on the topic: trigonometric identities, the IMTs used an activity in the form of right-angled triangle to explain how trigonometric identities were derived using the unit circle approach. He explained to the students that with a pair of compasses and a rule, the students were to construct a unit circle of a centimeter (radius) using the origin as the center and pick any arbitrary point say $T(x, y)$ in the first quadrant. From their knowledge of tangent of an angle as the ratio of the opposite side to the adjacent side, they were asked to find $\tan \alpha$ from the diagram as shown on the students' worksheet (see Figure 6).

In addition, the IMTs tasked the students to find the relation that connects all three sides of the right-angled triangle (applying the Pythagoras theorem). Finally, a student each from the different groups was asked to write down the Pythagoras theorem (students write the Pythagoras theorem mathematically as: $x^2 + y^2 = 1$).

The IMTs then guided the students to substitute the expression for the vertical and horizontal sides of the of the diagram above into the mathematical form of the Pythagoras theorem (thus, students obtained $\sin^2 x + \cos^2 x = 1$). Also, the IMTs guided students to write equivalent forms

Table 4. IMTs pre- & post-test on trigonometry in the first phase of the study

	M	SD	Paired differences			
			Md	T	df	p
Phase I pre-test scores	4.87	.895	3.28	9.527	7	.000
Phase I post-test scores	8.15	.896				

Note. Source: Fieldwork, 2021

of the relation above by dividing the equation by \sin^2x , then with \cos^2x to obtain other forms of trigonometric identities. The clarity of explanations offered by the IMTs attests to their improved content knowledge of the trigonometric identity using the CTLA. A further clarity was evidenced on the self-explanatory activity sheet designed by team 1 (T1T1 and T1T2) used during the conduct of lessons on the trigonometric identities as exhibited in their lesson notes.

The IMTs did not only offer a superior form of explanations in relation to the mathematical concepts taught (trigonometry) but presented lesson documents that reflected on the features of the CTLA. The IMTs used suitable annotated diagrams to augment the teaching and learning process. The clarity in the explanations offered and their capability in handling questions from the students as observed (from the field notes on the cooperative observation protocol), demonstrated an improved knowledge in the subject matter. This stems from the background of the poor performance of the IMTs during the pre-test (see **Table 3**) on some questions on trigonometric identities conducted prior to the PDA. Moreover, the IMTs inability to respond to questions and the tendencies of some struggle (as observed) when quizzed about trigonometric identities during the researcher's sample demonstration of the CTLA lesson depicted the gap in the understanding of the subject matter prior to the PDA. However, after a series of teaching and learning processes with the CTLA, the IMTs exuded confidence and mastery over the same concepts (trigonometric identities) they had difficulty with prior to the PDA. Consequently, the interview data gathered were also reflective of this observation. For instance, some of the IMTs revealed in an interview that, they benefited a lot. Thus, they gained better understanding of some concepts in trigonometry, which hitherto was difficult. Some of the remarks recorded post CTLA-lesson were in respect of the question: "What general comment can you make about using CTLA-intervention in teaching mathematics in SHS?"

T1T1: It improved my teaching style; I can confidently design an activity-based lesson on the content of trigonometry an area I thought was an abstract topic.

T2T1: I had difficulty in teaching some concepts in trigonometry especially using trigonometric identities, but I think I was able to overcome such anxiety. My lesson today was the best as compared to my classroom style of teaching.

Additionally, the improvement in IMTs content knowledge as a result of using the CTLA was further understood and explained from the perspective of the pretest and posttest that was conducted prior to and post implementation of the PDA. During the first and second study, the four groups of IMTs: team 1 (T1T1 and T1T2) and team 2 (T2T1 and T2T2) as well as team 3 (T3T1 and T3T2) and team 4 (T4T1 and T4T2) were administered pre- and post-tests consisting of the same items on the topics: angle of elevation and depression (application of trigonometric ratios) and compound angles and trigonometric identities before and immediately after implementing PDA. The test

Table 5. IMTs pre- & post-test on trigonometry in the second phase of the study

	M	SD	Paired differences			
			Md	T	df	p
IMTs pre-test scores	4.88	.895	3.27	9.987	7	.000
IMTs post-test scores	8.15	.896				

Note. Source: Fieldwork, 2021

required IMTs to detail the processes involved in arriving at the answers to the questions after which it was scored out of 100 using a marking scheme.

