Contemporary Mathematics and Science Education
2023, 4(2), ep23020
ISSN 2634-4076 (Online)
https://www.conmaths.com/

Research Article

OPEN ACCESS

Geometric perceptions of pre-service teachers and their geometric achievement

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Citation: Banson, G. M., Bonyah, E., Boateng, F. O., & Owusu, R. (2023). Geometric perceptions of pre-service teachers and their geometric achievement. *Contemporary Mathematics and Science Education*, 4(2), ep23020. https://doi.org/10.30935/conmaths/13341

ABSTRACT

The study of geometry enables students to think critically and make reasonable assumptions. In this paper, we investigate the relationship between pre-service teachers' (PSTs) perception and achievement in geometry. Convergent mixed method design, which is a mixed method approach based on the pragmatist paradigm of research was employed and a sample of 225 second year mathematics students participated in this study. Furthermore, the purposive sampling technique was used to select 10 students in order to gather data qualitatively. Questionnaire, geometry test and structured interview were used in collecting data. The perception and achievement of PSTs of this study showed a positively weak correlation. Exploring perceptions of PSTs, respondents indicated that geometry is an aspect of mathematics, and it entails shapes, angles, points, lines and many more. Participants also indicated that the content of geometry is not only about angles but there was one person who argued that geometry is all about angles. Since geometry is a broad aspect of mathematics, PSTs should be engaged in more of its content, even beyond angle properties and shapes. Some parts of geometry that deals with application of concepts in finding equations, making deductions and proving theorems should be stressed as vital in learning geometry.

Keywords: pre-service teachers, perception, geometry, achievement

Received: 28 Mar. 2023 ◆ Accepted: 27 May 2023

INTRODUCTION

Various aspects of mathematics are studied throughout the academic process and this situation is not so different in Ghana. One aspect of mathematics, that is geometry, is the main focus of this study and pre-service teachers' (PSTs) perceptions regarding it is prioritized in this study. As much as teaching and learning of geometry is concerned, the teacher's impact in the lessons of geometry cannot be underestimated. In Ghana, the content of geometry covered at the basic schools is centered around shapes or spatial attributes (Akayuure et al., 2016). Geometry covers a sizeable proportion of the syllabus and estimated at 17% of the mathematics syllabus. Teaching space and shape is rationally geared towards helping students to develop knowledge of shapes and space. This will enable them to acquire geometrical knowledge and to enhance their skills and spatial abilities in real life situations and prepare them for mathematics learning at a higher level. PSTs who are still in the process of training as prospective teachers may have their own perceptions in the area of geometry. Few studies have focused on PSTs' perception and achievement in geometry. Indeed, several studies have been done focusing on psychological constructs (like anxiety and motivation) and academic achievement, little has been done in the area of geometry, which is considered as an aspect of mathematics. This study aimed at examining the perceptions of PSTs as well as their achievement in geometry.

Research Questions

The study was guided by the following research questions:

1. What relationship exists between PSTs' perception of geometry and their achievement?
2. How do PSTs perceive geometry?

LITERATURE REVIEW

Lev Vygotsky's Sociocultural Theory

The sociocultural theory by Lev Vygotsky has played a vital role in education and especially social sciences. It depicts a clear knowledge of the role of socio-cultural influences in the learning environment. Vygotsky's theory has a strong link with this study if the perception and achievement of PSTs are to be clearly defined. When it comes to the sociocultural learning theory, learning happens in the environmental circumstance, and it occurs from interaction among the teacher and the peers of the learner (Abushariefeh, 2016). The sociocultural theory also lays emphasis on the fact that there is a zone that shows proximal development, where there can be an activity that cannot be solely accomplished by the student unless one gets help from other people. Teachers must be encouraged to employ scaffolding techniques, which requires that students acquire concepts or solve task by a systematic

approach to enable them to develop their cognitive dimensions (Ahmed, 2010). This therefore means that, the PSTs' methods and approach to solving geometrical problems is very important. The content of tasks in a geometry test and the accomplishment of PST can be judged depending on the person's strength of reasoning and the person's ability to apply various shapes and diagrams from one's environment. Teachers must internalize the geometry they learn, what to teach and how to teach. Vygotsky explains that the learner's action is mostly influenced by their socialization and association. It is justified that previous knowledge and experiences are not always forgotten by students, but they insensibly know of previous knowledge. As time goes on, they tend to build on the previous knowledge and experiences (Rusche & Jason, 2011).

