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Learning through virtual manipulatives: Investigating the impact of Gizmos-based lessons on students' performance in integers

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ABSTRACT

Understanding integers is critical for further learning in mathematics. However, most students have difficulty understanding integers, especially how to apply sign rules. This study adopted a mixed-method and action research approach to investigate the impact of a virtual manipulative tool (Gizmos) on students' performance in addition and subtraction of integers. It also explored students' perceptions of the Gizmos-based intervention. A multistage sampling was used to select 44 year 9 students in one of the secondary schools in Brunei, who were subjected to a Gizmos-based lesson intervention on addition and subtraction of integers. Data were collected through pre-test, post-test, and interviews. Our paired sample t-test showed that student performance after the intervention improved significantly. The results also revealed that the Gizmos-based intervention provided an opportunity for students to play, learn, and visualize integers, enhancing their confidence, understanding, and performance. However, they found it challenging to memorize the rules during Gizmos-based lessons. Based on our results, Gizmos can be an effective tool that can help improve student performance in addition and subtraction of integers when carefully implemented. It has dynamic representative features that provide quick feedback to students when dealing with integers. It is recommended that teachers should continuously guide students to understand and apply sign rules and instructions in using Gizmos.

Keywords: Gizmos, integers, addition, subtraction, virtual manipulatives

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INTRODUCTION

Students must have fundamental mathematical skills to understand and solve advanced mathematical topics (Khalid & Embong, 2019). An aspect of mathematics that needs to be mastered by students is abstract mathematics. It is a branch of mathematics that uses numbers, notation, and symbols (Ogunleye, 2019). An important aspect of abstract mathematics, and one of the most fundamental concepts that students should be able to grasp, is integers (Sahat et al., 2018). The ability to manipulate integers is a necessary skill for students to succeed in almost all mathematical concepts (Chong et al., 2022). Therefore, it is essential to develop students' conceptual understanding of integers.

The literature suggests that young people have long exhibited difficulties understanding abstract mathematics, such as integers (Piaget, 1965). For example, Bofferding's (2010) research in Northern California concluded that students had difficulties and lacked an understanding of integers, especially when subtracting integers. Similarly, in the context of Brunei, where the authors of this paper are familiar with its education system, it has been well established that students in lower secondary schools have problems with adding and subtracting integers (Goh et al., 2017; Sahat et al., 2018; Toh et al., 2017).

Notably, a teacher's teaching approach affects student understanding of integers. For example, Lim (2011) confirmed that exposing students to memorize the rule of integers makes them commit conceptual errors, which leads to the forgetfulness of instructional concepts. Sahat et al. (2018) shared that using rigid algorithms to teach mathematics concepts such as integers is not beneficial for improving student understanding and performance since it only encourages procedural skills. This practice only makes students follow what is taught; thus, they cannot elucidate the process of getting their answers (Sarwadi & Shahrill, 2014). Therefore, students need to have the fundamental skills and knowledge of integers for them to learn more advanced mathematics topics (Khalid & Embong, 2019).

It should be acknowledged that direct instruction is less effective, and students must be physically and interactively involved in their

This study is the Master of Teaching Research Work of Nur Fatin Haji Ismail, Masitah Shahrill is the research supervisor, and Daniel Asamoah was invited to contribute in the improvement of data analysis and initial editing for this manuscript.

learning. One approach that has been shown to improve student learning is using virtual manipulatives (Al-Balushi et al., 2020; Chong et al., 2022; Michaelides et al., 2019; Serin, 2019). This supports the modern era of technology and the current situation of COVID-19, where teachers need to adapt and implement technology in their teaching pedagogy. The use of ICT in teaching and learning, including visual manipulatives, aligns with Brunei Vision 2035, which focuses on training and equipping citizens with ICT skills (Ali et al., 2022; Finti et al., 2016; Japar et al., 2022). In addition, a student-centered approach where students learn independently with less teacher assistance and supervision is essential for students to take charge and responsibility for their mathematics learning.

Despite the reported lack of understanding of integers among lower secondary students elsewhere, particularly in Brunei, little is known about using virtual manipulatives such as Gizmos in teaching and learning integers. Therefore, this study investigated the effectiveness of Gizmos-based intervention on student performance in addition and subtraction of integers. The following questions were answered:

1. What is the effect of Gizmos-based intervention on student performance in addition and subtraction of integers?
2. How do students perceive Gizmos-based intervention as an instructional option in addition and subtraction of integers?

LITERATURE REVIEW

Misunderstanding of Students in Adding and Subtracting Integers

It has been well established that students face difficulties solving integers, especially in addition and subtraction. The research by Whitacre et al. (2015) concluded that students often have problems conceptualizing integers. Bofferding (2010) discovered that some students misinterpreted the double use of negative symbols where students performed the operation of subtraction twice or just ignored the negative sign. Most students lack the conceptual understanding to apply their knowledge and organise facts when solving integers and become confused when dealing with negative integers (Hiebert, 2013; Vlassis, 2008). For example, most students have difficulties understanding that (-4)-(-2)=-2. This is because they lack an understanding of sign rules and fail to interpret the negative signs correctly (Chong et al., 2022).

The literature suggests that the negative sign can be the nature of the integer itself or an operational sign (Fuadiah & Suryadi, 2019; Khalid & Embong, 2019). Therefore, in solving (-4)-(-2), most students may fail to recognise that the negative sign after (-4) is operational, while the one attached to 2, for example, expresses the nature of the integer 2. Another misconception of students about integers is the arrangement of integers from ascending or descending order. For example, most of them fail to recognise that -2 is greater than -4. They only read meanings to the positive integers and judge which numbers are greater or smaller without considering the negative signs attached to these numbers.

One of the contributing factors to the lack of student understanding of integers is the instructional approach of teachers. In particular, the use of student-centered approaches to teaching and learning of integers has been recommended. Cetin (2019) argued that teachers should no longer teach students the concept of integers through rules. He added that modelling has more significant benefits and helps students better understand integers. However, he highlighted that teachers need to describe the concept of integers before modelling to improve students' theoretical knowledge.

