




Measurement Model Testing: Adaption of Metacognitive Awareness Toward Mathematic Reasoning Among Undergraduate Education Students

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Citation: Tak, C. C., Zulnaidi, H., & Eu, L. K. (2022). Measurement Model Testing: Adaption of Metacognitive Awareness Toward Mathematic Reasoning Among Undergraduate Education Students. *Contemporary Mathematics and Science Education*, 3(2), ep22021. <https://doi.org/10.30935/conmaths/12510>

ABSTRACT

This quantitative research aimed to measure the metacognitive awareness model toward mathematics reasoning among 184 university students. Metacognitive awareness demonstrates convergent, and discriminant validity was performed, which includes six factors: conditional knowledge, declarative knowledge, procedural knowledge, monitoring, planning, and evaluation. Data analysis was using exploratory factor analysis. The results indicated that Cronbach's alpha coefficients demonstrated that metacognitive awareness was a reliable instrument researcher could use to evaluate university students' mathematical reasoning abilities. This research analysis revealed that positive relationship between metacognitive awareness and mathematics reasoning among university students.

Keywords: metacognitive awareness, mathematics reasoning, undergraduate education students, exploratory factor analysis validity

Received: 24 Jun. 2022 ♦ Accepted: 27 Sep. 2022

INTRODUCTION

Students often struggle the most with acquiring mathematical reasoning. Students encounter challenges in completing mathematics challenges as early as their first year of college (Subia et al., 2018). A low-efficiency belief system implies a low-performance position in mathematical logic, which causes difficulties engaging and putting in additional effort to study (Johnny et al., 2017). According to PISA and TIMSS 2013, approximately 65% of young Malaysians have an issue with mathematics reasoning, and roughly half have anxiety (Al-Mutawah & Fateel, 2018).

With enhanced skill in mathematical reasoning and metacognitive awareness toward mathematics come confidence and an increased willingness to learn the mathematical skills and contents and effective learning strategy to become a competent student of mathematics (Jeannotte & Kieran, 2017). According to the National Council of Teachers of Mathematics (2021), mathematical knowledge that supports students in developing mathematical skills and knowledge, with an obsessive focus on problem-solving and mathematical reasoning, is highlighted. Mathematics reasoning must integrate cognitive and metacognitive processes because a problem-solving individual must choose an approach and consider other solutions when encountering problems and changing settings. However, cognitive functions such as selecting appropriate strategies are not sufficient for

the answer; a metacognitive monitoring system that manages these mental activities and checks the effectiveness of applications of mathematics strategy is also required (Haryani et al., 2018).

Flavell (1979) invented the phrase "metacognitive." Flavell (1979) described metacognitive as "cognition and knowledge of cognitive activities" and regarded it as the learner's understanding of their cognition. As a consequence of Flavell's (1979) research, various researchers began to investigate metacognitively and classify it as a notion of multiple proportions. This case has shown that the concept may have many metacognitive components. In this scenario, many meanings have evolved. Meanwhile, Young and Worrell (2018) defined metacognitive as "understanding of several aspects of an individual's cognitive processes" and "editing capacities of persons concerning cognitive activities to comprehend more efficiently."

College students benefit from metacognition approaches and comprehend students' metacognitive abilities (Barenberg & Dutke, 2018). According to Young and Worrell (2018), individuals will be more inclined to apply various ways of learning, problem-solving, and thinking. Furthermore, Young and Worrell (2018) contend that there is a need to educate metacognitive knowledge completely. Two kinds of research have offered specific ways of improving metacognition (Abdelrahman, 2020; Harrison, 2018). However, Medina et al. (2017) discovered higher cognition knowledge among graduate students than undergraduates (Abdelrahman, 2020).

Metacognitive Awareness

Metacognitive awareness is a phenomenon that expresses itself in various ways as individuals engage with occurrences and events in their actual routine. Metacognitive information will assist individuals in organizing, sequencing, and monitoring their learning so that productivity improvements can be realized promptly (Ozturk, 2017; Tak et al., 2022a).

