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Pre-service Teachers' Perceptions of and Knowledge for Mathematical Modelling in Ghana

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ABSTRACT

Mathematical modelling, in general, improves problem-solving skills and can lead to innovation. Using an explanatory sequential mixed method approach, this study considered West African pre-service teachers' concepts of mathematical modelling as they tried to solve modelling-type application problems. The study was conducted in selected tertiary institutions in Ghana to determine pre-service teacher preparedness for teaching mathematical modelling at secondary schools. Modelling-type tasks were given to 30 undergraduate pre-service mathematics teachers who had completed several content courses. After attempting these tasks, questionnaires elicited further information about their views of and familiarity with modelling. A careful analysis of the written responses revealed that the majority understood some basic ideas of modelling. However, four of the respondents demonstrated minimal understanding of mathematical modelling concepts, and those four were interviewed about their perceptions. Analysis of the four selected pre-service secondary math teachers' written responses revealed both conceptual and procedural errors, and the interviews revealed that they had little familiarity with the mathematical modelling. Authors suggest that modelling should be part of the West African senior high school mathematics curriculum to build confidence and capabilities among students, who are next generation's teachers, and this will generatively propagate economic development.

Keywords: mathematical modelling, content knowledge, pedagogical knowledge, pre-service teacher

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INTRODUCTION

The 2015 "GAIMME report" (Guidelines for Assessment and Instruction in Mathematical Modeling Education) provides many examples of modelling tasks appropriate for various educational levels, from elementary to undergraduate learners, to describe what mathematical modelling looks like in practice. Although mathematical modelling cannot be comprehensively defined in one sentence, the GAIMME report does provide the skeleton of a definition. "Mathematical modeling is a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena" (CMA & SIAM, 2016, p. 8). Mathematical modelling in education has been well-researched (e.g., Huntley & James, 1990; Mudaly & Dowlath, 2016), and modelling authentically reflects practices from scientific research. Most discoveries are initially tested using modelling principles since models do not require much data before laboratory activities are carried out. Mathematical modelling is not new; however, little attention has been given to modelling in the standard curricular of developing countries, such as Ghana, Africa. For decades, mathematics education leaders (e.g., Levy, 2015; Pollak, 1969) have advocated that modelling should broadly be part of the teaching and learning process. However, the school curriculum in Africa mostly neglects modelling (Mudaly & Dowlath, 2016).

Recently, there have been calls for the re-introduction of mathematical modelling in the school pre-tertiary school curriculum. Recent curricular reforms have adopted new emphases on applications of mathematics to the real world, such as in the United States and in South Africa (De Villiers, 2007; NGACBT & CCSSO, 2010). Also in Ghana, the most recent mathematics curriculum places importance on the use of mathematics in solving societal problem (NaCCA, 2020). The demand of using mathematics to solve problems in real-life intersects with what mathematical modelling seeks to achieve (CMA & SIAM, 2016; Mudaly & Dowlath, 2016). Ostler (2003) suggests that there is the need to prepare teachers at the secondary school level for mathematical modelling instructions. There is a needed reform to include more mathematical modelling in Ghana and to include more learner-centered pedagogy to help students construct their own knowledge, but there is also a need to understand the current landscape of mathematics teacher education in Ghana, to initiate changes both effectively and ethically. To this end, this study seeks to investigate how the current pre-service mathematics teachers (PSMTs) in Ghanaian universities perceive mathematical modelling, in hopes that this will inform the assessment of needs–what should be changed and how it should be changed.

As another industrial revolution is envisaged in Ghana and as world technology changes rapidly, the 'real world' is quite complex. Thus, the PSMT requires complex teaching strategies that connect with reality. Mathematical modeling, as a method of teaching, makes mathematics more useful to the learner as it nurtures students' strategic competence in solving complex problems, like those they will find in the world. In Germany, mathematical modelling has been taken up as a primary way of teaching mathematics due to its importance in real life (Arseven, 2015; Blum & Ferri, 2015); however, few other countries concentrate on mathematical modelling as a way of teaching math. Teachers commonly perceive that teaching with modelling is too difficult or time-consuming, but if PSMTs were adequately prepared and understood what mathematical modelling looks like in a classroom, then pre-tertiary education could be improved with time, incorporating more modelling experiences for students (Arseven, 2015). Mathematical modelling as a teaching strategy provides learners the skills needed to solve real-life issues in mathematics and in other areas of human life (Blomhøj & Jensen, 2007; Blum, 2002; Crouch & Haines, 2004).