In Shanty's (2016) study that focused on improving student understanding of integers, it was discovered that students could make sense of the numbers on the thermometer with the idea of a number line, which also helped them add integers. When adding integers, students could decide the starting point of the first integer on the number line and which direction of quantity to add.

Stephan and Akyuz (2012) corroborated that using a number line and encouraging student participation improved student conceptual and perceptual skills. However, Liebeck (1990) has questioned the usefulness of this kind of approach. He claimed that the number line approach does not perform subtraction of operations, particularly by subtracting a negative number from positive or negative numbers. Other methods, such as the use of counters, have been argued by Sahat et al. (2018). They concluded that there was an increase in students' performance in integers when counters were used to teach and learn addition and subtraction of integers.

Technology and Virtual Manipulatives in the Teaching and Learning of Integers

It has been widely recognized that incorporating technology in mathematics promotes student motivation, active participation, and performance (Eyyam & Yaratan, 2014; Freeman et al., 2014; Goh et al., 2017; Raines & Clark, 2011). The key among these technologies is interactive video clips, presentation software, and graphical calculators. To improve students' conceptual understanding of integers, teachers must create teaching episodes that address the gaps in student learning of integers. One such approach is the use of virtual manipulatives.

Moyer et al. (2002) mentioned that the incorporation of virtual manipulatives becomes apparent due to the modernization of technology. Manipulatives have been recognized as one of the best approaches to learning arithmetic since they help build mental representations and skills; and help revise these representations to create new ones (Cai & Knuth, 2005). This allows students to manipulate learning content to create new knowledge using a computer with visual manipulatives (Gningue et al., 2014).

The use of virtual manipulatives as an instructional option is related to student understanding and performance in mathematics. Inman (2018) emphasized that the ability to physically and visually manipulate learning tools allows learners to have a deeper understanding of mathematical concepts. Learners who actively use manipulatives eventually grasp concepts through experience (Kolb & Kolb, 2008). In addition, visual manipulatives promote student-centered learning in various mathematics disciplines (Chong et al., 2022).

Furthermore, Durmus and Karakirik (2006) stated that students could improve their attitudes toward mathematics when visual manipulatives are used in teaching and learning. According to them, virtual manipulative creates an interactive environment that allows students to solve problems and pose meaningful questions during lessons quickly. They also reported that students could better reflect on mathematical concepts due to the immediate feedback features of virtual manipulatives. Lamb and Thanheiser's (2006) research that used software called "balloons and weights" to explain the addition and subtraction of integers concluded that students' understanding of integers improved after using the software.

The dynamic representations and immediate feedback students receive in the virtual manipulative lesson have also been validated in the literature. In the research conducted by Bolyard and Moyer-Packenham (2006), it was found that virtual manipulatives with definite features, such as dynamic virtual representations and feedback, significantly enhance student learning, especially in subtracting integers.

Goh et al. (2017) argued that the immediacy of feedback in virtual manipulation is critical as it helps build student confidence and performance. It also motivates them to practice and correct their mistakes if they are unsure about a particular procedure. Golafshani (2013) also confirmed that students become excited and enjoy learning mathematics concepts through visual manipulatives than through the traditional teaching style of chalk and talk, which improve the understanding of mathematical concepts among students.

In addition, Bouck and Park (2020) conducted a study on the addition of integers using the Brainingcamp Two-Color Counter application for students with mathematics difficulties. It was discovered that all students improved their understanding. There was an improvement in student achievement and participation (Al-Balushi et al., 2020) and motivation and understanding (Michaelides et al., 2019; Serin, 2019).

There has also been a growing interest in developing web-based instructions and applications in the teaching and learning of integers. This is because specific applications provide more detailed information on what works best in mathematics (Kay & Lauricella, 2018). One of those applications that have attracted widespread attention is Gizmos. It provides interactive simulations that help students develop a deeper understanding of mathematical concepts.

According to Raines and Clark (2011), Gizmos improve student understanding and achievement. Inman (2018), in his research on the use of Gizmos in the teaching and learning of percentages and proportions, revealed that student engagement improved. He concluded that using Gizmos simulation positively correlated with enhancing students' engagement and motivation. In their research, Kay and Lauricella (2018) concluded that using Gizmos significantly improved students' understanding and application skills. The current study is important to validate the effectiveness of using virtual manipulatives such as Gizmos in the teaching and learning of integers.

METHODS

Research Design

This action research study that investigated the effect of Gizmos-based intervention on student performance in addition and subtraction of integers was conducted using a mixed-method approach. Quantitative and qualitative data were collected sequentially to investigate the effectiveness of the intervention (Creswell & Creswell, 2017). First, the quantitative data was collected to answer research question one, which focused on the efficacy of the intervention. Finally, qualitative data was collected to answer the second research question, which focused on students' perceptions of the intervention. Since this study focused on investigating the effectiveness of an intervention to address student difficulties and misconceptions about integers, and at the same time, explore student views about the intervention using quantitative and qualitative data, a mixed-method design with an action research approach was considered appropriate (Cohen et al., 2007; Creswell & Creswell, 2017).

Participants and Sampling

Convenience and purposive sampling techniques were used to select 44 year 9 students from two classes at one of the government schools in Brunei. Convenience sampling was used because the two classes were easily accessible for the intervention. Purposive sampling was used because addition and subtraction of integers are taught at that year level. The first class consisted of 20 students (12 females and eight males), while the second comprised 24 students (14 females and 10 males).

The students in both classes were of mixed abilities and were exposed to the same intervention. The same instruments were used to collect data to analyze changes in their performance in the pre- and post-tests. Six students were then selected for the interviews. The students who availed themselves to be interviewed were conveniently selected from upper, middle, and low achievement groups. This was meant to obtain holistic and detailed information about the intervention. We used the six students for the interview because, after interviewing the fourth participant, we noticed a redundancy in the information they provided. This repetition was confirmed after interviewing the fifth and sixth participants, indicating that in-depth information was obtained (Saunders et al., 2016).