Metacognitive refers to an individual's awareness of their style of thinking, capabilities, and limitations while contributing to the learning process. Metacognitive awareness is necessary for reflection on one's views (Smith et al., 2017). For instance, a student may have a complete understanding of mathematical operations. Still, they would fail at problem-solving if they did not know how to apply these procedures to solve a problem (Yelgec & Dagyar, 2020).

Metacognitive awareness separates cognitive knowledge and cognitive regulation. Metacognition knowledge has three aspects; there is declarative knowledge (wisdom about why and when to know), conditional knowledge (knowledge about why and when to understand), and procedural knowledge (knowledge about when and why to learn). Meanwhile, cognitive regulation comprises five sub-components that aid in the process aspect: planning, monitoring, and evaluation (Yelgec & Dagyar, 2020).

Few researchers have suggested cognition knowledge and cognition regulation as fundamental elements/components of metacognition. Understanding cognition refers to how much learners learn through their learning processes and how much they know about their awareness (Erenler & Cetin, 2019; Moxon, 2022; Robillos & Bustos, 2022). Metacognition is knowledge created by the dynamic interaction of individual, task, and strategy parameters. The person remembers it with various cognitive aspirations, talents, and varied cognitive experiences. The three components of metacognitive knowledge are, as follows: declarative, procedural, and conditional knowledge. Regulation of cognition includes the techniques or talents that foster learning and give ways of achieving the goal and what is taught (Roick & Ringeisen, 2018).

A learner's understanding of why and when they are learning is conditional knowledge. Declarative knowledge is a learner's comprehension of what they are learning. Procedural knowledge is the learner's grasp of how they apply strategies while studying. This research also employs knowledge of cognition as one component of metacognitive awareness (Abdelrahman, 2020). Cognition regulation describes the actions that learners take to control and alter the trajectory of their cognitive activity. These stages develop as a consequence of domain-specific learning, with self-correcting activities as early antecedents. Cognitive regulation includes monitoring present activity, predicting an action or occurrence, validating action results, reality testing, and other behavior patterns for regulating and directing conscious attempts to learn and solve problems (Abdelrahman, 2020).

As a consequence of Flavell's (1979) research, various researchers began exploring metacognitive and classifying it as a theory in multiple configurations. This circumstance has demonstrated that the concept may have several metacognitive dimensions. Metacognitive awareness is an essential prerequisite for reflecting on one's views. As a result, when supported by understanding beliefs and personal theories, the reflective practice model is a potent driver for professional growth (Harrison, 2018).

Wafubwa and Csikos (2022) clarify metacognitive as information about different characteristics of an individual's behavior patterns and applicability aptitude of persons about cognitive activities to comprehend more efficiently. Metacognitive refers to an individual's awareness of their information processing and approaches and their ability to monitor and modify these processes. This approach demands students to assess, evaluate, and reflect on their learning and cognitive functions. Conceptualize the reflective thinking abilities involved in this process.

Moreover, according to Robillos and Bustos (2022), metacognitive is a debate between thinking and action at the core of the evolutionary transformation in approach, necessitating fundamental alterations in ideas, attitudes, and norms concerning instruction and accomplished learning if the change is to be through reflection. In addition, Karaoglan Yilmaz (2022) highlighted that "critical reflection is a process of in-depth examination that reveals unexamined ideas, assumptions, and expectations and makes evident our reflective loops."

Metacognition is fundamental in mathematics learning since it affects acquisition, understanding, retention, and application (Khodaei et al., 2022; Tak et al., 2021). It also impacts learning effectiveness, problem-solving, and critical thinking. Metacognitive awareness enables self-regulation of the control of thought and the learning process and results in mathematics. Knowledge of cognition refers to what a person understands about cognition in general or one's comprehension. Asy'ari et al. (2022) defined metacognitive awareness as the process of growing one's awareness of his own personal, strategy knowledge, and task in a setting through reflective thinking. According to Wafubwa and Csikos (2022), there are four strategies to promote metacognitive awareness in mathematics lesson structuring. They include raising public awareness of the need for metacognition, enhancing cognition knowledge, improving cognition control, and creating circumstances encouraging problem solutions. Metacognitive awareness enables self-regulation of the reign of thinking and learning process and consequences

