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Kindergarten children's combinatorial thinking without and with the use of ICT: The case of combinations

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ABSTRACT

Although the importance of combinatorial thinking development in mathematics education has been highlighted in various studies, the research concerning kindergarten children is very limited. The present research focuses on the development of combinatorial thinking in kindergarten children aged 4-6 years in two different learning environments, the first one involved only manipulative materials and the second one manipulative materials and ICT. Seventy-two kindergarten children, from a typical public kindergarten school in Greece, participated in a teaching experiment. The comparison of the two environments was based on the number of solutions the children found, their strategies and errors in combination problems with and without repetition, for two age groups (4-5 and 5-6 years old). SPSS software and qualitative content analysis were used for the analysis of the data. Based on the results, no statistically significant difference was found concerning the number of solutions of children in the combinations without repetition, while in the combinations with repetition, the children who worked with manipulative materials performed better. Moreover, the children produced the same strategies and errors in the two environments. The results allow us to discuss practical implications for the design of tasks concerning the development of combinatorial thinking in early childhood.

Keywords: combinatorial thinking, early childhood, ICT, manipulative materials

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INTRODUCTION

Combinatorics refers to the enumeration of all the possible manners in which many things can be combined, ordered, or joined to each other, so that it becomes clear that nothing has been omitted to the achievement of a goal (Bernoulli, 2006). There are three types of combinatorial problems: combinations with or without repetition, permutations and arrangements with or without repetition. This article focuses on combinations with and without repetition. More specifically, combinations refer to each selection of k items from the n items of the set, without the order being important. Repetition refers to the ability of each item to be used more than once in one choice (Fischbein & Grossman, 1997).

The importance of combinatorial thinking development in mathematics education has been highlighted in various studies (e.g., Batanero et al., 1997; Fischbein & Gazit, 1988; Krekic-Pinter et al., 2015; Lesh & Heger, 2001; Aini et al., 2020; Pamungkas & Khaerunnisa, 2020; Silwana et al., 2021; Tsai & Chang, 2008; Yuberti et al., 2019) as well as in a plethora of curricula worldwide (e.g., National Council of Teachers of Mathematics, 2000). The development of activities for combinatorics can greatly contribute to both the modernization and the surge of effectiveness when teaching mathematics (Krekic-Pinter et al., 2015). Batanero et al. (1997) have noted that many of the students' misconceptions occur due to a lack of combinatorial thinking, since they fail to correctly enumerate the space of a sample in a mathematical problem. Students' involvement with combinatorics encourages them to be more creative, curious and with confidence in problem solving (Tsai & Chang, 2008) and they can understand more easily other mathematic fields (Aini et al., 2020). As combinatorics does not require calculus prerequisites, it can be taught from early childhood, regardless of students' knowledge of calculus (Ammamiarihta et al., 2017). Towards this effort, research indicates that it is meaningful to engage students in combinatorial thinking from early school years (e.g., English 1991, 1992; van Bommel & Palmer, 2018a, 2018b, 2018c).

However, the research concerning the development of combinatorial thinking in kindergarten children aged 4-6 years is very limited. A few studies have investigated the strategies that the children use to identify solutions (English 1991, 1992; Shiakalli, 2013; van Bommel & Palmer, 2018a, 2018b, 2018c) as well as how these strategies affect the successful solution of a combinatorial problem (English, 1992). Moreover, little research exists concerning the role of ICT environments in early childhood and the few existed results seem contradictory (Kafoussi & Fessakis, 2009; van Bommel & Palmér, 2018a, 2018b). Moreover, no research has studied kindergarten children's strategies regardless of the number of solutions they identified and the possible differences between children aged 4-5 years old and 5-6 years old.

Our research focused on a systematic investigation of two different learning environments, in order to highlight whether there were differences between them concerning the development of kindergarten's children combinatorial thinking in different types of combinatorial problems. The first environment involved only manipulative materials and the second one manipulative materials and the use of ICT. The two environments were developed in the same way, so that a comparison between them could be feasible. The research investigated the strategies that children developed when engaging with the combinatorial problems according to the number of solutions they identified and the errors they made. Moreover, it was investigated whether there was a difference between children, according to the age group they belonged to (4 to 5 and 5 to 6). The contribution of the research is related to expand our understanding on the development of kindergarten's children aged 4-6 years combinatorial thinking in different learning environments for the design of appropriate tasks and the development of a suitable mathematics curriculum. In this article we present our research results concerning the case of combinations, the most usual problem in which the children are engaged in early childhood.

LITERATURE REVIEW

According to Dahar (2011, see Ammamiarihta et al., 2017), combinatorial thinking involves thinking that encompasses all possible combinations of objects, ideas, or proposals. Combinatorial thinking is a process of finding a number of alternative solutions to solve a distinct problem.