Instruments

This study used two instruments to collect data: an achievement test and a semi-structured interview guide. The achievement test comprised pre- and post-tests on addition and subtraction of integers. The purpose of the pre-test was to assess students' entry behavior and baseline performance, while the post-test was used to judge students' performance after the intervention. The two tests contained 15 similar questions, each with the same difficulty level. The questions tested the understanding of students on how to add and subtract positive and negative integers. For example, in the pre-test, students were asked to compute questions such as $4+(-4)$, $-5+(-2)$, and $-9-3$, while in the post-test, they computed questions such as $-15+7$, $-4-(-12)$, and $7+(-5)$. The total score a student could obtain on each test was 30 marks, with two marks for each question.

A semi-structured interview guide was developed to collect interview data. It asked questions about the perceptions of students after the intervention. Samples of the questions were: How did the use of Gizmos in teaching and learning integers improve your understanding and performance? How did the use of Gizmos in teaching and learning integers present benefits and challenges to your learning of integers?

In the design of the instruments, we ensured some steps to contribute to their validity and reliability. For example, the test items were aligned with the lessons' objectives. The tests and the interview guide were also given to three mathematics education experts with more than 15 years of teaching experience. They judged both instruments in the context of clarity and representativeness. Their suggestions were used to improve the nature of the questions.

Intervention Process

The study involved three lesson interventions that consisted of six periods. The duration of each period was 25 minutes. Gizmos online simulation that provides interactive laboratories in mathematics and science for 3rd to 12th graders was used as the virtual manipulative tool. Before the first intervention session, we guided the students to register on the Gizmos online application from ExploreLearning.com. They were also given an exploration sheet that contained some questions on the addition and subtraction of integers and the steps to solve the questions using Gizmos.

To guide the lesson interventions, a 50-minutes lesson plan was developed on addition and subtraction of integers. The lesson focused on helping to differentiate between integers and operations, understand the result of zero pairs and apply integers in real life. In addition, students were told to read and remember the instructions for using Gizmos. We gave them a sheet that contained all the instructions to facilitate their understanding and recall of the process and steps involved in solving integers using Gizmos.

The first lesson started with a mathematical vocabulary activity using PowerPoint slides. Students were then required to determine suitable mathematical terms for operation symbols. Meanwhile, the starter lesson was used to interpret and reinforce the meaning of mathematical symbols to students. For the first part of the main lesson, an introduction to the Gizmos functions was explained in an ICT room. In addition, they were taught the basic definition of integers and how to distinguish between positive and negative ones. This part of the lesson was explained so students could fully understand the concepts of adding and subtracting integers.

We demonstrated to the students the role of virtual two-colored counter chips, including zero-pair effects. For example, a yellow chip represents one positive integer, and a red chip represents one negative integer. Therefore, in question 1+(-1), one yellow chip and one red chip would be dragged in the models or the whole region of zero pairs (see **Figure A1** in **Appendix A**, steps for Gizmos for zero pairs). When one yellow chip is dragged over one negative chip, it will not change the sum. This helped the students understand that the result is zero when one yellow chip and one red chip are combined.

In the second lesson, a real-life application of a simple word problem was given as a starter to test students' problem-solving skills involving integers. In this lesson, the addition of integers with different patterns was also introduced to students using examples in the Gizmos application. It was expected that students would use their relevant previous knowledge in lesson one to model the chips needed, cross out the zero pairs, and write the sum in the space given. For example, in 3+(-2), 3 is a positive integer, and three yellow chips could model this, and -2 could be represented by two red chips and subsequently pulling out two red chips (see **Figure A2** in **Appendix A**, modelling three yellow chips and two red chips).

During the intervention, students were asked some specific questions. For example, they were asked the number of zero pairs that needed to be removed. In this example, two zero pairs were identified and then removed by dragging one positive yellow chip and one negative red chip on top of each other, leaving only one positive yellow chip in the modelling area (see **Figure A3** in **Appendix A**, modelling one positive yellow chip left after removing the zeros). Feedback and comments were also given to students when approaching the final answer (see **Figure A4** in **Appendix A**, feedback given on the correct answer). Students were given the opportunity and time to practice additional problems in Gizmos and answer questions on the exploration sheet simultaneously.

In the final session, students were introduced to the demonstration of the counter chips in subtracting integers in Gizmos. For example, in -5-(-7), students would initially start with five red chips (see **Figure A5** in **Appendix A**, modelling five negative red chips). However, since there were insufficient red chips to deduct seven red, two zero pairs were added to have enough red chips removed (see **Figure A6** in **Appendix A**, adding two zero pairs before subtracting). Therefore, two yellow chips remained after removing all seven red chips (see **Figure A7** in **Appendix A**, two positive yellow chips after removing seven negative red chips). Similar to the second lesson, students were guided to independently solve questions using the Gizmos app and answer questions in the exploration sheets. A reinforcement session was conducted on student learning on addition and subtraction of integers using Gizmos. The students simultaneously illustrated using manipulatives drawing as they worked on the practice sheet.

Data Collection

Before the intervention, a pre-test on addition and subtraction of integers was conducted. All 44 year 9 students from both classes took the pre-test. The test lasted for 30 minutes. Appropriate testing conditions, such as a well-arranged and conducive environment, were ensured during the test. The students were also invigilated throughout the test. The post-test was administered immediately after the three intervention lessons. The same time duration and testing conditions were used for the post-test. The pre- and post-tests were marked, and the scores obtained by every student were recorded. Three days after the post-test, the interviews were conducted through zoom with the selected participants. All conversations between the participants were recorded on audio tape. The average duration of the interviews was 10 minutes.