Widana et al. (2018) defined metacognition as learning, planning, awareness, problem-solving of the individual's reasoning, and organizing their thinking process. Metacognition is the awareness of how someone learned, accomplishes an aim, and how to utilize knowledge when he does not comprehend and is aware of this, the capacity of judgment in the cognitive demand in a specific task, strategic knowledge related to the objectives, and the appraisal of someone's both during and after the process. Metacognition is the ability to manage cognition utilizing knowledge and regulatory skills (Sulistiyowati et al., 2017).

As a result, metacognition is a broad emotion used to classify a variety of components. All of these elements correlate with unique ideas, assumptions, thoughts, and actions, as well as mathematical reasoning skills and metacognitive awareness, which may be said to be owned by the person. Individual growth and the earning process are essential for both notions (Adinda et al., 2021). As a result, there is a link between metacognitive and mathematical reasoning abilities. In this research, undergraduate education students in levels of mathematical reasoning and metacognitive awareness explored various factors and attempted to discover the link and correlation between each construct.

Mathematics Reasoning

Generalization is a type of reasoning that involves several contexts, but the emphasis is not solely on the context but also patterns, procedures, structures, and relationships between the forms. The mathematical activity involves two types of reasoning: logical reasoning, generated by conjecture, and deductive reasoning, developed by proven mathematical knowledge. There are four categories of mathematics reasoning: providing evidence, identifying patterns, making conjectures, and providing unsubstantiated arguments (Zayyadi & Kurniati, 2018).

Malaysian education has made the development of scientific talent an essential national priority. For that purpose, the Malaysian mathematics and science education integrated curriculum in secondary school emphasizes developing critical thinking and creativity, reasoning, and higher order thinking abilities. Higher-level mathematics needs the application of abstract notions, which necessitates formal logical reasoning based on ideas and concepts (Arshad et al., 2017).

Johnny et al. (2017) discovered that minorities of students are capable of analyzing information, drawing inferences, and generalizing while solving complex problems. However, Singh et al. (2020) point out that reasoning abilities represent low levels of higher-order thinking engagement. These findings indicate that pupils grasp fundamental mathematical ideas but cannot use that knowledge in non-routine problem settings (Singh et al., 2020).

Some research describes reasoning as a combination of two different processes: deductive and inductive reasoning abilities (Hidayah et al., 2020). Much reasoning research has been performed over the years to bridge the gap between inductive and deductive arguments (for example, suggestive reasoning, abductive reasoning, and plausible reasoning). Such definitions and concepts of reasoning differ in detail. Still, they all deal with a method or process of various cognitive abilities and habits essential to mathematical problem-solving (Artemov & Fitting, 2019).

Mathematics reasoning is considered essential and solves mathematical problems. However, the relationship between reason and the subject matter has highlighted issues about the ubiquitous basis of learning. Reasoning abilities are universal, and arguments support the assumption (Sukirwan et al., 2018).

For example, before solving algebra word problems, a student concludes that specific general problem-solving strategies they have employed with word problems have improved performance. Students could use techniques such as transforming the world issue into a symbolic form, classifying it according to the solution approach, and checking and validating each result (Olson & Johnson, 2022; Tak et al., 2022b).

Mathematical reasoning has experienced several definitions and terminology. The term "reasoning-and-proving" describes a series of activities that look into how and why "things operate" in different fields of mathematics, including algebra, geometry, and more. The definition of reasoning is "the process of drawing inferences from facts or expressed views" (Battista, 2017). Generalizing about mathematical occurrences and conjectures about their links are necessary for reasoning. Overall, reasoning focuses on introspective processes that help students build and improve their mathematical understanding (Artemov & Fitting, 2019).

This research used a mathematics reasoning question from Calvin and Duane (2002) to examine these students' reasoning aptitude and mathematical abilities. In addition, four mathematical reasoning topics, including critical thinking, whole numbers; fractions; geometry, are utilized in measurement analysis.