Geometry

From historical times, geometry was actually seen as the measurement of earth (Dillon, 2018). The developments in mathematics and various progressive knowledge in the study of geometry has made us to understand that geometry deals with curves and surfaces of various dimensions and due to that there has been several branches within geometry; some of which are algebraic geometry, differential geometry, and others (Dillon, 2018). Geometry also addresses shapes and their associations among themselves. According to Salim and Tiawa (2015), these shapes can be solid shapes or plane shapes. Plane shapes have been grouped as shapes with flat surfaces and this means they have no thickness since they are generally two-dimensional. Plane shapes can be bounded by lines and curves sometimes. Basically, some examples of plane shapes are triangle, rectangles, pentagon, trapezium, etc. On the other hand, solid shapes are three-dimensional shapes. They have breadth, length and height. Examples are cube, cylinder, prism, pyramid, cuboid, cone, and many others.

There are several branches in geometry but for the purpose of this study and the background description of the participants, Euclidean and analytical geometry was the major content of geometry considered in this study. Euclidean geometry in the early parts of mathematics begun with postulates and definitions about shapes, which were generally accepted to be true. Some definitions, which have stood the test of time encompass that about lines, a point, a plane, angles and so on (Singmaster, 1982). For instance, Euclid defined a point as having no part and he also speaks of lines as length with no breadth. Many definitions are also given on angles. Euclid defines an angle as two lines that meet and inclined to one another, which do not lie in a straight line. He explains obtuse angle as that bigger than a right angle and that which is less than a right angle is termed acute angle. Euclid also explained a figure as that which has a boundary or boundaries. He further defines a circle as plane figure bounded by a single line with straight lines lying on it but are the same from one point to the other. This definition of circle makes the clarity that a circle has some parts like the center and diameter.

Analytic geometry constitutes more of algebraic equations. This means geometrical shapes like circles and lines are additionally described by equations algebraically (Ayre, 1965). Analytic geometry is an evident blend between algebra and geometry and by this, algebraic relationships can be drawn in geometry. These algebraic relationships are mostly explained by already known discoveries of geometry of which much attention has not been given. For instance, establishing a mathematical relationship between diameter and radius of a circle can be generally interpreted as $d=2r$, where d is diameter and r is the radius. Similar conceptual relations can be derived as $r=d/2$. Proportionally, we can also have $r=50\%d$. The geometry task at hand can determine the mathematical relation to be deduced. Additionally, Ayre (1965) indicates that in analytic geometry, a major impact is made in algebra as there is a visual display of algebra. Analytic geometry does not leave out algebra to stand alone neither does it omit geometry rather it shows the geometry displayed in algebra and conversely shows the algebra displayed in the geometry.

Perception Towards Geometry

Perception has been used in literature to imply the idea one has about something (Al Meslamani, 2019). Basically, learners' views about a subject refers to learners' perception about the subject. Naidoo and Kapofu (2020) explain perception as one's understanding of our surroundings based on sensual experiences and the processes the mind goes through to understand the environment. The learners' own understanding of a situation is greatly subjective based on individual exposures. Perception toward geometry refers clearly to the mental view a person has in the teaching and learning of geometry, but attitude goes beyond one's mental view. Learners are unequal so they hold different perceptions about geometry and the perceptions they hold can shape their behaviour toward geometry (Naidoo & Kapofu, 2020).

Geometry perception may not be attributed to learners' present view, but it can also be due to previous experiences in geometry or mathematics. Supportively, Lewis (1999) makes it clear that previous and current experiences can make someone have a unique perception about geometry. Past experiences of teaching methods, the teacher, classroom environment and many others have influential role in perceptions of learners. Perceptions in this present study focus on PSTs' views toward geometry.