To prepare to use mathematical modeling as a pedagogical tool, PSMTs should concentrate on activities that encourage conceptualization, development, and modelling competencies (Haines & Crouch, 2001). Mathematical modeling can be a complex activity, and different mathematical concepts can be necessary to implement them. The PSMT must have sufficient knowledge for any modelling task to be undertaken (English, 2009). Unfortunately, PSMTs often lack the appropriate pedagogical skills that allow the learner to construct their own knowledge in the classroom (Shulman, 1986). In most cases, the PSMT is guided to acquire content knowledge in many mathematical concepts and models, in hopes that they will be able to apply them to real-life problems they face in the future, but more practice for PSMTs with authentic modelling experiences and more development of pedagogical content knowledge would benefit future generations of learners.

MATHEMATICS EDUCATION IN GHANA

After gaining independence from British Colonial rule in 1957, the Ghanaian government prioritized education by introducing compulsory education policies. Then, the number of schools increased, with government schools joining the already-existing mission-based schools. However, political turmoil in Ghana hindered progress as well. Then, in 1996, the civilian government implemented the Free Compulsory Universal Basic Education (Dei, 2011), and Ghana strives today to enact free compulsory education even with minimal resources as a developing country. Understandably, these much-needed education reforms have outpaced teacher recruitment and teacher education expansion in Ghana (Abreh et al., 2018). The 1987 Education Sector Reform helped Ghana produce more qualified teachers in primary education, but Blunch (2014) argues that this policy was too narrow and that future policies should include higher levels of education in Ghana as well. Mathematics education research is growing in Africa. While a meta-analysis showed almost double the number of studied from 2007 to 2015 compared to a similar review from 2000 to 2006, Adler et al. (2017) reveal that this increase is in primary mathematics. More reform and more research need to be implemented in higher levels of mathematics. In 2003, when Ghana participated in the trends in international mathematics and science study (TIMSS) for the first time, the average Ghanaian student scored 276 compared to the international average of 466 (Frempong, 2010). Poor performance of Ghanaian students in mathematics was found to be partially attributable to inadequate teacher preparation and impoverished teacher knowledge in mathematics (Ansah et al., 2020; Butakor et al., 2017).

In addition to the need for more teacher preparation in Ghana, there is also a need for reform to the pedagogies emphasized in teacher preparation programs. Most mathematics teachers in Sub-Saharan Africa deliver content via teacher-centred lectures and emphasize rote memorization (Abreh et al., 2018; Dei, 2011). The 2011 poor mathematics performance on the TIMSS is partially attributable to teachers' tendencies to emphasize lower rather than higher thinking skills in Ghanaian schools (Butakor et al., 2017). Ghanaian students are not encouraged to challenge teacher's ideas or methods for problem solving, and teachers encourage them to obtain answers more than pose questions (Dei, 2011). Because mathematical modelling requires students to make choices, create systems, pose questions, and challenge ideas in iterative cycles (CMA & SIAM, 2016; English & Watters, 2004), modelling may be difficult to implement in the current pedagogical landscape. Many researchers call for a change in teacher education, to emphasize student-centred learning, to provide students opportunities to explain their mathematical ideas, and to teach for understanding (Akyeampong et al., 2013; Frempong, 2010; Nabie et al., 2018).

Mathematics pedagogy in Ghana should also improve the degree to which they are able to connect mathematics to the real world. Mathematics in Ghanaian schools is not typically contextualized using students' life or culture (Dei, 2011), and when materials are borrowed from other countries, the contexts of the mathematics are culturally incongruent with Ghanaian learners (Cole, 2012). Fredua-Kwarteng and Ahia (2015) recommend that mathematics be connected to Ghanaian students' lived experiences and communities. They specifically suggest that students bring cultural artifacts, tools, and games to incorporate into learning. Sitabkhan and Ampadu (2021) conducted a study that revealed Ghana's teachers were more likely to change their practices when they were provided sets of manipulatives to use in their classrooms. Because modelling activities are often paired with manipulatives (Lesh et al., 2000; Moore et al., 2015), introducing mathematical modelling could facilitate much-needed pedagogical reform. Many mathematics classes are taught in English and sometimes translated to Ghanaian languages, which makes it more difficult to do applications of math problems (Dei, 2011). Fredua-Kwarteng and Ahia (2015) recommend the development of mathematics registers for the major Ghanaian languages and that the mathematics at basic school should be conducted in students' primary languages.