These parameters were assessed at the assumption of the academic session so that students may reflect on their experiences with their statistics course and the teacher since several questions referred to previous learning practices. According to Lestari and Jailani (2018), mathematical reasoning applies to various logical thinking abilities. The learner can clarify the conclusion from the earlier steps' logic in a logical chain of reasoning is an example of deductive reasoning.

The desire of mathematics students to comprehend mathematics and evaluate validity in mathematics leads to inductive and deductive reasoning. Promoting the development and discussion of deduction reasoning in the classroom may significantly impact mathematics learning (Maiti, 2017).

Mathematical reasoning abilities are required for a person to explain and compare similarities of mathematical patterns properly. Widya et al. (2019) state that students' capacity to articulate reasons for each interpretation is crucial to the abstraction process. The application of mathematical concepts is a plan to use solid assertions. Additionally, one of the primary purposes of learning activities is to build the capacity to provide coherent reasons for inferring mathematical values. Saleh et al. (2018) defined mathematical reasoning as a mental action that requires mathematical reasoning abilities.

METHOD

Research Sample

The researcher chose the maximum variation sampling approach among the deliberate sampling strategies employed in selecting the participant group. A purposeful sampling approach is a sampling strategy used to locate and select samples that are rich in information (Creswell & Creswell, 2018). Participants in this research were chosen based on their academic course and willingness. Participants were involved in this research during the first and second semesters of the 2020-2021 academic years.

This research included 184 undergraduate students from the faculty of education in a few public universities around Klang Valley, Malaysia. Gender and stream were the demographic factors evaluated to reflect the backgrounds of research participants. Regarding gender, 75 of the research participants (40.8 percent) were female, while 109 participants (59.2 percent) were male. Regarding the stream, 159 (86.4 percent) of the research participants were science stream students, while 25 (13.6 percent) were non-science stream students.

Instrument

This research collected gender demographic data and the present participants' stream of undergraduate students from the faculty of education. This research adopted mathematics reasoning questions from Calvin and Duane (2002) to measure students' mathematics reasoning. Includes topics of critical thinking, geometry, fraction, and set and whole numbers.

Table 1. Variable and item for metacognitive awareness instrument

Item	Variable
Sub-construct: Declarative knowledge	
C1	I know which information is essential to learn.
C2	I can organize information well.
C3	I can remember information well.
C4	I can control my learning whether I have learned well or not.
C5	I can decide whether I have understood something well.
Sub-construct: Procedural knowledge	
C6	I try to use strategies that have worked previously.
C7	I have a specific purpose for each strategy I use.
C8	I realized and learned the type of strategy used.
C9	I found myself using appropriate strategies spontaneously.
Sub-construct: Conditional knowledge	
C10	I can learn better when I know something about the topic.
C11	I use different strategies depending on the situation of the given question.
C12	I can motivate myself to study if necessary.
C13	I use intellectual strength to balance my weaknesses.
C14	I know the strategy I use is effective.
Sub-construct: Planning	
C15	I think about what I have learned before starting an assignment.
C16	I set certain information before starting a task.
C17	I will choose the best way to solve the problem.
C18	I read the instructions carefully before starting the task.
C19	I manage my time as best I can to achieve my goals.
Sub-construct: Monitoring	
C20	I often ask myself if I have achieved the learning goal.
C21	Before responding to a situation, I explore numerous possibilities.
C22	I question whether I have examined all possible techniques for accomplishing the task.
C23	I learned and reviewed periodically to help myself understand the essential connections of information.
C24	I found analyzing the usefulness of different strategies when studying.
C25	I find that I always pause to reflect on my understanding.
Sub-construct: Evaluation	
C26	I know the extent of self-achievement after completing a test.
C27	After completing the task, I asked myself if there was an easy way to complete the job.
C28	After studying, I always summarize what I have learned.
C29	After completing a task, I ask myself how far I have achieved my goals.
C30	I asked myself if I had learned any additional knowledge after completing the assignment.