Piaget and Inhelder (1958, 1975) were the first researchers to address the development of children's combinatorial thinking (Krekic-Pinter et al., 2015). Their studies included the investigation of arrangements, permutations and combinations and involved ages 6 to 15 years. According to Piaget and Inhelder's (1958, 1975) findings, children's strategies emerged in three main steps: initial "empirical combinations" (identifying combinations through trial-and-error process), "searching for a system" (looking for patterns) and "discovering a system" (finding an appropriate systematic strategy). Piaget and Inhelder (1958, 1975) argued that a child is capable of solving combinatorial problems only at the level of formal functioning (11-15 years of age).

English (1991,1992) has conducted two studies concerning the young children's combinatoric strategies (aged from 4 to 9 years old). The children were involved in dressing bears with two objects. According to her results, six increasingly sophisticated strategies emerged by the children in order to solve a problem of combinations:

(a) two strategies without planning: strategy A in which they selected random items without discarding those that have been shown to be inappropriate; strategy B when they selected random items and discarded those that have been shown to be inappropriate;

(b) two transitional strategies, when they used a pattern in item selection: in strategy C they produced an emergent pattern, while in D a complete and consistent pattern; and

(c) two odometer strategies, when an odometer pattern appeared, that is children selected an item repeatedly so that all combinations containing it were formed: in strategy E, an odometer pattern with object rejection appears, while in strategy F, a complete odometer pattern without object rejection is discernible.

It should be noted that according to English's (1991) research, younger children aged 4 to 6 years usually followed A and B strategies (trial and error), with a small number of them making some attempts to follow a pattern in item selection (strategy C) but rarely continued to complete it. Moreover, in her later research (English, 1992), she supported that young child (from 4 years old), were able to solve combinatorial problems throughout a meaningful context for them.

Shiakalli (2013) studied the ability of children 4 to 5 years old to solve permutation problems. The problem involved a fish and bubbles in three different colors, where the children had to place the bubbles in all different ways–the number of possible solutions was given. The children were asked to solve the same problem before and after a teaching intervention. Four kindergarten children's strategies were identified. In the 1st strategy named "random solution detection", children selected items randomly, looking at the recording sheet to avoid repeating the same solution. In the 2nd strategy named "combination of deliberate alternation of items and their random placement", children found the first three solutions, placing a different item in the first position and randomly selected items for the second and third positions in order to find the remaining solutions. In strategy 3 named "detecting solutions by non-consecutive pairs", children followed the second strategy for the first three solutions, while for the remaining solutions they returned to a solution, they had already identified by placing the same color in the first position and reversed the items in the second and third positions. Finally, in the 4th strategy named "detecting solutions by consecutive pairs", children placed the same item consecutively in the first position and reversed the items in the second and third positions. Then, they did the same to find the remaining solutions, choosing a new item in the first position each time. According to her research, an improvement was noticed in the children's strategies, when they faced with the same type problems repeatedly. Furthermore, as the exact number of possible solutions was given, the children had a motivation in order to identify all the possible solutions.

In a study of van Bommel and Palmer (2018a), children aged 6 years old were asked to help three bears who were arguing about their position in a sofa. The children had in their disposal pencils in different colors in order to find all the possible solutions. According to their results, few children found all the solutions. The children followed three strategies, trial and error (with and without duplications), transitional strategies (with and without duplications) and odometer (no duplication–not all solutions/all solutions). The most children used the trial an error strategy and a few children, at about the same rate, used transitional and odometer strategies. The same researchers (van Bommel & Palmér, 2018b, 2018c) expanded their study by introducing a carefully designed digital version of the task as a complementary one to pencil and paper. The first research (van Bommel & Palmer 2018b) was realized in three design cycles: in the first and second cycles only paper and pencil was used, while in the third cycle both a digital version of the task and paper and pencil were used. Two difficulties were identified in the analysis of the first and second cycles. The first one was that children did not seem to structure and systematize their solutions, for example when they repeated a solution, they didn't understand it. Moreover, few children succeed in finding all the solutions. The researchers investigated if the digital version of the task, which offered systematization as well as comments for repetition, could help the children to structure their recordings and look for possible solutions. The principle of completion (and therefore the principle of exhaustion) was incorporated into the digital version, as it indicated when all solutions were found. Children who used both a pencil and paper and a digital version found more permutations and did fewer repetitions than groups that had not worked with the digital version, yet only fewer children solved the task. It was pointed out that the digital version influenced the children's written responses, but it cannot be argued that the digital version alone could work efficiently, as only a digital version was not used. The second research (van Bommel & Palmer, 2018c), was realized in two cycles: in the first cycle only paper and pencil were used, and in the second only digital version was used. According to their results, the children who used the digital version had fewer repetitions and were helped to systematize their solutions.