In Ghana, connections between different types of mathematics are not emphasized (Dei, 2011). More mathematical modelling would provide students with more opportunities to notice the interconnectedness of mathematics. Currently, 70% of Ghana's senior high school (SHS) content emphasizes algebra to align with the West Africa Examination Council's common exams, without any dedicated preparation for mathematical modelling (WAEC, 2009). Thus, learners often graduate secondary school and enter the tertiary level to study mathematics as PSMTs without any prior knowledge in mathematical modelling and find it difficult to organize and reorganize complex mathematical problem and apply appropriate mathematical tools. Dogbey and Dogbey (2018) monitored WAEC's core mathematics assessment, given in Ghana, over a 20-year period, and it was found to be mostly routine and abstract. Particularly, 80% of the items assessed students' ability to either recall basic facts or perform routine procedures. While 70% of the items were framed in abstract contexts, 15% were framed in semi-realistic contexts, and 15% in realistic contexts. Also, the content of the exam had not changed much over the 20-year period. Efforts have been made toward integration of technology into Ghanaian mathematics classrooms, but lack of professional development and technological knowledge among teachers has slowed the implementation of technology in classes (Agyei & Voogt, 2011; El Yacoubi, 2013). Oduro (2015) found, however, that teachers' practices in Ghana are influenced by both professional development and by institutional policies, both of which will be useful for broadening the use of mathematical modelling.

MATHEMATICAL MODELLING AT PRE-TERTIARY LEVEL

Modelling fits nicely in primary education because young children are generally open, inquisitive, and creative (CMA & SIAM, 2016). Children may be more likely to develop a productive disposition toward mathematical modelling if they have been exposed to open-ended tasks because they learn that problems can be solved innovatively and in varying ways (CMA & SIAM, 2016). Because children build their own knowledge based on their experiences (Piaget, 1971), their exposure to mathematical modelling encourages innovation, perseverance in problem solving, and creativity. Placing modelling in the forefront of pre-tertiary mathematics curriculum will promote critical thinking and problem solving (Arseven, 2015), which then, in turn, positively affects advancement in technology and engineering in society (Cheng, 2001).

Many countries in Sub-Sahara have implemented educational reforms (Mudaly & Dowlath, 2016); however, thus far, these reforms have largely missed the opportunity to infuse mathematical modelling into classes. Perhaps African curricula has not seen the needed turn toward modelling because of a deficit in teacher preparation. If Africa is to be industrialized, then teacher preparation will lead the charge. Mathematics is the catalyst for engineering and for other sciences. Teachers can teach new concepts, such as mathematical modelling, provided they have been prepared adequately at the tertiary level (Bruner, 1986; Vygotsky,1987). In the 21st century, technological knowledge is crucial in the quest for solving complex problems inside and outside the formal classroom setting, and mathematical modelling can be integrated with technology to foster quality teaching and learning.

Prior research on the pre-tertiary teaching and learning of mathematical modelling has been conducted in different nations, such as in Germany (Blum & Ferri, 2009), in China (Fu & Xie, 2013), in Denmark (Jankvist & Niss, 2019), and in South Africa (Mudaly & Dowlath, 2016). In all these studies, the mode of instruction greatly influenced students' success in problem solving. Also, in all these studies, it was noted that teaching mathematical modelling is not easy. Students face challenges with modelling, as with other mathematical approaches. However, the ever-growing research base provides ideas and recommendations for how to teach modelling effectively, overcoming challenges. We argue that the beneficial affordances that accompany mathematical modelling outweigh the constraints that may need to be overcome. When considering a nation like Ghana, a developing country, with limited resources, with poor national academic performance, with a newly established institutionalized education system, and with a national teacher shortage, one might wonder if Ghana should embrace ambitious practices in science, technology, engineering and mathematics (STEM) education, like the institutionalization of mathematical modelling, or whether it would be safer to settle for lower hanging fruit. As Galbraith (2007) argues, the lack of advantages only adds urgency, not complacency, to the call for broad implementation of mathematical modelling in Ghana. The desired destination is evident. The journey ahead will be difficult. This study serves to provide a snapshot of the present, a starting place for progress.

METHODS

Design

This study examined PSMTs at tertiary institutions in Ghana. Mixed method design was adopted for this study, particularly an explanatory sequential mixed method (Creswell, 2011). This design involves the collection of quantitative data via a written test and is followed by interviews for collecting qualitative data. The quantitative and qualitative data were analyzed to obtain perceptions respondents held about mathematical modelling.

Participants

The target population for the study consisted of third year undergraduate PSMTs in public universities in Ghana. Three public universities in Ghana were selected for this study because they are offer mathematics education programmes. At the time of this study, those three universities had a total of 120 PSMTs who had already completed studies in ordinary differential equations, calculus, geometry, and statistics. Of those 120 PSMTs, 30 were chosen for this work based on their voluntary consent to participate. Permission was obtained from the participating universities to administer a three-question written test to respondents after the universities' first term of the academic year had concluded. The researcher, who was not concurrently an instructor for any participants, administered all the tests. The respondents took approximately one hour to complete the written test. Respondents' tests were marked according to the researcher's rubrics. For each question, points were awarded for correctness and completeness of solutions, with a maximum of 10 points signifying a correct and complete response. Based on these scores, four participants' tests were chosen for further analysis because of lower marks. To investigate the nature of misconceptions, errors, and deficits in PSMTs' modelling, the researcher chose four respondents who failed to demonstrate significant evidence of procedural knowledge or conceptual understanding for any of the three questions. This prompted the researcher to explore the marked script of these four respondents. The researcher denoted the respondents using the letters A, B, C, and D, and their marked scripts were subsequently analyzed in this study.