The researcher adapted the instrument from Schraw and Dennison (1994) to measure students' metacognitive awareness. Metacognitive awareness inventory is a five-point Likert scale from strongly agree to disagree. Each statement on the questionnaire will be asking must be completed by participants. Metacognitive awareness scores are determined by evaluating all items in the metacognitive awareness inventory scale. The score on the perceived value of mathematics reasoning describes participants' metacognitive belief in mathematics reasoning experience. **Table 1** shows the variable and item for metacognitive awareness instrument.

Data Collection

Before administering the instrument to the participants in the research, the researcher had to seek permission from the academic staff involved. The relevant academic staff received application letters with a brief description of the research purpose and method. Following approval, the researcher contacted the course instructor to seek permission and set up an appointment to administer the instrument to

the students in their class. The researcher gave the research purpose, and respondents also were given 45 minutes to complete questionnaire.

Exploratory Factor Analysis

Before conducting the research, the researcher evaluated data input reliability, missing values, normality, and outliers. All items in this research were reasonably normally distributed, with skewness and kurtosis statistics indicating that all constructs were within the value ranges of ± 2 . Meanwhile, the standard score Z for each item was in the field of ± 4 , indicating no extreme instances and no outliers in the data. As a result, the data is acceptable for further analysis since there is no significant dispute in the data research (Awang, 2018).

Three factors had to evaluate if the data was suitable for factor analysis to be considered. The data used the Kaiser-Meyer-Olkin (KMO) and Bartlett's test of sphericity (BTS) to assess the sampling adequacy of factor sample size. Awang (2018) recommended limiting sample sizes to 100 or greater. This research included a sample size of 148 respondents based on these assumptions for assessing the adequacy of the sample size for factor analysis.

The KMO values between 0.8 and 1 reflect adequate data sampling. In other words, the KMO and the BTS assess whether the sample was acceptable to persist with factor analysis (Tabachnick & Fidell, 2007). Additionally, the researcher must consider a few things, such as ensuring that the excellent range of anti-image correlation for all items is more significant than 0.5 (Hair et al., 2019). Conversely, the commonality for all items was higher than 0.3 (Cohen et al., 2012).

To evaluate the instrument's construct, the researchers employed varimax rotation and exploratory factor analysis (EFA), principal component analysis. Assessing the appropriateness of concluding an individual based on test results received inside a concept is what is meant by the term "construct validity" (Cohen et al., 2012). An instrument's utility in research relies on the validity elements that might provide value to the investigation. If the research data include data dropouts, outliers, and analyses of normality, an EFA should be conducted (Cohen et al., 2012).

The researcher employs three approaches during the EFA to determine the number of factors: the Kaiser-Guttman criterion (eigenvalue > 1), the parallel computation, and the screen plot; the approach's goal is to compute the number of elements more genuinely than a single method. It's also essential to pay great attention to the KMO indicator while evaluating exploratory variables to determine if the data are appropriate for research. Exploration with EFA generates accurate and distinct factors nearing the KMO value of 1. It is also possible to assess if the variables analyzed are factorable by the results of the BTS (Cohen et al., 2012).

For the EFA, the researcher tested the effects of loading factors ranging in size from 0.3% to 0.6%. Hair et al. (2019) mention that can achieve an EFA research best finding to determine the proper sample size for the research's empirical and theoretical parallels. As a consequence of the outcomes of the factor analysis, Hair et al. (2019) recommend several criteria that the researcher should consider before deciding whether or not to retain or get eliminate a particular item, including:

1. items that are highly reliance on two or more factors (cross-loading),
2. items that have a loading factor more minor than the size of an essential loading factor,

Table 2. EFA of variable for metacognitive awareness

Item	Factor	LF	CR	AVE	CA
C1		.591	.810	.972	.770
C2		.591			
C3	Declarative	.493			
C4	Knowledge	.604			
C5		.573			
C7	Procedural	.610	.713	.527	.712
C9	Knowledge	.610			
C12	Conditional	.543	.817	.772	.726
C13	Knowledge	.614			
C14		.532			
C15	Planning	.635	.813	.903	.775
C16		.567			
C17		.548			
C19		.530			
C20	Monitoring	.504	.805	.594	.741
C22		.544			
C23		.553			
C24		.605			
C26	Evaluation	.553	.789	.667	.714
C27		.597			
C28		.526			
C29		.634			
C30		.613			

Note. LF: Loading factor & CA: Cronbach's alpha

- items that have a critical loading factor but have a low communality value, and
- items that meet the theory underpinning (Hair et al., 2019).