Fessakis and Kafoussi (2009) studied the capabilities of children aged 4 and 5 years old, as they engaged in problem solving regarding the production of combinations of two items from a set of 4 objects with repetition, utilizing microworld software and educational materials. Two groups of children initially solved the problem using two microworlds with or without feedback and building mechanisms, respectively. A few days later, all the children who participated in the experiment solved a similar problem using manipulative materials. Based on the results, the children were able to produce combinations in all learning environments (microworlds, manipulatives) despite the fact that they did not follow any systematic pattern. Kindergarten children who used the second microworld (with feedback) performed slightly better in terms of the number of combinations they produced, but this difference was not statistically significant. Furthermore, the overall performance of all the children using the software was significantly lower than their performance with the manipulatives. It seems that regardless of the microworld they used in their first activity, the children's performance was significantly better when they worked with manipulatives.

In a study by Frantzeskaki et al. (2020), the contribution of a microworld designed for kindergarten children aged 4 to 6 to the development of their combinatorial thinking was examined. The children were presented with three problems and asked to identify all the solutions in pairs from the elements given to them, the first with two elements (4 possible solutions), three elements (6 possible solutions) and four pieces (16 possible solutions). The children engaged with the combinatorial problems in a digital Scratch environment. Based on the results, the children, without help or direct instruction, having only contact with the microworld, managed to produce combinations. This finding provides the impetus for kindergarten children to participate in combinatorial problems. In addition, it appeared that the children's contact with feedback information, specifically with the boxes that gave the number of solutions, played an important role in finding all the solutions. Finally, it was found that children aged 5-6 seemed to understand feedback better and used it to find new solutions, compared to children aged 4-5.

Research Questions

Echoing the above findings, the aim of our research was to study the development of combinatorial thinking in children aged 4 to 6 years in two different learning environments. The first environment included only tasks with manipulative materials, while the second one included both, with manipulative materials and digital tasks. In this paper we will present a part of our research that concerned problems about combinations with and without repetition.

The research questions were the following:

1. Was there any difference in relation to the number of solutions identified by the children between the two learning environments?
2. Which strategies did the children produce? Was there any difference concerning children's strategies between the two learning environments?
3. Was there any difference between children aged 4 to 5 years old and children aged 5 to 6 years old concerning the number of solutions and their strategies?
4. What errors did the children make when they involved with combinations problems in the two learning environments?

METHODOLOGY

Participants

The research was conducted in a kindergarten school, in the City of Rhodes in Greece, with students aged 4 to 6 and it was carried out over a period of approximately six months. The children attended a typical public school, with families from a middle socio-economic status. Overall, 72 children, 40 aged 5-6 and 32 aged 4-5 participated, of which 29 were boys and 43 girls. Two groups were organized, as the first group (group A) worked only with manipulative materials, while the second group (group B) worked with both manipulative materials and ICT. In group A, 37 children (16 children aged 4-5 and 21 children aged 5-6) participated, while in group B 35 children participated (16 children aged 4-5 years and 19 children aged 5-6 years). Each group was divided into two classrooms, according to the kindergarten school organization. The two groups did not have any differences concerning their prior arithmetical knowledge. The scratch application was chosen as an accessible and easy-to-use application for early childhood about the design of the digital tasks (Bers, 2018; Oliveira & Lopes, 2011; Talan, 2020).

The Phases of Research

The research was realized through a teaching experiment (Chronaki, 2010) during three phases. In the first and third phase, the children were engaged with two problems individually, concerning combinations without and with repetition and in the second phase a teaching intervention was applied simultaneously to all the children in both groups, A and B.

More specifically, in the first and third phase the children were asked to solve two combinatorial problems, and they recorded their answers on a worksheet that was given to them, using concrete materials (see **Figure 1**). In the first problem, a hero that loved the juices was presented to the children. The hero had 3 fruits and the children were asked to make juice and spot all the possible combinations of 2 fruits out of the three they were given. In that problem, concerning

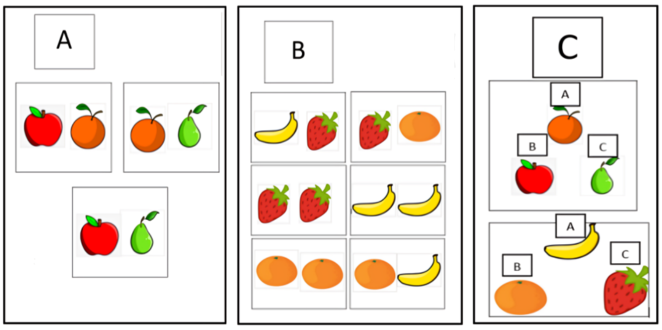

Figure 1. (A) Completed worksheet for combinations without repetition, (B) completed worksheet for combinations with repetition, (C) the cards given to the children [Source: Frantzeskaki (2025, p. 83)]