Data Analysis

The quantitative data obtained through pre- and post-test were analyzed with a paired sample t-test to evaluate the mean difference for the two combined classes. R-studio aided the analysis. Using both tests as dependent variables, this statistical approach was suitable because the pre- and post-test scores were analyzed on the same group of students who experienced the same intervention (Coman et al., 2013). Since the paired sample t-test is a parametric test, we performed and fulfilled the normality assumption. We observed that pre-test ($p=0.807>0.05$) and post-test scores ($p=0.467$) were approximately normally distributed (Fisher & Marshall, 2009), providing enough evidence to support that the use of the statistical approach is considered appropriate.

The interview data were transcribed and analyzed thematically following Braun and Clarke (2012). This was done by carefully reading familiarizing ourselves with the data. Initial codes were generated, and potential themes were searched, reviewed, and defined before producing interview report. To validate findings from the interview, the excerpts representing the participants' views were stated verbatim.

Table 1. Paired sampled t-test on pre- and post-test scores

	Descriptive statistics		MD	SD	SE	95% CI of MD		t	df	Significance	Cohen's d
	Mean	SD				Lower	Upper				
Pre-test	6.930	2.970	2.523	3.099	0.467	-1.581	-3.465	-5.3997	43	0.000	0.400
Post-test	9.450	2.780									

Note. SD: Standard deviation; MD: Mean difference; SE: Standard error; CI: Confidence interval; n=44; & the mean difference is significant at the 5% alpha level

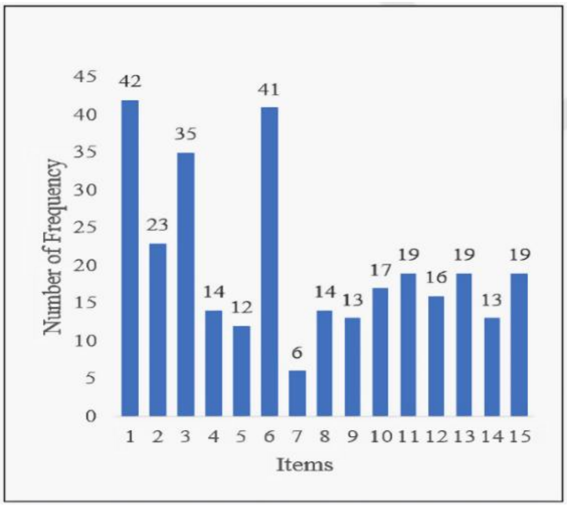

Figure 1. Number of correct responses pre-test (Source: Authors)

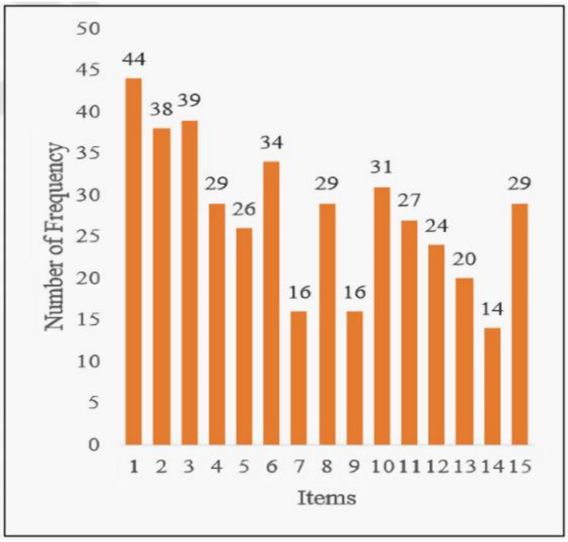

Figure 2. Number of correct responses post-test (Source: Authors)

RESULTS

The Effects of Gizmos-Based Intervention

Table 1 presents the results of the paired sampled t-test on the effectiveness of virtual manipulatives on students' performance in adding and subtracting integers. It focuses on the mean difference between pre- and post-test to judge effectiveness of the intervention.

The paired sampled t-test shows a statistically significant difference between the pre- and post-test scores (**Table 1**). Students scored higher in the post-test (M=9.45, SD=2.78) than the pre-test (M = 6.93, SD = 2.97), with t(43)=-5.4231, p<0.001. The results indicate that the students' performance improved after the Gizmos-based virtual manipulative intervention. Furthermore, **Table 1** shows that the intervention accounted for about 40% of student performance (Cohen's d-value of 0.4). **Figure 1** and **Figure 2** represent the correct pre- and post-test responses, respectively.

Figure 1 shows that most students (n=42, 41, and 35) scored items 1, 6 and 3, respectively. This is expected because these questions focused on simple addition and subtraction of integers. Apart from item 2, more than half of the students scored relatively low on the rest of the items. After the intervention, there was a significant improvement in performance: 44, 38, and 39 students, scored items 1, 2, and 3, respectively.

Especially for item 2, 15 more students scored correctly compared to the pre-test. Apart from items 7, 9, and 14, which did not record significant improvement, more than half of the students scored higher in the post-test compared to their pre-test. This justifies the effectiveness of the Gizmos-based intervention in improving student performance in addition and subtraction of integers. We were particularly interested in most students' common pre- and post-test errors. **Figure 3**, **Figure 4**, and **Figure 5** represent samples of these errors.

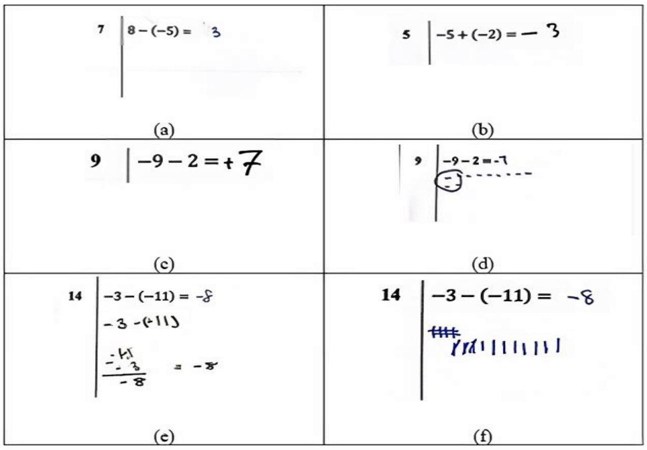

Figure 3. Correct responses pre-test (Source: Authors)

It is observed in **Figure 3** that most of the students were able to compute simple negative and positive integers. This suggests that most students had a satisfactory understanding of how to add and subtract integers. However, most students struggled to subtract a negative integer from a positive integer (item 7 in **Figure 4**).