Instrument

A three-question written instrument was comprised of two non-routine questions and one routine question. Its content included series, sequences, chemistry applications, and operation research. These application problems do not represent mathematical modelling projects in their entirety, as those found in the GAIMME report; however, these three tasks measure how the PSMTs can make connections to

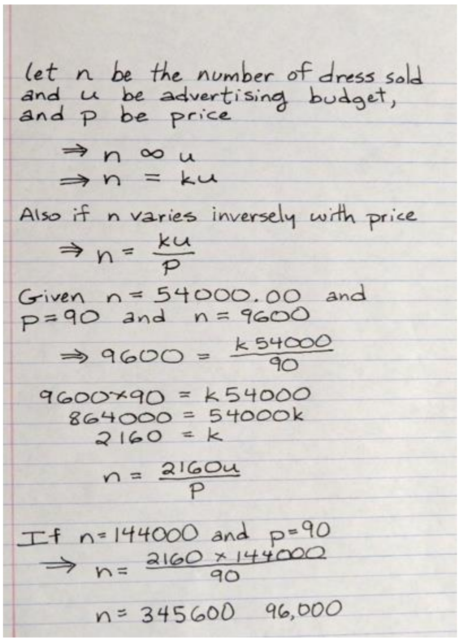

Figure 1. A sample answer to Q1 with partial correctness

applications, a necessary precondition for modelling. Application-laden word problems, which should not be confused with modelling itself, as Galbraith (2007) warns, can be used to enhance research about modelling capabilities in studies such as these, where the subjects have no prior experiences with modelling. The three questions were carefully reviewed to meet the standard of the respondents as well as the objective of the work. A marking scheme was constructed to help reliably and consistently measure the performances of the respondents. Ten points were allocated for each question for analysis. Specifically, for four selected participants, whose lower marked responses were purposively chosen, the researcher analyzed the particular questions where the respondents did not do well.

Objectives

This study attempted to attend to the following questions:

1. To what extent can Ghanaian pre-service mathematics teachers solve modelling-like real-world problems?
2. What are some types of errors, misconceptions, and/or deficits that exist in Ghanaian pre-service mathematics teachers' understandings related to mathematical modelling?
3. What are some of Ghanaian pre-service mathematics teachers' perceptions or ideas about the subject of mathematics modelling?

RESULTS

The first question (Q1) requires understanding of variation, which is a concept in Ghana's senior high school core mathematics syllabus. Q1 was, as follows:

> Designer Dolls found that the number of its Dress up Doll sold varies directly with their advertising budget and inversely with the price of each doll. When GHs 54,000.00 is spent on advertising and the price of each doll is GHs 90.00, then 9,600 units are sold. Determine the number of dolls sold if the amount of advertising budget is increased to GHs 144,000.00.

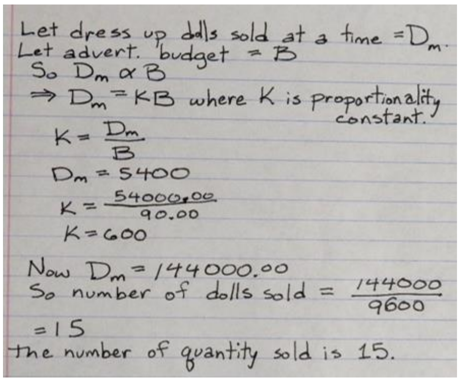

Figure 2. A substantially incorrect sample answer to Q1

Of the 30 respondents, 13 (45%) of the respondent did not make any attempt to answer the question at all. Of the 17 (55%) of the respondents who attempted Q1, six of those responses (36%) were marked with 10 points because they were completely correct, while the other 11 of them (64%) responded with only partial correctness. The respondents who tried Q1 utilised algebraic methods, without demonstrated any novelty in solution strategies. **Figure 1** is a sample where the respondent tried Q1 but made an error that was typical among those who responded at a varying degree of correctness. In **Figure 1**, the PSMT correctly defined variables, interpreted the situation into correct equations, but then incorrectly solved the equation 864,000=54,000*k*.

As Ostler (2003) asserted, PSMTs should be adequately prepared in mathematical content to meet the expectations as a teacher in the classroom, and the error shown in **Figure 1** demonstrates how understanding of modelling procedures must still be paired with attention to precision and fidelity to algebraic rules. Because the WAEC mathematics syllabus does not have significant modelling content (WAEC, 2009), it was not surprising that respondents' success rates on Q1 were low. We believe that this deficit, exhibited by the proportion of PSMTs who did not attempt this problem, indicates that the pre-tertiary level of mathematics should emphasize learners doing, rather than the teacher being in the center of teaching and learning.