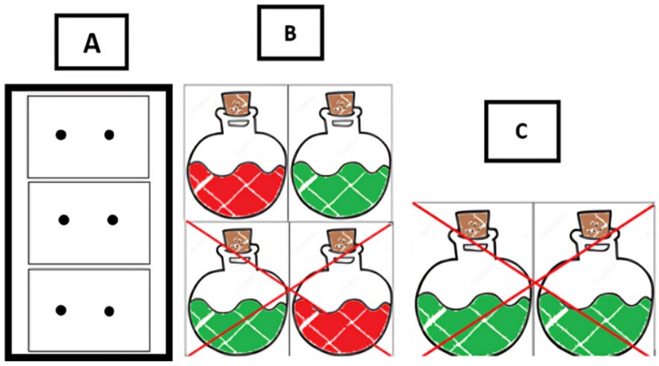

Figure 2. The worksheet for the first problem [Source: Frantzeskaki (2025, p. 90)]

combinations without repetition, the children could not put together two fruits with the same content (e.g., orange-orange), so there were three solutions in total (see in part A in **Figure 1** a completed worksheet). In the second problem, concerning combinations with repetition, the scenario was the same and the children could put together two fruits with the same content, so there were six solutions in total (see in part B in **Figure 1** a completed worksheet). The children had been provided with cards on stacks (pictures with objects from the problems, see part C in **Figure 1**) and were asked to fill in their answers on a worksheet separately for each problem. Children had at their disposal more cards (three more for each fruit) than they needed in order to find all solutions, so the last solution was not revealed. The corresponding number of possible solutions was given on the worksheet (Shiakalli, 2013). This phase lasted from five to ten minutes for each child and the children involved in the problems without any intervention.

In the second phase, the children were engaged in solving problems about combinations without and with repetition in their classrooms. All problems during the teaching experiment were designed by the researcher with the aim of actively engaging children in a meaningful context, according with their interests and their daily life (English, 1992).

During the teaching intervention, in the first problem, the children were called to help an animal which wanted to be transformed to another one. The children had on their disposal pictures with filters in three different colors (red, green, and yellow). The children were asked to make magic filters and spot all the possible solutions of 2 out of the three colors. A worksheet was used for all the children (see part A in **Figure 2**: three solutions to the problem, one in each box. The box had

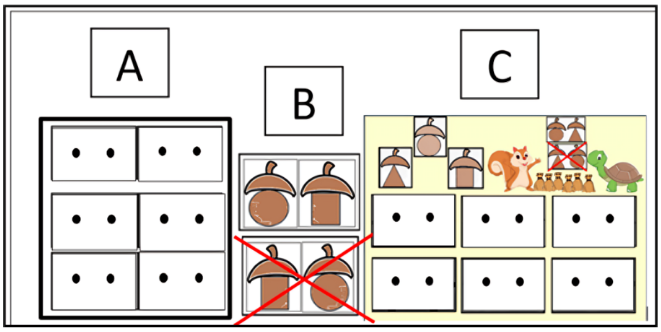

Figure 3. The worksheet for the second problem [Source: Frantzeskaki (2025, pp. 92-93)]

two positions where children were asked to place the pictures). The children were informed that they could not have two magic filters with the same colors (e.g., red-green and green-red, see part B in **Figure 2**) and they could not put together two pictures with the same color (e.g., green-green, see part C in **Figure 2**). Both groups worked with manipulatives materials.

In the second problem, the children were called to help the animal, by making sacks that each of them should have two different acorns. Acorns had three different shapes (triangle, circle and square) and the children should put them in pairs of two, finding all the possible solutions (see part A in **Figure 3**). The children were informed that they could not put in a sack two acorns with the same shapes (e.g., circle - triangle and triangle-circle, see part B in **Figure 3**), but they could put together two acorns with the same shapes (e.g., triangle-triangle).

In the second problem, group A worked only with manipulatives materials (see part A and part B in **Figure 3**) and group B only with ICT (scratch, see part C in **Figure 3**). The instructions were the same in both environments. Every time the children identified a new solution, they were asked if that solution was different. In the first environment (manipulative materials), the question was asked by the teacher, while in the second environment (using ICT) it was asked by the scratch application.

Data Gathering Methods

Data collection was based on the completed worksheets of the children from the first and third phase of the research. Specifically, the number of solutions found by each child was recorded, as well as the way that children used in order to find the different solutions. The pictures of the different fruits were called A, B, and C items (see part C in **Figure 1**). Each solution consisted of two items that the children placed in the box. If a child placed an Apple-Orange in the box, then the decoding was BA (see part C in **Figure 1**). Decoding was based on which item was placed in the first position of the box, not which item was placed first on the box. Furthermore, the order of the boxes was not taken into account, but rather the order in which they produced the solutions. For example, the first box did not mean that it was the first solution the children found. Overall, 288 complete worksheets that collected before and after intervention were analyzed.