Perception and Achievement

Studies in between perception and achievement have always looked at relationships or effects. For instance, Ahmad et al. (2017) studied relationship between mathematics perception of students and achievement and findings showed that there was negative relationship between students' perception and their achievement. Some studies have rather sought to investigate whether perception affects achievement or whether achievement affects perception (Maat & Zakaria, 2010). A study by Hagan et al. (2020) assessed the effect of perception on students' mathematics achievement and by making perception a predictor variable, mathematics achievement was made the criterion. The findings showed that perception did not significantly affect mathematics achievement although there existed a negatively weak association between the two variables. Hagan et al. (2020) concluded that the perception of students did not have any significant effect on their performance in the subject.

METHODS

Research Design and Paradigm

The paradigm for this study was based on the pragmatic paradigm and the design was a mixed method approach, specifically the convergent mixed method design. Sample for the data collection was not necessarily the same and quantitative and qualitative data were taken together. In a mixed method design a comprehensive and in-

depth data can be obtained due to the broad nature of the design (Creswell & Plano Clark, 2011). In a mixed method design, Cohen et al. (2017) indicate that the methodological dimensions need to cover both the quantitative and qualitative aspect, so it becomes vital that the quantitative and qualitative methods are explicitly justified. Basically, this design is structured in such a way that qualitative data and quantitative data are taken separately, and the analysis of the data is done separately. Interpretations can be given to the analyzed data, but the discussions can be done in a manner, where the qualitative findings confirms/disconfirms quantitative findings (Creswell & Creswell, 2017, p. 300). Creswell and Creswell (2017, p. 300) further explained that the quantitative findings may also confirm/disconfirm qualitative findings.

Participants

This study used 225 second-year mathematics education students of Akenten Appiah Menka University of Skills Training and Entrepreneurial Development. The systematic sampling technique was used to collect quantitative data from mathematics students in the second year. This was done by selecting students systematically from the entire population list. Additionally, the purposive sampling technique was used to select 10 students in order to gather additional data for the qualitative aspect. This process of sampling helps to collect sufficient data and to extensively explain the quantitative data too (Cohen et al., 2017).

Data Collection Procedures

Based on the aim of this study three major instruments were used for data collection. Questionnaire on perception was designed as well as achievement test on geometry, which was based on van Hiele levels of geometric thinking, and the geometry course taught at university. A structured interview was conducted to seek the views of participants on their perception of geometry. Although quantitative data was collected on perception, some of the participants (especially those who did not do well) were interviewed to obtain more information on their perception of geometry.

Validity and Reliability Analysis

For validity and reliability purposes, the questionnaire and structured interview were designed in accordance with accepted instruments, that is geometry perceptions as investigated by Ozerem (2012) and Sudihartinih and Wahyudin (2019). Two expert researchers also made some suggestions to properly design the questionnaire and the contents of the interview. The achievement test was also a sample of questions from van Hiele accepted questions on geometry and the content of the geometry course at university.

Trustworthiness

There was the need to ensure trustworthiness of the structured interview used for the qualitative data collection. Here, the trustworthiness of the data collected, and its credibility were ensured as these elements also helped establish the consistency of an instrument for a qualitative study (Golafshani, 2003; Kaya & Aydin, 2016). To do this, the participants were given the freewill to answer the questions and they went through their responses to ensure they clearly stated what they wanted to say. The themes that emerged from their responses were first summarized in **Table 1** for two expert researchers to go through. This confirmation from the expert mathematics education researchers made the instrument dependable even before the reports from the data were presented.