Figure 2 shows another solution to Q1 submitted by a third-year undergraduate PSMT who made more substantial errors. The researcher noticed numerous errors, misconceptions, and omissions in this work sample. First, this PSMT did not attempt to model the inversely proportional relationship between advertising budget and price. Instead, the respondent only represented the directly proportional relationship between advertising budget and quantity sold. It is possible that the respondent did not understand how to represent an inversely proportional relationship or did not deem it relevant. The work sample also includes an error in transcription, where 5,400 was written instead of 54,000. Also, the PSMT mistook price for advertising budget, and then later mistook the advertising budget for the quantity sold. Finally, the PSMT failed to check the reasonableness of their solution with the real-world context. In the given problem, 9,600 units were sold before increasing the advertising budget. After more than doubling the advertising budget, it would not make practical sense for the sales to drop to only 15 units. In this example, we see that there is some confusion of quantities with terms, such as confusing quantity and budget; however, there is also evidence of a lack of broader understanding of real-world context, allowing erroneous solutions to be egregiously unreasonable.

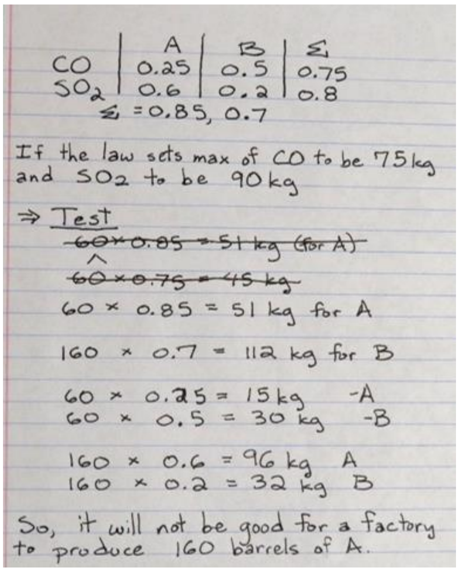

Figure 3. A substantially incorrect answer to Q2

Although these respondents had taken many mathematics courses, their mathematical modelling skills were impoverished. Perhaps they had received lecture-based, teacher-centred instruction that had not previously allowed these PSMTs the opportunities to create their own models. In modelling courses, lecturing should be limited to only a portion of the course modality, and rather, the learners should be encouraged to learn mathematics by doing. According to Haines and Crouch (2001), the PSMT teacher should see a curriculum that embraces conceptualization, development, and modelling competencies. Social and cognitive constructivist theories should be the focus of educator preparation programs, preparing PSMTs so that they can link the basic mathematics concepts to solve modelling problems. The social aspect of learning fosters integration of transfer of learning from mathematics to other closely related disciplines.

The second problem (Q2) given to students was based on chemistry; however, the mathematical concept required to solve is linear programming. Q2 was, as follows:

> A factory produces two agricultural pesticides, A and B. For every barrel of A, the factory emits 0.25 kg of carbon monoxide (CO) and 0.60 kg of sulfur dioxide (SO_2), and for every barrel of B, it emits 0.50 kg of CO and 0.20 kg of SO_2. Pollution laws restrict the factory's output of CO to a maximum of 75 kg and SO_2 to a maximum of 90 kg per day. Would it be legal for the factory to produce 60 barrels of A and 160 barrels of B per day?

Q2 measures the ability to apply mathematics to other physical sciences. Only 10 (35%) of the respondents attempted Q2. None of these respondents got the solution completely correct. **Figure 3** shows an incorrect solution provided in this study.

The response in **Figure 3** shows evidence that this PSMT failed to understand the meaning of the quantities they were calculating. This PSMT summed pesticide A's kilogram-per-barrel rates for both carbon monoxide and sulfur dioxide to get 0.85 kg per barrel and then multiplied that sum by 60 to arrive at 51 kg of gas for pesticide A. However, this quantity represents how much in total emissions, a combination of carbon monoxide and sulfur dioxide, being produced by 60 barrels of pesticide A. If this PSMT had realized what this quantity represented, then the student would have chosen a different strategy to answer this question because the legal maximums were articulated for the two gasses separately instead of aggregately. The student made the same conceptual error and summed the rates for pesticide B as well. It is possible that this PSMT suspected the need to perform separate calculations for each gas because they then performed some separate calculations; however, they included superfluous calculations as well and did not include any combinations of amounts to address Q2, indicating the PSMT was uncertain how to choose which calculations corresponded to the relevant question. The PSMT concluded in **Figure 3** that "it will not be good for a factory to product 160 barrels of A;" however, because the original Q2 text only inquired about producing 60 barrels of pesticide A, there is evidence that the PSMT completely lost sight of Q2's objective.