Pending the outcome of the EFA, it is necessary to undertake an instrument reliability exploration. The reliability of the measurements determines the degree of consistency between several measurements of an attribute. The researcher performed a Cronbach's alpha instrument reliability analysis to establish the degree of instrument reliability in the investigation. Researchers can use the approach to determine if the measuring objects are the same or different from one another, as well as ways other researchers often employ. The correlation between items with items surpassing the value of 0.3 and the Cronbach's alpha value exceeding 0.7, as suggested by Hair et al. (2019), should be met to determine the degree of consistency in the established instrument.

FINDINGS

The researcher then used the statistical software AMOS and SPSS for the reliability analysis. The value of Cronbach's alpha, loading factor, composite reliability, and average variance extracted (AVE) established the research instrument's degree of reliability. The findings of the EFA are summarized below.

As Awang (2018) recommended, the Cronbach's alpha criterion should have a value greater than 0.70. **Table 2** demonstrates that the metacognitive awareness constructs matched the criterion requirements; Cronbach's alpha values ranged from 0.71 to 0.77.

According to Awang (2018), the AVE value should be less than 0.50, and for the construct validity (CR) criterion, the criteria must have a value greater than 0.60. The CR value ranges between 0.71 and 0.81, and the AVE for metacognitive awareness ranged from 0.52 to

Table 3. The goodness of the fit index of metacognitive awareness

Statistic fit	Value	Explanation
χ^2/df	2.396	Model vs. saturated
GFI	0.798	Comparative fit index
CFI	0.804	Tucker-Lewis index
RMSEA	0.087	Root mean square error of approximation

Table 4. Goodness of fit index of metacognitive awareness and mathematics reasoning

Statistic fit	Value	Explanation
χ^2/df	1.754	Model vs. saturated
GFI	0.760	Comparative fit index
CFI	0.859	Tucker-Lewis index
RMSEA	0.064	Root mean square error of approximation

0.97. Overall, the EFA of the metacognitive awareness variables meets the specific requirements.

A goodness-of-fit model utilized the statistical value of mean square root error of approximation (RMSEA) and the good-of-fit, χ^2 , in the factor analysis. More than 0.10 indicates model rejection; however, less than 0.08% values are considered acceptable (Awang, 2018). Comparative fit index (CFI) is related to analysis and Tucker-Lewis goodness of fit indexes (TLGFI). A fair value for the indicated model is one with a value more than or equal to 0.90. For this research, were eliminated the items with a loading factor lower than 0.60. The following results have been revised for the CFA research as it has.

Utilizing **Table 3**, it is apparent that metacognitive awareness in the model is good. Model's Chi-square/df=2.396; GFI=0.798; CFI=0.804 and RMSEA=0.087; the model's Chi-square correspondence index is good at the level of significance (Awang, 2018).

Loading factors of less than 0.40 are removed in this research instrument since it positively impacts the result (Awang, 2018). Model's Chi-square/df=1.754; GFI=0.760; CFI=0.859; and RMSEA=0.064; the model's Chi-square correspondence index is good at the level of significance (Awang, 2018). Utilizing **Table 4**, it is apparent that the model structure is good.

The finding also showed university students could use varied techniques such as transforming the world issue to a symbolic form, classifying it according to the solution approach, and checking and validating each result in mathematics reasoning questions. Besides, the finding also reveals that university students could analyze information, draw inferences, and generalize while solving complex problems in mathematics reasoning.

DISCUSSION

This research aimed to examine the role of mathematical reasoning in understanding the relationship with metacognitive awareness. The research purpose necessitated the implementation of SEM since it permits conceptual testing assertions about the relationship between mathematics reasoning and metacognitive awareness constructs.