Table 1. Correlation between PSTs' perception & achievement

Statistics	Value
Spearman's rho	.190**
Sig. (2-tailed)	.004
n	225

Note. Correlation is significant at p<0.05 (2-tailed)

FINDINGS

Relationship That Exists Between Pre-Service Teachers' Perception of Geometry and Their Achievement in Geometry

To identify the relationship between PSTs' perception of geometry and their achievement, Spearman's correlation (non-parametric test) is used as an alternative to Pearson correlation, which is a parametric test. Spearman's correlation was used because both variables were not normally distributed hence did not satisfy the prior assumptions in using the parametric test for correlation. Efforts to transform the variables to be normally distributed is also an alternative as suggested by Marshall and Samuels (2017). The process of transformation aided in transforming the test scores to be normally distributed but that of Perception did not work indicating a non-suitability for parametric test. **Table 1** shows the results from the correlation analysis between PSTs' perception and achievement. The results show a positive correlation between perception and achievement in geometry (rho[225]=.190, p=.004<.05). The correlation is positive, but the correlation coefficient (rho=.190) represents a weak correlation. This means the strength of association between the two variables is very low (Schober, 2018).

Pre-Service Teachers' Perception of Geometry

A structured interview was designed to explore the views and perception of PSTs. The responses of 10 PSTs were used in this part of the study, and they are presented subsequently.

What is geometry?

The question on what geometry was asked and some respondents indicated that it is an aspect of mathematics. Other responses further explained that it talks about lines, points, distances, relations and properties. Some of the responses are outlined below:

> "It is the branch of mathematics that deals with lines and shapes" (PI-2).

> "Geometry is the branch of mathematics concerned with the properties and relations of points, lines, surfaces, solids, and higher dimensional analogies" (PI-10).

> "The study of shapes, figures and measurement is termed as geometry" (PI-4).

> "Geometry is the study of points, lines, shapes and its related properties and spatial figures" (PI-6).

Do you think geometry should be taught at the tertiary level and why do you think so?

All respondents supported that geometry should be taught at the tertiary level and some of the reasons were that they see geometry as a branch of knowledge that requires in-depth application of knowledge. One other response was that it is an interesting subject, and some

admired the complex nature of geometry. Others further highlighted that it improves one's problem solving skills and can be applied to real life situations. Also they explained that it is vital in their possible future career (the 8th respondent gave an example that it is applied in construction job). One last response was that geometry as a branch of mathematics is also needed to learn other parts of mathematics as a whole. Some of their responses are noted below:

> "Geometry should be taught at the tertiary level because knowledge gained can be used at work such as construction" (PI-8).

> "Yes, geometry should be taught at the tertiary level. This is because we go to tertiary institutions in order to be exposed to the world of work and service to the society. Knowledge of geometry therefore will help in accomplishing that" (PI-5).

> "Yes, geometry should be taught at the tertiary level because study of geometry is crucial in studying mathematics in general and also has many applications in our daily life" (PI-6).

> "Yes, geometry should be taught at the tertiary level to prepare the individual for the study of science and advanced mathematics" (PI-10).

How would you evaluate the difficulty level of geometry?

The keyword in this question was "difficulty", however responses on how easy it is also explained the difficulty level of geometry. One of the respondents chose to answer by the use of percentage score and he responded that geometry is 70% difficult. Some also simply used the words, "very abstract", "not too difficult", "moderately difficult", "less difficult", and "normal" to describe the level of difficulty. Some also judged the level of difficulty by comparing with other aspects of mathematics. For instance, a response like

> "it is not difficult as compared to other fields of mathematics" (PI-5).

He further gave a reason that

> "since it is about shapes, it is always easy when the object or the shape is sketched when solving question under geometry" (PI-5).

Some key responses are captured, as follows:

> "I would regard geometry as a less difficult aspect of mathematics" (PI-7).

> "Geometry is moderately difficult" (PI-6).

> "I sometimes find geometry challenging because it involves abstract thinking to make concrete decision" (PI-4).

The content of geometry is only about angles. What is your view on this assertion?

Some of the responses with regard to the assertion that the content of geometry is only about angles were based on how true the statement holds. Four of the respondents made a strong claim that it is not true that the content of geometry is only about angles. Their responses were supported by clear justifications. For instance, one answered that

> "it is not only about angles" (PI-7).

He further explained that

> "there are studies like parabola, ellipse and circles of which they do not consist of angles" (PI-7).