Data Analysis

The data analysis was based on quantitative and qualitative methods. More specifically, the data were analyzed with SPSS (IBM SPSS software) concerning the number of the solutions the children found. To check whether there was a statistically significant difference before and after the intervention per group, a non-parametric Wilcoxon test was performed, while for the comparison between group A and group B, a non-parametric Mann-Whitney U test was performed. The Mann-Whitney U test was also used to test between children aged 4 to 5 and 5 to 6. The reason for choosing the non-parametric test was because the normal distribution did not apply to the sample (Shapiro-Wilk $p < 0.05$).

Moreover, the method of content analysis (Bryman, 2016) was used for the description of the children's strategies and errors. The categories of strategies were created based on the related literature (English, 1991; Shiakalli, 2013) and a particular emphasis was placed on the item that children selected in the first position in their solution. The above analysis was carried out both by group (with and without ICT) and by age (4 to 5 and 5 to 6). Furthermore, the number of solutions found was correlated with the emerged strategy.

RESULTS

The results for the two combination problems, in which children were engaged, are presented below. The results include the number of solutions identified by the children, the strategies that emerged in the two learning environments and the errors that they made. In the tables we use percentages (%) for the number of the children.

Combinations Without Repetition

Number of solutions

Table 1 presents the number of solutions that the children found in the combination problem without repetition, before and after the intervention.

Table 1. Number of solutions in the combination problem without repetition

Number of solutions	Group A		Group B		Total	
	Before	After	Before	After	Before	After
3	68	60	54	63	61	61
2	16	40	26	37	21	39
1	8	0	0	0	4	0
0	8	0	20	0	14	0
Total	**100**	**100**	**100**	**100**	**100**	**100**

The majority of the children in group A, in group B and in total, before the intervention, identified 3 solutions, at a rate of 68% (25 children), 54% (19 children) and 61% (44 children), respectively. After the intervention, most of the children identified 3 solutions, at a rate of 60% (22 children), 63% (22 children) and 61% (44 children). A significant increase identified to the children that found 2 solutions after the intervention, from 16% (6 children) to 40% (15 children) in group A, from 26% (9 children) to 37% (13 children) in group B, from 21% (15 children) to 39% (28 children) in total. It should be mentioned that there is no child who found one solution or no solution after the intervention, in both groups.

Based on the results, no statistically significant difference was found before the intervention between the two groups ($p = 0.242$), nor after the intervention ($p = 0.769$). When testing between children aged 4-5 and 5-6, before and after the intervention, no statistically significant difference was found within both groups separately.

In the whole sample, before the intervention, no statistically significant difference was found, but after the intervention, a statistically significant difference was found ($p = 0.028$), with children aged 5-6, performing better in the number of solutions identified.

Strategies

Table 2 presents the strategies produced by the children. For both groups, three strategies were identified:

- Using a fixed item: The children choose the same item consecutively in a row in the first position, alternating the secondary item (e.g., BC, AB, AC or AB, AC, BC).
- ABC items: The children choose a different item each time A, B or C in the first position in a row or in a random way (e.g., BC, AB, CA).
- Random selection: The children chose the items at random (e.g., BC, AC, BA).

Table 2. Children's strategies in the combination problem without repetition

Strategies	Group A		Group B		Total	
	Before	After	Before	After	Before	After
Fixed item	51	32	23	43	37.5	37.5
ABC items	8	41	43	34	25.0	37.5
Random selection	41	27	34	23	37.5	25.0
Total	**100**	**100**	**100**	**100**	**100**	**100**

Before the intervention, most children in group A, used a fixed item at a rate of 51% (19 children), since most children in group B, used ABC items, with a percentage of 43% (15 children). A significant number of children, in both groups, chose the items at random (41%, 15 children in group A and 34%, 12 children in group B). In total, the children used both a fixed item and a random selection at the same percentage (37.5%, 27 children).

After the intervention, in group A, most children used ABC items, with a percentage of 41% (15 children), marking a significant increase of this strategy, as the corresponding percentage before the intervention was 8% (3 children). Unlikely, in group B, most children used a fixed item (43%, 15 children, before the intervention 23%, 8 children). In total, children used a fixed item, maintaining the same percentage as before the intervention (37.5%, 27 children). Additionally, the same percentage 37.5% of children used ABC items (before the intervention 25%, 18 children). In both groups less children randomly selected items.

With regard to the strategies used by children aged 4 to 5 before and after the intervention, most children, before the intervention, used both a fixed item and a random selection in an equal percentage (37.5%). Similarly, most children aged 5-6 used both a fixed item and a random selection (37.5%), respectively. After the intervention, most children aged 4-5 used a fixed item (41%), while most children aged 5-6 used ABC items (43%).