The findings showed a weak but significant relationship between students' metacognitive awareness and mathematics reasoning. Students' mathematics academic performance was significantly related to metacognitive awareness (Karaoglan Yilmaz, 2022; Tak et al., 2021; Wafubwa & Csikos, 2022).

The direct effect of metacognitive awareness findings is consistent with earlier research findings (Abdelrahman, 2020; Erenler & Cetin, 2019; Flavell, 1979; Ozturk, 2017; Smith et al., 2017; Tak et al., 2022b; Yelgec & Dagyar, 2020). However, some research does not show an essential of justifying in mathematics reasoning association between metacognition awareness. This result is mainly consistent with the finding that metacognitive awareness is positively significant, which engages mathematics reasoning performance (Harrison, 2018; Sulistyowati et al., 2017). Moxon (2022) mentioned that the inconsistent results of cultural difference learning approaches might be attributed. More specifically, the data indicated a significant positive relationship between metacognitive awareness for Malaysian undergraduate students in mathematics reasoning performance.

The findings are aligned with previous research findings and revealed that university students could analyze information, make conclusions, and generalize when solving challenging problems in mathematical reasoning (Hidayah et al., 2020; Johnny et al., 2017; Maiti, 2017; Singh et al., 2020; Sukirwan et al., 2018; Zayyadi & Kurniati, 2018). Besides, findings indicate that university students grasp fundamental mathematical reasoning ideas with their metacognitive strategies (Adinda et al., 2021; Arshad et al., 2017; Artemov & Fitting, 2019; Lestari & Jailani, 2018; Olson & Johnson, 2022). The finding also showed university students able to use a few techniques, such as transforming the world issue to a symbolic form, classifying it according to a solution approach, and checking and validating each result in mathematics reasoning questions (Battista, 2017; Saleh et al., 2018; Sulistyowati et al., 2017; Tak et al., 2022b; Widya et al., 2019). Therefore, findings also showed that adaptation metacognitive awareness instruments are suitable for measuring their strategies in mathematics reasoning.

Implication and Suggestion

This research on metacognitive awareness evaluation contributes to mathematical reasoning. Otherwise, some findings suggest that metacognitive awareness positively impacts mathematics reasoning achievement. The current research findings indicated that high metacognitive awareness affects mathematics reasoning performance. The researcher also suggested that implementing reasoning elements in a mathematics lesson boosts students' metacognitive awareness to enhance their mathematical reasoning skills. The following research should conduct further research on model research with more extensive and varied student populations. Moreover, the various learning approaches recommended for mathematics reasoning enhance students' metacognitive awareness.

CONCLUSION

This research successfully evaluated university students' metacognitive awareness instruments for mathematical reasoning. Findings indicated that metacognitive awareness of mathematics reasoning correlation is statistically significant. Other researchers are Adinda et al. (2021), Karaoglan Yilmaz (2022), and Wafubwa and Csikos (2022). The positive relationship between metacognitive awareness and mathematics reasoning was similar finding previous researchers such as Haryani et al. (2018), Lestari and Jailani (2018), and Tak et al. (2021, 2022a, 2022b). The reliability analysis verified all six aspects of metacognitive awareness constructs. A metacognitive awareness factor excludes seven items yet retains the characteristics of

components envisaged by researchers based on empirical evidence and the perspectives of the Malaysian educational context. Analyses of instrument reliability using Cronbach's alpha coefficients demonstrated that the metacognitive awareness instrument was reliable. In conclusion, the researcher can use the questionnaire to measure metacognitive awareness of university students' mathematical reasoning abilities.

Author contributions: All authors contributed equally to this work and were involved in concept, design, collection of data, interpretation, writing, and critically revising the article. All authors approve final version of the article.

Funding: The authors received no financial support for the research and/or authorship of this article.

Ethics clearance: Universiti Malaya Research Ethics Committee (UMREC) approved the study on June 3, 2022 (Reference number: UM.TNC2/UMREC_1907).

Declaration of interest: Authors declare no competing interest.

Data availability: Data generated or analysed during this study are available from the authors on request.

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