Another misconception is that most students cannot add two negative integers. They tend to ignore the additive sign, treat the second integer as positive and use the negative sign attached to the second integer as an operational sign (item 5 in **Figure 4**). In item 9a, the students ignored the negative sign for the first integers and subtracted the second integer from the first integer. In item 9b, the students did not ignore the negative sign attached to the first integer but subtracted the second integer from the first integer.

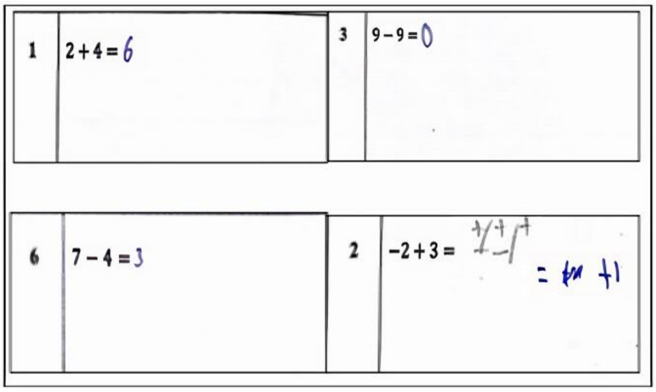

Figure 4. Correct responses post-test (Source: Authors)

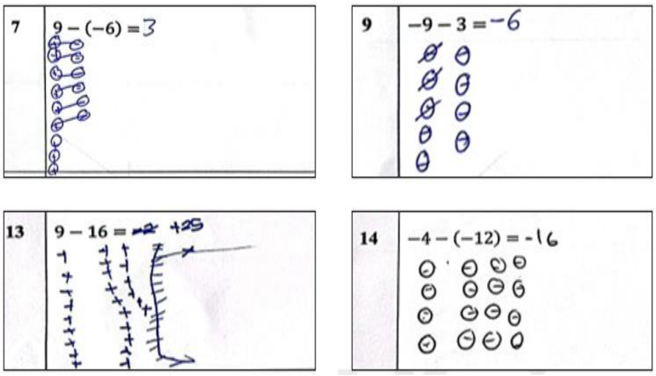

Figure 5. Common errors after intervention (Source: Authors)

In item 14, the students could realize that the two negative signs make the second integer positive. Therefore, although most of the students had a good basis of adding and subtracting integers, they also had difficulties applying the negative rules of integers. After the intervention, most students could add and subtract integers that involved both positive and negative numbers, as our results have shown. But, there were common errors. This is presented in **Figure 5**.

Of all the items in **Figure 5**, most of the students had problems with zero pairs. In item 7, for example, they were required to subtract a negative integer from a positive integer. To do this in Gizmos, they were to obtain 9 by modelling nine positive chips. To remove -6, six zero pairs need to be added so that there are enough six negatives to be subtracted from 9. Removing six negative chips would leave it with 15 positives, giving an answer of 15. Most students mistakenly added and modelled -6 with six positive chips and then cancelled the apparent zero pairs, suggesting that they applied the concept of addition.

In item 9, the students did not realize that the second integer was positive since they were supposed to subtract positive 3 from -9. In Gizmos, three zero pairs were to be added to obtain three positive chips and once obtained; the students could remove the three positive chips, which gives a final answer of 6. They did not realise that three positive chips could be obtained by adding three zero pairs. In item 13, the students could not subtract a bigger integer from a smaller integer and made a procedural error by adding a positive operational sign to the wrong answer. In item 14, the students could not subtract two negative integers. In Gizmos, they were to model four negative chips and decide whether they had enough 12 chips to remove. They needed to be aware that they had four negative chips, which was insufficient. Therefore, eight zero pairs were required to be added to remove 12 negative chips.

Table 2. Experiences of students with integer tests

Experience
I: Before I ask you about Gizmos, what do you think of tests before & after interventions?
A: The first test was challenging, but after the intervention, the second test was easy.
I: First, what did you think of the test before and after the intervention?
B: Little difficult before.
I: I see. How about after the intervention?
B: It's easier to answer the questions.
I: So, how did you find the tests?
C: I felt nervous when I did the first test.
I: Why did you feel nervous?
C: Because I was not confident.
I: How about after the intervention?
C: It was okay. I am more confident doing the second test.
I: I'm going to start with question. How did you find test before & after intervention?
D: Um … before the intervention, it was very difficult to learn. It was very difficult to answer the questions.
I: Have you studied integers before?
D: Yes. But I still did not understand.
I: How do you feel after the intervention?
D: I was excited because I understood it. I was able to answer the questions easily.
I: How did you find the test before and after the intervention?
E: When I did the first test, I was confident, but after the intervention, I felt less confident during the test.
I: Why did you feel less confident?
E: I got confused when drawing integer chips because I'm used to the multiplication rules.
I: How did you find the test before and after the intervention?
F: In the first test, I got confused, and it was hard.
I: Why?
F: Because of the bracket involved.
I: Why about after the intervention?
F: It was easy.

Note. I: Interviewer; A: Student A; B: Student B; C: Student C; D: Student D; E: Student E; & F: Student F

Once removed, the remaining chips or the final answer should be 8. It is suspected that they modelled -12 instead of adding the required zero pairs and then summed all the negative chips as the answer. These results suggest that despite the improvement in student performance after the Gizmos intervention, they still had difficulties subtracting integers involving ordered pairs.

Students' Perceptions of the Gizmos Intervention

Three themes emerged after analyzing the interview data. These are student opinions about the test on integers, their positive perception of the Gizmos intervention, and instructional features of Gizmos lessons.