With regard to the correspondence between the strategies used by the children and the number of solutions they found, the following could be noted:

1. The children who identified all the solutions, before the intervention, used a fixed item (52%) and a significant number of them (36%) selected items randomly. After the intervention,

most children used a fixed item (41%) and ABC items (from 12% before the intervention to 34% after the intervention).

2. The children who identified two solutions, before the intervention, randomly selected the items (53%). After the intervention, most children used ABC items, at 43% (the rate before intervention was 20%). A significant decrease was found in the proportion of children who randomly selected the items (from 53% before intervention, to 25% after intervention).
3. The children who identified one solution, before the intervention, randomly selected items at 67%, with the remaining 33% using ABC items. The children who did not identify any solution before the intervention used ABC items, at 90%.

Errors

Table 3 presents the errors that the children make when they solved the problem of combinations without repetition. In general, four types of errors were identified:

- Same item twice: the children chose the same item twice in one selection, (e.g., AA, BB, CC).
- Same order: the children chose the same items in the same order (e.g., AB, AB).
- Permutation: the children chose the same items in a different order (e.g., AB, BA).
- Same order and permutation: the children chose the same items both in the same and different order.

Table 3. Children's errors in the combination problem without repetition

Errors	Group A		Group B		Total	
	Before	After	Before	After	Before	After
Without errors	68	62	54	63	61	63
Same item twice	13	8	20	3	17	5
Same order	13	6	14	8	14	7
Permutation	3	24	12	26	7	25
Same order & permutation	3	0	0	0	1	0
Total	**100**	**100**	**100**	**100**	**100**	**100**

In group A, the most common errors, before the intervention, were both: selecting different items twice in the same order and selecting the same item twice in one option (13%, 5 children in both cases). In group B and in total, the most common error was choosing the same item twice in one option, at a rate of 20% (7 children) and 17% (12 children), respectively. After the intervention, the most common error was the use of the same items in permutation, in both groups, marking a significant increase, from 3% (1 child) to 24% (9 children) in group A, from 12% (4 children) to 26% (9 children) in group B and from 7% (5 children) to 25% (18 children) in total.

Combinations With Repetition

Number of solutions

Table 4 presents the number of solutions that the children found in combinations with repetition.

Table 4. Number of solutions in the combination problem with repetition

Number of solutions	Group A		Group B		Total	
	Before	After	Before	After	Before	After
6	11	51	0	34	5	43
5	11	30	9	11	10	21
4	16	5	11	23	14	14
3	54	11	74	32	64	21
2	8	3	6	0	7	1
Total	**100**	**100**	**100**	**100**	**100**	**100**

Before the intervention, most children in both groups, identified 3 solutions (54%, 20 children in group A, 74%, 26 children in group B) and in total 64% (46 children). After the intervention, most children identified all the solutions, marking a significant increase, from 11% (4 children) to 51% (19 children) in group A, from 0% (0 children) to 34% (12 children) in group B and from 5% (4 children) to 43% (31 children) in total.

According to the statistical analysis, there was no statistically significant difference before the intervention ($p = 0.150$) between the groups. However, after the intervention, there was a statistically significant difference between the groups ($p = 0.027$), with group A performing better in the number of solutions. Moreover, there was no statistically significant difference before and after intervention between children aged 4-5 and 5-6, within both groups and in total, concerning the number of solutions.

Strategies

Table 5 presents the strategies produced by the children in the combination problem with repetition. The four identified strategies were the following:

- Using a fixed item. This strategy included:
  - Odometer: The children chose the same item consecutively in the first position, alternating the second ones, until all the combinations containing it were exhausted. The same procedure was repeated for the rest of the items (e.g., AC, AB, AA, BC, BB, CC).
  - Partial use of odometer: They used only one item with all secondary items (e.g., AA, BB, BC, BA, CA, CA).
  - Two fixed items: They chose at least two fixed items in the first position and alternate the secondary ones (e.g., CB, CA, AC, BA, BC, AB).
- Using a pattern. This strategy included:
  - Pattern of two or three items: They first selected items based on a pattern, either with two items (e.g., BA, BC, AB, BC, BC, AB), or with three items (e.g., AB, CA, BC, AA, CC, BB).
  - Pattern of two items alternately: They chose in the first position two items alternately, applying it to all the solutions (e.g., AB, BC, AC, BC, AC, BC), or some solutions (e.g., AB, BC, AB, BC, BA, AC).
- Use of ABC items. This strategy included:
  - Repetitive ABC: The children choose in the first position each time a different item for the first three solutions and they repeat it, not with the same order (e.g., BC, AC, CB, AA, CC, BB).

- Initial ABC: They chose in the first position each time a different item for the first three solutions (e.g., BA, CB, AC, BC, BA, CC).
- Final ABC: They chose in the first position each time a different item for the last three solutions (e.g., AB, CB, AC, AA, BB, CC).
- Random ABC: They chose in the first position each time a different item, in a random way (e.g., BA, CC, BB, AB, CA, AC).