Teaching methods at the pre-tertiary level in Ghana typically do not emphasise learners' constructing their own knowledge, and the secondary school curriculum has little bearing on mathematical modelling. Thus, because secondary school mathematics curriculum does not emphasize the types of problem-solving skills that support mathematical modelling skills, it was not surprising that only 35% of the PSMTs attempted Q2. Some respondents thought it was meant for chemistry students, rather than mathematics students. The fact that no respondents got the solution to Q2 correct is worrying because PSMTs will be expected to use this linear programming concept to teach at the senior high school level in Ghana after they complete their programme. This leads to the question of what content knowledge of linear programming should teachers possess to be able to guide students at the senior high school level. We believe tertiary institutions with mathematics teacher preparation programmes should provide at least two courses that are related to operations research.

Practical questions that are like Q2 should be presented to PSMTs more frequently. Blum (2002) found that mathematical modelling provides the required skills for solving real-life problems, which will support PSMTs in their abilities to guide secondary mathematics students in linear programming. Exposure to technological content knowledge also enhances abilities to solve more complex linear programming problems, and modelling is an appropriate avenue to accomplish this in teacher preparation. Teacher educators should continually link school mathematics to varying applications and different fields, throughout tertiary curriculum.

The third problem (Q3) should have been familiar to the respondents because it includes a popular topic taught often in senior high school. Q3 was, as follows:

> Mr. Mensah starts a job with an annual salary of ₵6,400.00, which increases by ₵240.00 every year. After working for eight years, Mr. Mensah is promoted to a new post with an annual salary of ₵9,500.00, which increases by ₵360.00 every year. Calculate Mr. Mensah's salary in the fifteenth year of service.

All respondents attempted Q3, and 25 of the 30 respondents (85%) got Q3 completely correct. Only 5 (15%) of the respondents got some aspect of the solution wrong. Although this sequence-related task was somewhat familiar to PSMTs in general, the researcher chose incorrect samples to analyze in order to uncover the nature of the constraints and misconceptions that might correlate with this type of modelling task. **Figure 4** depicts an incorrect result one of the respondents obtained.

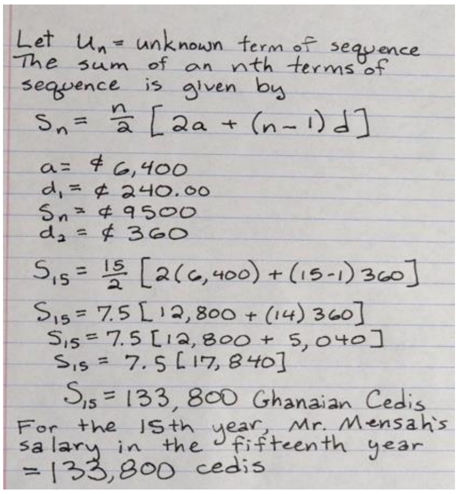

Figure 4. A substantially incorrect answer to Q3

This PSMT made two errors, shown in **Figure 4**. First, the respondent chose a formula for the sum of sequence terms. Although this PSMT correctly labels this formula as appropriate for summations of sequences, Mr. Mensah's salary in the 15^{th} year would be represented by the 15^{th} term of a sequence, rather than the sum of the first 15 terms. Also, this PSMT used 6,400 Ghanaian Cedis as the initial term and 15 as the number of terms; however, in this scenario, the subsequence that begins with 6,400 has only seven terms and spans the years after Mr. Mensah's promotion. This demonstrates that PSMTs may have formulas memorized and may even be able to state what those formulas produce in mathematical language, such as "sum of terms," but PSMTs may still struggle to apply the correct formulas to real-world scenarios and may mis-identify the quantities that are signified by variables in those equations.

Calculating sequences is not a difficult topic in the pre-tertiary mathematics curriculum, and therefore, teachers are generally confident in teaching this topic in high schools. However, modelling tasks like Q3 still demand a connection of mathematics to real life. The formation of a correct mathematical model by a PSMT influences their application of mathematics to real-life problems. Realistically, many mathematical applications surround Ghanaian learners in real-world arenas, such as in market and in church. Such cultural knowledge provides students with clarity when being taught about sequences in class. According to Bruner (1986), learners learn better when what is being taught is directly related with the immediate learning environment. Teachers can build learner confidence by exploiting what they already know so that the learner can explore new problems.

Students' Perceptions of Mathematical Modelling

To obtain information on students' perceptions of mathematical modelling, four of the respondents (A, B, C, and D) were purposely selected for guided interviews in order to learn about students' mathematical knowledge in modelling.