One other female respondent also asserted

> "the content of geometry is not only about angles, but also guide us through other things such as points, lines, planes, trigonometry, transformations, circles, and area" (PI-10).

Some other responses were, as follows:

> "Even though geometry involves angles in most of its discourse, it also concerns itself with lines and shapes" (PI-4).

> "It is not only about angles. It also consists of lines and position of objects. Example, center of a circle, focal points as applied in parabola" (PI-5).

However, one of the respondents fully supported that geometry is only about angles and this was her justification:

> "This is true because it seems all the topics under geometry involves angles" (PI-8).

What could be your reason for the learning of geometry at every level of education?

As geometry is a course at the university and colleges of education in Ghana, the researcher sought to obtain the perceptions of PSTs on reasons or the need for the learning of geometry at the tertiary level. Some of the reasons given by the participants were that it helps to develop critical thinking skills; it is needed to be applied to our daily lives and some further supported that it helps in enhancing reasoning abilities. One respondent asserted that

> "to provide many foundational skills of logic, deductive, reasoning, analytical reasoning, and problem solving" (PI-10).

Her response emphasizes not only a skill acquired through learning geometry but rather she outlined many skills that can be acquired through the learning of geometry. Another response was also on the applicative nature of geometry in other subjects, and it was, as follows:

> "I learn it at all levels so that it will groom me well for its numerous advantages at higher level such as applying it in graphic design and in ICT" (PI-5).

One other response that was captured by the researcher was when one respondent remarked

> "According to the view of Pierre van Hiele and his wife, theorems (in geometry) should be learned to develop critical thinking" (PI-2).

Have you heard of geometric thinking before & how do you understand it?

Six of the participants said they have not heard about geometric thinking before but one of them was able to say something about this concept. He said he has not heard about it, but he can understand that it is a type of thinking, which deals with points, lines, shapes, angles and

so on. Most of those who said they have heard about geometric thinking centered their explanations around geometric figures, properties, shapes and other geometric concepts. Some statements are, as follows:

> "It means using your cognitive mind to deduce the shapes and angles of an object" (PI-2).

> "I think it is the ability of a student to analyze a geometric problem and provide a good solution to that problem" (PI-7).

> "Yes, geometric thinking is concerned with how people reason using the properties of a geometric figure and spatial relationships" (PI-10).

One other participant also said that

> "it is all about your ability to visualize mathematical statements as shapes or diagrams for easy understanding" (PI-4).

DISCUSSION

The results of correlation between perception and achievement in geometry shows a positive relation although it is weak. The findings differ from a study by Kanafiah and Jumadi (2013), which had an objective of finding out the link between perception and attainment in mathematics. Their study found out that there was a negative relationship between the learners' perception and their attainment in the subject. Their result indicated that the relationship was not just negative, but it was moderate (r=-.496) and it was significant. Hagan et al. (2020) also conducted a study, which sought to find the association between students' perception and their academic attainment. Their results were in contrast to the findings of the present study because they found a negative relationship between their perception and achievement. The correlation coefficient in their finding was very small (r=-.027) indicating very weak correlation. Comparably, the correlation was negatively weak, but the present study had positively weak correlation. Also the present study had significant relationship between perception and achievement but the study by Hagan et al. (2020) found that the correlation determined was not significant.

From the present study, learners described geometry as a field of mathematics, which consists of points, lines, angles, shapes and relationships between shapes. These responses are in line with themes that emerged from a study by Ozyildirim Gumus et al. (2021). Their study encompassed the teaching of geometry with several tasks to help examine learners' attitude, perceptions and self-capacity. It was identified that learners defined geometry to be shapes and also some said it is a field of mathematics or a subject in mathematics. Present study also found out that learners identified geometry as an aspect of mathematics. Ozyildirim Gumus et al. (2021) conducted post-interviews and learners further explained geometry to be a subject that talks about angles and deals with geometric terms.