- Random selection: They chose the items in a random way.

Table 5. Children's strategies in the combination problem with repetition

Strategies	Group A		Group B		Total	
	Before	After	Before	After	Before	After
Fixed item						
Odometer	0	0	0	3	0	1
Partial use of odometer	3	5	0	3	2	4
Two fixed items	5	11	0	3	3	7
Pattern						
Pattern of two/three items	16	5	20	17	18	11
Pattern of two items alternately	8	3	14	11	11	7
ABC items						
Repetitive ABC	16	16	20	9	18	13
Initial ABC	19	25	20	11	19	18
Final ABC	11	16	3	17	7	17
Random ABC	8	14	14	26	11	19
Random selection	14	5	9	0	11	3
Total	**100**	**100**	**100**	**100**	**100**	**100**

Before the intervention, the majority of the children in group A used the initial ABC strategy (19%, 7 children), the pattern of two or three items (16%, 6 children) and the repetitive ABC (16%, 6 children), while in group B, the children used the same strategies at a rate 20% (7 children), respectively. After the intervention, most children in group A used the ABC strategy; the initial 25% (9 children), the repetitive and the final, 16% (6 children), respectively, while in group B, they used the random ABC strategy, at a rate 26% (9 children) as well as the final ABC 17% (6 children) and the pattern of two or three items 17% (6 children). In total, before the intervention, most children used initial ABC, 19% (14 children) and pattern of two or three items and repetitive ABC at the same rate 18% (13 children). After the intervention, most children used random ABC strategy, at 19% (14 children) and initial ABC at 18% (13 children). It is worth noting that few children used an odometer strategy and the children who used it was mainly after the intervention.

Most children aged 4-5, before the intervention, used the repetitive ABC (22%), while most children aged 5-6, used the initial ABC (25%). After the intervention, most children aged 4-5, used the initial ABC (25%), while most children aged 5-6, used the final ABC (22.5%). Thus, most children in both age groups (4 to 5 and 5 to 6), used ABC items. Moreover, most children who used odometer were 5 to 6 aged and most children who used pattern, were 4 to 5 aged. Most children aged 4 to 5 used the random selection.

Furthermore, analyzing the strategies of the children in relation to the number of solutions, we could notice that most children who identified all the solutions before the intervention used the initial ABC (75%) and after the intervention, the random ABC (29%), respectively. It is worth noting that 3% of children used odometer and 7% used partial odometer strategy, after the intervention. Most children who identified 5 solutions before the intervention used the initial ABC, at a rate of 57%. After the intervention, most children used the same strategy, at a rate of 27%. Most children who identified 4 solutions before the intervention used the initial ABC strategy at 40%. After the intervention, most children used the pattern of two or three items, at a rate of 30%, while a significant number of children (20%) used the initial and the final ABC. Most children who identified 3 solutions, before the intervention, used both the pattern of two or three items, at a rate of 24%, and the repetitive ABC, at a rate of 22%. After the intervention, most children used the pattern of two or three items, with a percentage of 27%, while a significant number of children, 20%, used a two-item pattern alternately, as well as using initial and final ABC. Most children who identified 2 solutions, before the intervention, selected items randomly at 60%. After the intervention, only one child identified two solutions and selected the items randomly.

Errors

Table 6 presents the children's errors when they solved the problem of combinations with repetition. The types of errors were the same as the combination problem without repetition, except of the use of the same item twice in one selection, as at the problem of combination with repetition it was allowed to select the same item twice in a solution.

Table 6. Children's errors in the combination problem with repetition

Errors	Group A		Group B		Total	
	Before	After	Before	After	Before	After
Without errors	11	56	3	34	7	46
Same item twice	43	14	43	32	39	22
Same order	14	16	3	11	8	14
Permutation	32	14	51	23	42	18
Same order & permutation	11	56	3	34	7	46
Total	**100**	**100**	**100**	**100**	**100**	**100**

The most common error in group A, before the intervention, was the use of the same items in the same order (43%, 16 children). A significant number of children (32%, 12 children), put the same items in the same order and permutation. The most common error in group B, was the use of the same items in the same order and permutation (51%, 18 children) and a significant number of children (43%, 15 children), who chose the same items in the same order. After the intervention, in group A the most common error was choosing the same items in permutation (16%, 6 children), while in group B was choosing the same items in the same order (32%, 11 children). In total, the most common error, before the intervention, was choosing the same items in the same order and permutation (42%, 30 children) and choosing the same items in the same order (39%, 28 children), while after the intervention, choosing the same items in the same order (22%, 16 children).