Initially, the researcher asked respondents, "*have you heard about mathematical modelling before?*" Three of the respondents (A, B, and D) said they had heard about mathematical modelling, and one of respondents (C), said they had no idea about mathematical modelling. This was followed by a second question, *"If yes, where did you hear it from?"* The three respondents who had heard of it reported that they had heard of mathematics modelling from different sources including the internet, television, and radio.

The next question asked, *"How do you define mathematical modelling?"* The four interviewees responded as shown in **Table 1**. Participants A, B, and C used the word model to define modelling, signifying a surface-level understanding of mathematical modelling. Participant C described modelling as a connection to real-life problems, and participant D described modelling as a tool used in research. The definitions presented by the respondents during the interview failed to capture all that constitute mathematical modelling possesses. Although modelling is the certainly a connection of mathematics to real life, not every real-life mathematical situation constitutes or calls for mathematical modelling. For instance, "*Kwame was given GHs 600.00 by his father; his father asks him to give one-third of the money to her sister Ama.*" This problem is a real-life problem, but it does not merit a mathematical modelling approach to provide the answer. Mudaly and Dowlath (2016) define mathematical modelling as a process that should include conjecturing, modifying, and adapting mathematical theories to fit the real-world situation under consideration. The modified theories can be tested to see whether they supply the required information or solution.

Lastly, the researcher asked the interviewees to consider the items Q1, Q2, and Q3 tasks. All four interviewees described the problems as word problems under the specified content area of mathematics. When asked whether Q1, Q2, and Q3 tasks constituted mathematical modelling, their abilities to judge were limited because some had not attempted some of the written problems. Only respondent C perceived that Q1 and Q2 were mathematically modelling tasks. This deficit in the PSMTs' abilities to evaluate a type of mathematical task reveals that their prior exposure to mathematical modelling problems was thin. This indicates that their low scores were not a function of lower abilities or lower intelligence, but rather a lack of exposure to modelling in their studies. Comparing and contrasting the interviewees' responses in **Table 2** shows a more complete profile of these PSMTs' perceptions of modelling. Interestingly, the only interviewee who recognized that modelling has a connection with real-life problems was also the only one who claimed to have not previously heard about modelling and was also the only one confident enough to attempt a judgment about the written tasks. It is possible that this participant had heard about modelling from an academic setting, instead of from television or radio as the others had.

DISCUSSION

The pre-tertiary level of mathematics curriculum does not have modelling as a content in Ghana's educational system. Linking mathematics with real life situation is mostly not accounted for in the school system. De Villiers (2003) asserts that a learner's ability to solve mathematical problems does not guarantee ability to solve word problems. This was demonstrated in this study; selected tertiary PSMTs in Ghana, where basic application of variation which is taught at the senior high school, could not apply their mathematical ability to Q1. It is interesting to note that 13 (45%) of the selected respondent did not make any attempt to solve the Q1. Deficits in problem solving skills only hinder mathematical modelling skills.

At the senior secondary school level, teachers should let learners construct their own knowledge and avoid teaching learners only for examination purpose. In South Africa, the pre-tertiary mathematics curriculum, according to Mudaly (2004), needs modelling-based pedagogical reform. He believes that modelling should be the first stage

Table 1. Respondents' definitions of mathematical modelling

Respondent	Response to "how do you define mathematical modelling?"
A	*"Mathematical modelling, a kind of mathematics where the problem is solved using models."*
B	*"Mathematical modelling is a situation where mathematicians model with mathematics."*
C	*"It is the identification of mathematical models in real-life problems."*
D	*"A course that is studied in some universities for research purposes."*

Table 2. Interview responses comparison

Respondent	Description of modelling	Heard of modelling before?	Classification of Q1, Q2, and Q3
A	A kind of mathematics	Yes	Unable to judge
B	Something mathematicians do	Yes	Unable to judge
C	Making models from real-life problems	No	Q1 and Q2 are modelling
D	A course for research	Yes	Unable to judge

of introducing students to abstract mathematics. The connectivity of mathematics with real application in everyday life is sparse in most developing countries in Africa. In one provided work sample for Q1, the respondent committed a procedural error by applying direct proportion instead of indirect proportion in solving the problem. This respondent possibly could not analyze the mathematics because they could not understand the question. Concept map as a technique of teaching new concepts should be encouraged at the pre-tertiary level of the Ghanaian mathematics curriculum since it helps develop strategic competence, improves learners' problem-solving skills, and eventually leads to higher mathematics learning outcomes.