Table 4 and **Table 5** show the results of the paired sample *t*-test that was used to determine if the mean differences in the pre-test and post-test of the scores obtained by the IMTs prior to and post introduction of the PDA were significant in both the first and second studies.

From **Table 4**, there was a significant mean difference in the pre and post test scores of the IMTs ($M=3.28$, $t[7]=9.527$, $p=.000$). In addition, eta squared was calculated to determine the effect size. Consequently, an eta squared statistic of 0.91 was obtained from the analysis which depicts a 'large' effect size in the differences in the IMTs pre-test and post-test scores obtained before and after the introduction of the PDA. This shows that the IMTs improved their content knowledge in the post-test after the using the CTLA intervention. Similar findings were also observed in the results in the second phase. Thus, IMTs performance improved post introduction of the CTLA intervention to the second cohort of IMTs at the PDA.

Table 5 shows the results of the paired sample *t*-test of the scores obtained prior to and post introduction of the CTLA lessons to IMTs. **Table 5** shows the paired sample *t*-test with a significant (mean difference=3.27, 95% CI) mean difference. A very large eta squared (eta-squared=0.95) was obtained from the analysis which depicts that, there was significant difference in the IMTs pre- and post-test scores obtained prior to and post introduction of the CTLA lessons during the PD. Thus, IMTs performance in relation to their content knowledge improved after they were introduced to the CTLA intervention on the concepts of trigonometry. Even though there was a balance (equilibrium) in the assimilation and accommodation processes in the minds of the IMTs developed CTLA, they encountered some challenges with the use of CTLA-intervention for trigonometric lessons. Some of the challenges they faced during the initial stages of the peer-teaching try-outs were the difficulty in integrating the new method of teaching into their existing (old method of teaching) one. However, most of the teachers corrected this by broadening their horizon (schemas) to be reflective of the new method of teaching (accommodation) through the series of the teaching try-outs conducted in the study.

In conclusion, the IMTs showed improvement in the post self-assessment on the use of the CLTA as a teaching method in the classroom context as was measured quantitatively in the study. This assertion was qualitatively supported through the IMTs interviews, lesson plan document analysis and classroom observations. That, the IMTs developed the competencies in the affordances of the five features of CTLA to enact CTLA-lessons in the classroom context. As a result of which, an improved content knowledge of the IMTs prior to and post introduction of the PDA was observed in the study.

DISCUSSION

Notwithstanding the copious quantitatively biased cause and effect research on the use of CTLA in most studies (Bosson-Amedenu et al., 2021; Edekor & Agbornu, 2020; Kaymak et al., 2021; Nurhuda et al., 2018; Teng, 2022;) very few employs case study (Fernández-González & Franco-Mariscal, 2021) as research design as was used in the current study. Thus, many studies concentrate on the effect of CTLA. However, this study sought to explain the use of CTLA as a teaching strategy in the real classroom context (SHS classroom) post PDA. The study reports of an improvement in the knowledge and skills of the IMTs on all the features of CTLA. With some major change occurring in CTLA-related sub-scales: positive interdependence (gain=2.87), promotive face-to-face interaction (gain=2.25), individual accountability (gain=2.05), and group processing (gain=2.05). This observation as recorded in this study is in line with the results of recent studies (D'Eon & Zhao, 2022; Fernández-González & Franco-Mariscal, 2021; Jurkowski & Hänze, 2015; Nam & Zellner, 2011; Vernon et al., 2022).