DISCUSSION AND CONCLUSIONS

Echoing the need for more research concerning the development of combinatorial thinking in early childhood, the aim of our research was to study and compare the development of combinatorial thinking in children aged 4 to 6 years in two different learning environments. One group of kindergarten children worked only with tasks with manipulative materials (group A) and the other one worked with manipulative materials and digital tasks (group B). The children were asked to solve problems through a meaningful context for themselves concerning combinations without and with repetition. For the comparison of the two environments we investigated if there were differences between the two groups, and between children aged 4-5 and 5-6, in relation to the number of solutions identified, the strategies they produced and the errors they made. It should be noted that children's strategies were recorded regardless of the number of solutions they had identified. According to our results, the two learning environments did not seem to influence the development of kindergarten's children combinatorial thinking in relation to the issues investigated.

Concerning the number of solutions, in the combination problem without repetition, no statistically significant difference was found in the number of solutions produced by the children in the two groups. However, in the combinations with repetition, it was found that the children who worked with manipulative materials performed better after the intervention comparing to the children used ICT environment. This finding is consistent with the results of the study by Fessakis and Kafousis (2009) and contradicts the results of van Bommel and Palmer (2018b, 2018c). Moreover, no significant difference was found in the number of solutions identified by children aged 4-5 and 5-6, before and after the intervention, in the combination problems within the groups. An exception was the combination problem without repetition, after the intervention, where there was a statistically significant difference with children aged 5-6 years having better performance than the children aged 4-5 years in total.

Moreover, the two different learning environments did not seem to significantly affect the strategies that children produced in combinatorial problems. The strategies emerged were the same for both groups:

(a) use of a fixed item,

(b) use of ABC items, and

(c) random selection.

In the problem of combination with repetition there was one more strategy that emerged from the children, the use of pattern of two or three items and the use of pattern of two items alternately. In addition, the already existing strategies that appeared in the problem of combination without repetition were categorized into new subcategories in the problem of combination with repetition (the use of a fixed item: odometer, partial odometer and two fixed items and the use of ABC items: repetitive, initial, final, random). It should be stressed that the strategy of ABC items was the mostly used in different ways in order to find the solutions, either with repetition, or at the beginning, at the random, or at the end. Perhaps the children thought that using a different item each time in the first position would lead to a new solution. In particular, in the problem of combinations without repetition, both groups of children used mainly two strategies after the intervention (the use of a fixed element and the use of ABC items). The group that worked with manipulative materials showed a significant increase in the use of ABC items after the intervention. In the combination problem with repetition, most children in both groups used ABC items. It is worth noting that the children who worked with the ICT environment made more use of patterns in their choice of items.

According to English (1991), young children aged 4-6 years usually follow trial and error strategies, randomly selecting items, with a small number of children using a pattern but rarely completing it. Based on the results of the present study, children aged 4-6, use a variety of strategies. More specifically, the children used four strategies:

(a) *fixed item* (odometer, partial use of odometer and two fixed items),

(b) *pattern* (pattern of two or three items and pattern of two items alternatively),

(c) *ABC items* (repetitive ABC, initial ABC, final ABC, random ABC), and

(d) random selection.

Moreover, the findings about the strategy of ABC items are consistent with the research of Shiakalli (2013), concerning the use of three items in the first position, for finding the first three solutions (see strategy 2 and strategy 3 in the section of literature review). However, our results showed that the use of three items (ABC), is not only connected by children for the first three solutions, but also with a variety of ways like repetitive ABC, initial ABC, final ABC and random ABC, in order to find the remaining three solutions (combination problem with repetition).

With regard to the strategies used by children aged 4 to 5 and 5 to 6, no significant differences were found. However, the number of children aged 5 to 6 who used an odometer or partial odometer was greater than that of the children aged 4 to 5. Analyzing the strategies of the children in relation to the number of solutions, it should be noted that the strategy used in all solution categories was the use of ABC items, while the use of an odometer or partial use of odometer was observed only in children who identified all or most solutions. In addition, the use of patterns was mainly observed among children who identified the fewest solutions.

Finally, concerning the children's errors, there were the same for both problems in the two learning environments:

(a) placing items in the same order,

(b) placing items in permutation, and

(c) placing items both in the same order and in permutation.

In total, the most common error, before intervention, was the selection of items both in the same order and in permutation, while after the intervention, was the selection of items in the same order.

The above results offer evidence for the development of a mathematics curriculum in early childhood including combinatorial problems as well as the design of combinatorial tasks. Tasks with manipulative materials and digital tasks can offer equal learning opportunities as kindergarten children engaged with combinatorial problems. A limitation of the study is connected with the fact that its sample consisted of only one school; further research could be carried out including children from different schools and cultural groups, in order to bring out more evidence on the development of combinatorial thinking in early childhood. Moreover, the influence of the development of combinatorial thinking in children's performance later in mathematics is an open question.

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