Student opinions about integer tests

When students were asked to describe their perceptions about the tests on integers before and after the intervention, most (n=5) shared that they had difficulty attempting the pre-test. This was because they lacked understanding and were confused about trying the pre-test correctly. Most of them reported that the pre-test was challenging. However, after the intervention, most of them found the post-test easy since they had the understanding and confidence to answer it. The experiences of tests on integers of six students are shown in **Table 2**.

Table 2 shows that the students expressed positive views about the Gizmos intervention. Students A, B, and F found the post-test easy, while student C gained more confidence in answering the post-test items than the pre-test. Student D was excited and had the understanding to answer the post-test, reporting that the post-test was easy. However, student E got confused when using the integer chips in Gizmos and did not prefer to use Gizmos.

These findings suggest that using virtual manipulatives such as Gizmos can improve student understanding, confidence, and

Table 3. Students' positive perception of the Gizmos intervention

Perception
I: How did Gizmos improve your learning of adding and subtracting integers?
A: It made me focus.
T: Do you think Gizmos improved your understanding of adding & subtracting integers?
B: Yes.
I: How did it improve your understanding?
B: It helped me understand how to apply zero pairs correctly.
I: Did Gizmos improve your understanding of the lesson?
C: Yes.
I: In what way?
C: During the test, I could imagine that I was answering the questions in Gizmos because it felt like I was playing while studying.
I: Oh, that's interesting. In what part can you relate it to learning?
C: I can relate to how and when to remove or add zero pairs. I read the instruction when I got stuck.
I: Which part of the lesson intervention helped you?
D: When using Gizmos.
I: How did it help you?
D: I could visualise the numbers by drawing the chips. The instruction also helped me to decide what to do next. When to add or when to remove zero pairs. I felt like I was playing while studying.
T: How did Gizmos improve your understanding of adding and subtracting integers?
E: It helped me to visualise the questions better with the chips.
T: How did Gizmos improve your understanding of the lesson?
F: It was okay. But I was confused about when to add zero pairs. I still prefer the rules.
T: I see. Did you understand why when a positive sign meets with a negative sign, it becomes negative?
F: I do not understand it. I just follow the rules.

Note. I: Interviewer; A: Student A; B: Student B; C: Student C; D: Student D; E: Student E; & F: Student F

excitement when solving tests involving integers' addition and subtraction. However, they may also be confused when adding and subtracting integers using Gizmos.

Students' positive perception of the Gizmos intervention

Most of the students (n=5) had a positive perception of Gizmos, as they understood addition and subtraction of integers. Their perceptions are shown in **Table 3**.

Several positive experiences were shared by most of the students (**Table 3**). For example, student A reported that Gizmos made him focus when finding solutions to integer questions. Students C and D shared that using Gizmos helps to play and learn simultaneously. They expressed that Gizmos improved their understanding of zero pairs in solving integers. Student B confirmed this and mentioned that Gizmos helped apply zero pairs correctly. Students D and E also said that Gizmos helped them visualize integer questions, including differentiating between integers and operations. In contrast, student F was confused when dealing with the addition of zero pairs in Gizmos and preferred using rules.

These findings suggest that using Gizmos in the teaching and learning of addition and subtraction of integers helps improve student focus. It may also introduce play into lessons, improve student understanding, and visualize integer questions to differentiate between the integers themselves and the operations used. However, students' understanding can be affected when using Gizmos because they have to memorize rules without understanding the concept behind those rules.

Instructional features of Gizmos intervention

The students found the Gizmos intervention was beneficial due to its features. They reported that Gizmos has step-by-step features for them to follow and guide them in solving questions on integers. **Table 4** presents the students' views on the instructional features of Gizmos.

Table 4. Student views on the instructional features of Gizmos

View
I: How did the use of Gizmos benefit or affect your learning?
A: Umm ... If I make mistakes in modelling the chips, I cannot undo them.
I: How did the use of Gizmos benefit or affect your learning?
B: The instructions helped me understand what was taught.
I: Oh, that's interesting. In what part can you relate it to learning?
C: I can relate it to how and when to remove or add zero pairs. I read the instructions when I get stuck.
I: In what way do you like or dislike using Gizmos?
D: I liked it because I felt happy when I got the correct answer. It made me want to do it again and again.
I: How did you benefit from using Gizmos?
D: I just needed to read the instructions to avoid mistakes.
I: Can you think of any benefits or problems when using Gizmos?
E: It helped improve my understanding.
I: Anything to add about Gizmos?
F: I like how the instructions were step-by-step until we got the correct answer. And any time we reached the wrong answer, Gizmos provided feedback or comments on why the answer was incorrect.

Note. I: Interviewer; A: Student A; B: Student B; C: Student C; D: Student D; E: Student E; & F: Student F

From **Figure 4**, student D shared that the rules in Gizmos help avoid making mistakes. Student F also voiced that Gizmos provides feedback for every answer, including the incorrect ones. This also made student D happy because the correct answers encouraged her to attempt more questions. Student C was able to link the action made with the correct answer. Despite the benefits and features of Gizmos, student C reported that Gizmos do not have undo features, hindering the amendment of answers where necessary. These findings suggest that using Gizmos in teaching and learning integers decreases the chances of making procedural mistakes when rules are followed. It increases prompt feedback to questions, encouraging students to solve more questions. It also helps students to link their procedural steps to their final answers. However, wrong answers cannot be amended.

DISCUSSION

This study aimed to investigate the effectiveness of Gizmos-based intervention on student performance in addition and subtraction of integers. The study found that students' performance improved significantly after the Gizmos intervention. This result aligns with previous findings that emphasized that virtual manipulatives improve student understanding and performance in mathematics (Chong et al., 2022; Gningue et al., 2014; Inman, 2018). The result also agrees with the literature that argues that using Gizmos in integers teaching and learning improves students' remembering, understanding, and application skills (Kay & Lauricella, 2018; Raines & Clark, 2011).