Learners evaluated the difficulty level of geometry and the responses showed that geometry was not too difficult. Other responses were that it was moderate, and some said it was normal. Some of the respondents made strong assertion that it is easy. Similarly, the study by Ozyildirim Gumus et al. (2021) obtained varying results, where some learners said it was difficult, but others described geometry as easy. Some even perceived geometry as funny (Ozyildirim Gumus et al., 2021). Others also said it can be taught easily but can be very difficult when learning. This confirms some findings of the present study as learners said it can be difficult when learning.

Additionally, the present study found out that some students were not fully sure of the difficulty in geometry neither did they fully justify the difficulty in learning this branch of mathematics. They simply said it neither difficult nor easy. A study in South Africa by Naidoo and Kapofu (2020) also identified that some female students found geometry to be difficult based on some reasons. For instance, a student responded that geometry becomes difficult when it has to deal with proofs. For the present study there were similar reasons given by students and some said the teacher teaching geometry will determine whether geometry will be difficult or not. Some were of the view that geometry involves formulas and procedures so when followed it can be easy to learn.

Learners were also asked whether geometry involved only angles and most of the respondents said it was not a true statement. They viewed geometry to be an aspect of mathematics that has a lot to do beyond the concept of angles. Some emphasized that points, lines, shapes and properties are also part of geometry. There was a surprising remark as one student supported the assertion that geometry is all about angles. Naidoo and Kapofu (2020) in their study also asked similar question but they made the statement that geometry is all about proofs. In the students' responses, one of the respondents said it is all about proving but it has to be linked with angles and has to be linked with applying theorems. In the present study none of the respondents emphasized proofs in geometry but they spoke of the involvement of theorems and formulas in geometry. Some learners in the present study rather see the presence of theorems and formulas as a phenomenon that makes geometry learning easy instead of making it difficult to learn.

Some of the major points outlined the need for geometry in real life activities, opportunities for further academic pursuit and career opportunities like construction and industrial activities. Some of them said it helps in the development of critical thinking skills. These reasons given are in line with several studies in mathematics. For instance, Tieng and Kwan Eu (2014) supported that geometry helps to develop reasoning skills that helps in solving real life problems. Mokgwathi et al. (2019) also found that students valued mathematics and realized its need in solving real life problems and in job/careers.

Finally, PSTs shared diverse opinions on their knowledge of geometric thinking. Evidently, some of them responded that they have not heard about it before. Those who confirmed their knowledge of it explained that it involves reasoning based on the application of geometric, figures, properties, theorems and relationships. This indeed affirms van Hiele's stages of geometric reasoning, which explains thinking based on geometric ideas but further groups these reasoning abilities into levels (Hourigan & Leavy, 2017; Vojkuvkova, 2012).

CONCLUSIONS AND RECOMMENDATIONS

This study concludes that there was positively weak relationship between PSTs' perception and achievement in geometry. Further exploration on their perception also revealed that geometry is an aspect of mathematics, and it entails shapes, angles, points, lines and many more. It was also concluded that geometry is needed in daily life activities, job/careers and further studies. Diverse opinions also reasoned that geometry is not an entirely difficult subject, but its level

of difficulty can be attributed to external factors. Also, it was concluded that geometry is not only about angles. Lastly, it was concluded that geometric thinking deals with the kind of reasoning, where relations are drawn from applying the knowledge of geometry, specifically shapes, properties, relationships and other geometric concepts, to solve problems. Since geometry is a broad aspect of mathematics, PSTs should be engaged in more of its content, even beyond angle properties and shapes. Some parts of geometry that deals with application of concepts in finding equations, making deductions and proving theorems should be stressed as vital in learning geometry.

Author contributions: All authors were involved in concept, design, collection of data, interpretation, writing, and critically revising the article. All authors approve final version of the article.

Funding: The authors received no financial support for the research and/or authorship of this article.

Ethics declaration: Authors declare that the ethics committee approves of this study since it is a thesis for all postgraduate students at the institution of the authors. No participant names were stated in the findings – to ensure anonymity and confidentiality.

Declaration of interest: Authors declare no competing interest.

Data availability: Data generated or analyzed during this study are available from the authors on request.

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