On the challenges of the implementation of the CTLA by the IMTs, the current study contrasts the findings of Gillies and Boyle (2010) who delineated a number of issues of implementation associated with CTLA in his work on "Teachers' reflections on cooperative learning". Again, the issues of implementation sharply contrast with the studies of Johnson and Johnson (1998) and Sharan (2010). This is because, through the series of teaching try-outs, reviews, and feedback the problems encountered were overcome. Thus, registering the improvement in the teacher's knowledge and skills in the enactment of lessons anchored on the CTLA. Further, the study recorded a significant improvement in the content knowledge of the IMTs. This is because, some concepts in the area of trigonometry which the IMTs had difficulty explaining prior to the study was taught with ease after the PDA. In addition, an estimation of eta squared after the t-test analysis depicted an improved content knowledge prior to and post introduction of the PDA.

In particular, this finding is synonymous to a similar study by Nam and Zellner (2011) who revealed an improvement in the participants content knowledge after examining the effects of some of the features of the CTLA on student achievement and attitude in an online tuition. Moreover, the findings in the current study further affirm the results of the studies of Johnson et al. (1991, 2007), Kyndt et al. (2013), and Roseth et al. (2008). In relation to the initiation of PDA in the current study, the improvement in the IMTs knowledge and skills in the design and enactment of CTLA-lessons post introduction of the PDA aligns with Ho et al. (2001). Also, the use of the PDA in the study affirms the study conducted by Guskey (2002), where he asserts that PDA brings about changes in teachers' attitudes and beliefs.

Implications, Research Limitations, and Future Work

The findings and conclusions drawn from this study cannot be absolved from the inherent errors and limitations of the tools used in collecting the data, however the diverse methods were complementary as a result of triangulating them; thus, improving confidence in the findings. The sequential case study design that was employed also allowed for a limited number of participants (eight IMTs) who were pre-service teachers specifically in the area of mathematics in the current study. Consequently, generalizing of the findings in this study to a larger population was also limited. Thus, a replication of this study in future with different subject teachers in other subject disciplines such

as biology, physics, and chemistry as well as art related subjects will provide a better understanding of the deployment of CTLA as a pedagogical strategy at the SHS level.

The limitations of the study notwithstanding, the findings provide some insights into features that need to be maximized for effective deployment of the CTLA as a pedagogical tool in the classroom context in Ghana and countries with a similar context. Our results pose an interesting challenge to the acceptance and use of CTLA by the IMTs. The findings imply that, the conduct of the PDA and provision of readily available teaching and learning materials reflective on the features of the CTLA in subject specific areas should be a priority. Accordingly, the study indorses some ongoing national initiatives in education such as the introduction of the new curriculum which intend to use CTLA to facilitate teaching and learning at all levels of the educational cycle primarily at the basic and high schools.

CONCLUSION

We sought to explore and enhance effective ways of employing CTLA as a teaching method in the high school mathematics classroom context. This stems from the loophole created in relation to instructional theory and practice in the teaching and learning of mathematics especially in the area of trigonometry. Conditioned on the operationalization of the five main features of CTLA and commitments from teachers, heads of school authorities and management, the implementation of CTLA will broaden the educational horizon especially in the teaching and learning of mathematics to the understanding of multitudes of student.

In addition, it will serve as an innovative pedagogical strategy in the classroom context. Thus, based on the result of the study, we recommend that teacher education institutions consider training prospective teachers on how to enact mathematics lessons using CTLA, prioritize professional development training to sharpen high school teacher's competencies in using the CTLA in subject specific content pedagogies and admonish high school mathematics teachers to make conscious efforts to adapt the CTLA in order to increase the proficiency of high school and basic school students in the area of mathematics.

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Ethical clearance: The study was approved by the University of Cape Coast Institutional Review Board (UCCIRB) on November 11, 2021 (Protocol ID: UCCIRB/CES/2021/75).

Declaration of interest: Authors declare no competing interest.

Data availability: The research data used to support the findings of this study may be released upon application to the Institutional Review Board, Ghana-Ethical Clearance Form ID-UCCIRB/CES/2021/75, who can be contacted at University of Cape Coast, Ghana.

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