This result is expected because critical steps were implemented in the Gizmos intervention to improve student learning. Comprehensive Gizmos-based lessons on addition and subtraction of integers were developed and implemented, along with a well-designed worksheet and step-by-step instructions on how to add and subtract integers using Gizmos. Students were guided to have a preliminary understanding of the basics of integers before using Gizmos. Key concepts such as two-colored counter chips and zero pair effects were explained.

In addition, open-ended and real-life questions were asked so the students could reflect and develop knowledge. Students were allowed to discover knowledge on their own. Timely feedback was given during the Gizmos intervention. Students were given more time to practice additional problems. Therefore, it is not surprising that performance improved significantly. This suggests that teachers who may want to use Gizmos as an instructional option in integers can consider these approaches to enhance students' performance.

This study also found that students expressed varied views about the integer test. Before the intervention, most students found the pre-test complex and confusing. This confirms previous findings, arguing that most students have problems understanding how to add and subtract integers (Bofferding, 2010; Goh et al., 2017; Sahat et al., 2018; Toh et al., 2017). However, after the Gizmos intervention, most students improved their understanding, confidence, and excitement when solving integer tests, strengthening their positive perceptions of the Gizmos intervention. This is expected because students' attitudes towards mathematics improve when visual manipulatives are used in teaching and learning (Durmus & Karakirik, 2006).

Generally, the Gizmos lessons captured the students' attention, improving their focus and awareness of the instructional concepts. They also found lessons as fun as they played and learned simultaneously, motivating them to participate in the lessons. These improved their performance and enhanced their positive perceptions of virtual manipulatives like Gizmos. These findings confirm the findings shared by Golafshani (2013). He found that students become excited and enjoy learning mathematics through visual manipulatives. In addition, the results are consistent with the literature that reported that incorporating technology in mathematics promotes student understanding, motivation, active participation, and performance (Al-Balushi et al., 2020; Bouck & Park, 2020; Eyyam & Yaratan, 2014; Freeman et al., 2014; Goh et al., 2017; Michaelides et al., 2019; Raines & Clark, 2011; Serin 2019). The students mentioned that they could visualise integer questions and draw a distinction between integers and operational signs. This agrees with the findings shared by Cai and Knuth (2005). They found that virtual manipulatives such as Gizmos build mental representation and skills among students.

Moreover, this study found that Gizmos limits students' procedural mistakes, especially when they can follow Gizmos rules. Students can receive feedback on questions, encouraging them to solve and practice more questions. This suggests that using virtual manipulatives is associated with increased student interest in solving integers due to the prompt feedback they receive. These findings align with previous studies that reported that the features of virtual manipulatives, such as dynamic representations and feedback, allow students to reflect, improving student learning on integers (Bolyard & Moyer-Packenham, 2006; Durmus & Karakirik, 2006). The findings also support the study by Goh et al. (2017), which concluded that virtual manipulatives provide immediate feedback that improves students' confidence and performance; and motivates them to practice and correct their mistakes.

Contrarily, using virtual manipulatives such as Gizmos can affect students' understanding. Some students reported going through Gizmos rules on adding and subtracting integers without understanding them. This caused most of the students to be confused when using Gizmos. This suggests that Gizmos lessons can force students to memorize rules, which teachers should not encourage when teaching and learning integers. Exposing students to memorize the rule of integers can lead to conceptual errors and forgetfulness of instructional concepts, which can affect their understanding and performance (Lim, 2011; Sahat et al., 2018; Sarwadi & Shahrill, 2014). Other students also shared that they could not amend their answers because Gizmos does not have the undo or erase options. This suggests that the developers of visual manipulatives such as Gizmos should include erase and undo opportunities to improve student learning on adding and subtracting integers.

CONCLUSIONS

This study investigated the effectiveness of Gizmos-based intervention in improving students' performance in addition and subtraction of integers. It also explored the perceptions of students about the provided intervention. It is one of the few studies investigating the effectiveness of virtual manipulatives as an instructional option. It is the first to use Gizmos to teach and learn integers in Brunei. Therefore, this study contributes to the literature on the effectiveness of virtual manipulatives in teaching and learning and provides essential information on the effective teaching and learning of addition and subtraction of integers.

Generally, it was found that Gizmos virtual manipulative intervention improved students' performance in adding and subtracting integers. It has critical and dynamic representative features that provide quick feedback to students when dealing with integers. Students reported positive perceptions because Gizmos teaching, and learning improve their understanding, performance, confidence, and excitement in integer lessons. Students play and learn and can visualize learning and assessment tasks and follow and apply rules to solve problems on integers. It decreases the procedural mistakes of students, motivates them to practice more questions, and relates their procedural steps to the answers they obtain.

Based on this study's results and the features of Gizmos, it can be an effective tool for improving the understanding and performance of students in addition and subtraction of integers when carefully implemented. Despite the improvement in student performance after the Gizmos intervention, there were still some difficulties and misconceptions in subtracting integers that involved ordered pairs. Some students were still confused and could not understand and apply the negative rules of integers. They could not memorize the instructions in using Gizmos and redo their mistakes in Gizmos since it does not have this option. This confirms that students lack the conceptual understanding to deal with negative integers and interpret sign rules, as the literature suggests (Chong et al., 2022; Hiebert, 2013; Vlassis, 2008).

Although most students in this study improved their understanding and performance after the Gizmos intervention, applying negative sign rules continues to be a problem for most of them, which needs continuous and critical intervention. Therefore, teachers should continuously guide students to have an in-depth understanding of applying sign rules and the alternative ways to understand and apply the rules in using Gizmos compared to memorizing these rules.

This study had some limitations. It used a small sample size to analyze the effectiveness of the Gizmos intervention. The results and conclusions are also based on addition and subtraction of integers among Year 9 students in Brunei. Therefore, generalizing the results of this study to other areas in mathematics and different contexts should be done with care by considering the context of generalization. Despite these limitations, this study contributes to a critical understanding of improving students' performance in addition and subtraction of integers. This study is recommended to be replicated in other mathematical areas and educational contexts using larger samples to confirm our results. Future researchers can also consider exploring alternative ways for students to understand and apply the sign rules of integers.