In this study, PSMTs' skills in linear programming were measured by Q2, a question which applied mathematics to chemistry. Linear programming is a topic taught in the Ghanaian Senior High School Elective Mathematics Curriculum. The respondents had also taken algebra courses that included linear programme as a topic. However, only 10 (35%) made an attempt to answer Q2. This result does not bode well for Ghana's quest for improving mathematics learning outcomes. This observation is consistent with the finding, by Fredua-Kwarteng and Ahia (2015), that Ghanaian mathematics does not connect with learners' immediate environment and culture. The respondent found it difficult to connect with the known concept in different environments and had little ability to transfer the concept of linear programming to real applications. Malambo (2021) believes that learners' (PSMTs) understanding of mathematical concepts influence the way they will teach in the future. Most of the mathematics concepts taught at this level are based on deductive rather inductive approach because of the nature of the external examination in Ghana. This indicates that improving mathematics learning outcomes at the pre-tertiary level will be a tall task if the content knowledge of the PSMTs are not improved substantially. The entire pre-tertiary mathematics curriculum must be revised to focus on analytic and synthetic approaches which lead to problem solving skills.

Only 15% of PSMTs in this study made errors on Q3, which hinged on series and sequences. The salient error was a failure to understand the model; the 15th term was misinterpreted as the sum of the 15th term of the sequence. This topic is one of the fundamental pre-tertiary mathematics curriculum that is taught in Ghana's core and elective mathematics. Nonetheless, some deficits are not surprising because Ghanaian teachers generally do not emphasise mathematics language and communication in problem solving skills. This has a negative effective on learners' ability to understand key mathematical terminologies, such as the word sum. The findings of this study buttress Akyeampong et al.'s (2013) view that most Ghanaian mathematics teachers use deductive approaches in classrooms and that students therefore find it difficult to construct their own knowledge, rather than focusing on drill learning to memorization of mathematical facts. Through series of workshops and seminars, teachers should be provided a wide range of real-life applications of every mathematics topic taught in class. The school inspectorate division should visit the schools frequently to ensure that learner-centred approaches are being employed in the classroom at all times. The nature of school assessment should be revised to include problem solving as a compulsory component to sharpen the critical skills of the learners. Majority of the teachers at the pre-tertiary level use teacher-centered and deductive methods in teaching mathematical concepts, which exacerbates problems in poor mathematical modelling skills.

The few selected respondents interviewed indicated poor knowledge about the concept of mathematical modelling. They found it difficult to appreciate the existence of the concept of mathematical modelling in the Ghanaian mathematics curriculum. Pehkonen et al. (2013) suggest that teaching methods should be based on constructivist learning theory and that this approach is likely to improve student modelling skills. The new mathematics curriculum for the junior high school in Ghana seeks to utilize enquiry-based learning, and this could eventually improve the teaching and learning of mathematics. Currently, the entire pre-tertiary mathematics curriculum contains no content on mathematical modelling as a concept. Mathematical modelling can be framed from some of the topics taught the junior high schools; however, the issues of the implemented curriculum stem from the teaching methods and the nature of the final examination leading to certification of the learners.

CONCLUSION

Currently, Ghana's PSMTs have limited exposure to mathematical modelling in tertiary preparation. In the current educational landscape of Ghana, with the newness of free compulsory education and with a national teacher shortage, we do not seek to add more coursework requirements to a PSMT's tertiary journey; however, we believe that there is a dire need to incorporate mathematical modelling, as both a topic and a teaching mode, into existing coursework. We also believe that inclusion of mathematical modelling in pre-tertiary curricula would serve to (a) better equip students for meaningful mathematical modelling in tertiary school in both knowledge and disposition, (b) stimulate economic and industrial progress, and (c) orient future Ghanaian citizens to creative solution-seeking for societal problems.

Although traditional teaching approaches, such as lecture, are still broadly used in Ghana's mathematics classrooms, such methods do not encourage creativity and problem-solving skills to the extent that mathematical modelling would. Although the need for reform toward more learner-centred pedagogies may be a foregone conclusion, change is difficult. There may be a broken road of brave trials and errors that will bring Ghana from Point A, a state of teacher-centred lecture and rote memorization, to Point B, a state of learner-centred inquiry and mathematical modelling. This study provides a profile of Ghanaian PSMTs' existing perceptions about mathematical modelling and capabilities with modelling-type tasks. Developing PSMTs' modelling-based content knowledge and pedagogical content knowledge will move Ghana forward. Thus, strengthening modelling-based teacher preparation is a generative move toward long-term transformation. Mathematics educators should take ownership as stakeholders and identify the missing links needed to connect the ideal reformed mathematical classroom to the immediate needs in Ghana. The results of this study, which revealed specific shortcomings in PSMTs' understandings of applied mathematics problems, recommend that basic modelling should be a part of the secondary school mathematics curriculum. In-service mathematics teachers would, in turn, need to be provided a series of professional development workshops to update their understandings of mathematical modelling principles.

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