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Declaration of interest: Authors declare no competing interest.

Data availability: Data generated or analyzed during this study are available from the authors on request.

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APPENDIX A

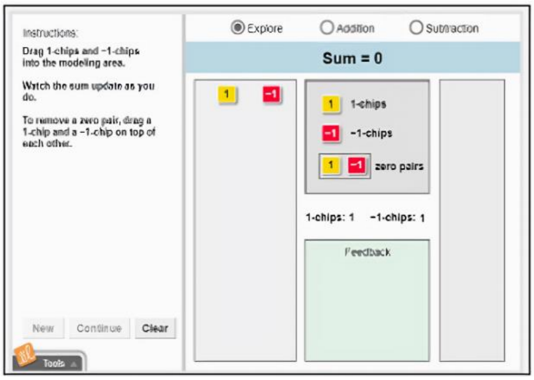

Figure A1. Steps for Gizmos in zero pairs (Source: Gizmos free online application from https://gizmos.explorelearning.com/)

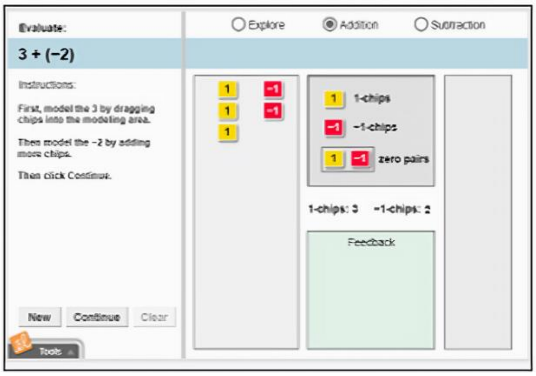

Figure A2. Modelling of three yellow chips & two red chips into modelling area (Source: Gizmos free online application from https://gizmos.explorelearning.com/)

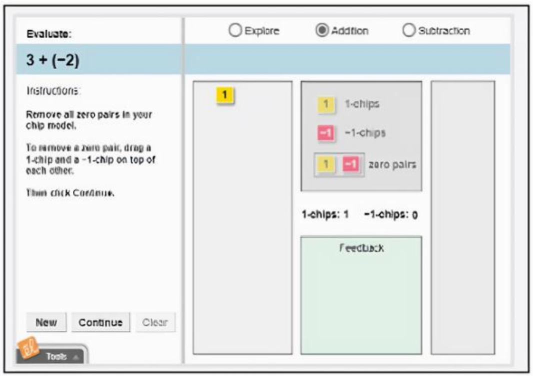

Figure A3. One positive yellow chip was left after removing zero pairs (Source: Gizmos free online application from https://gizmos.explorelearning.com/)

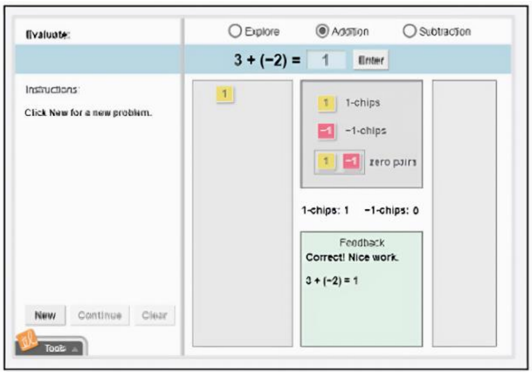

Figure A4. Feedback is given on the correct answer (Source: Gizmos free online application from https://gizmos.explorelearning.com/)

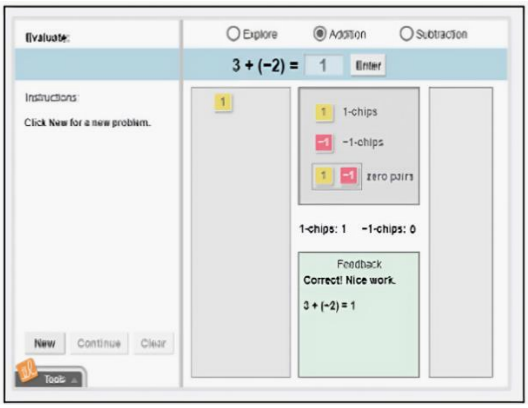

Figure A5. Modelling of five negative red chips into modelling area (Source: Gizmos free online application from https://gizmos.explorelearning.com/)

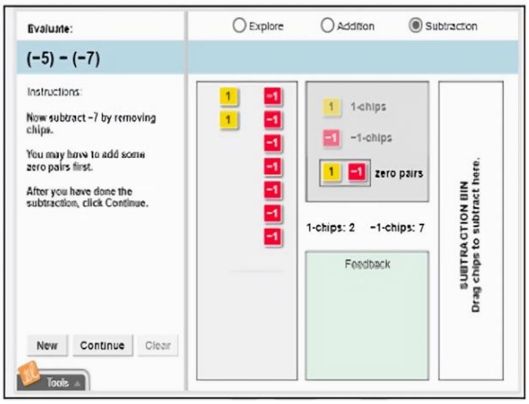

Figure A6. Addition of two zero pairs in modelling area before performing subtraction (Source: Gizmos free online application from https://gizmos.explorelearning.com/)

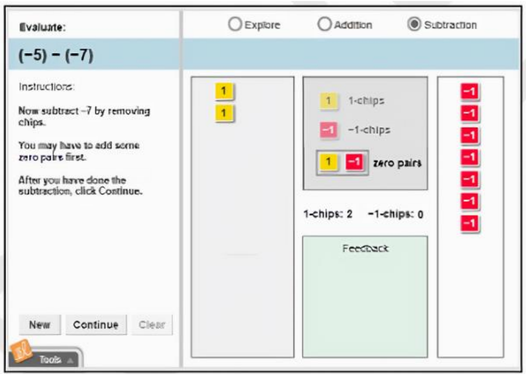

Figure A7. Two positive yellow chips were left after removing seven negative red chips (Source: Gizmos free online application from https://gizmos.explorelearning.com/)