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Quadratic Functions and PhET: An Investigation from the Perspective of the Theory of Figural Concepts

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ABSTRACT

This work aims to present the results of an investigation in the teaching of quadratic function with the help of the PhET Colorado simulator, analyzed from the perspective of the theory of figural concepts in the context of hybrid teaching, using the teaching methodology flipped classroom. The research methodology used was the case study, which was developed with a group of 45 high school students from a Brazilian public school. The proposed activity was developed from the simulation called "graphing quadratics", available in the PhET, and was developed in two stages, one in a virtual way and the other in person. The results show us the need to explore the study of the quadratic function using technology from a more dynamic perspective. We reinforce the importance of the manipulation performed in the simulator to understand the relationship between the coefficients a, b and c of the function and the behavior of its graph, being a potential resource in the learning of this subject by the students.

Keywords: 2nd degree function, mathematics teaching, blended learning, flipped classroom, theory of figural concepts

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INTRODUCTION

With the new Coronavirus pandemic, several sectors, including education, have undergone profound changes in their work format due to the social isolation imposed by the health crisis around the world. Remote teaching on an emergency basis was implemented in schools through the use of technologies as a way to minimize the damage caused by the suspension of face-to-face classes, continuing the school year and school activities (Arruda, 2020).

However, currently, many Brazilian schools are in a moment of transition, migrating from the model of remote classes to hybrid learning, in which part of the study routine takes place at school and the other part in an environment different from the classroom. According to Bacich et al. (2015, p. 13) "blended learning is a pedagogical approach that combines classroom activities and activities carried out through digital information and communication technologies". Thus, in this teaching model, the aim is to develop a student with a more autonomous, active, and responsible profile for the construction of their knowledge.

Faced with a context of so many changes, student learning in mathematics, which was already facing difficulties even in the face-to-face teaching modality, has been compromised. As data from PISA for schools – *Programme for International Student Assessment*–show, mathematics is considered the biggest setback in basic education learning (National Institute of Educational Studies and Research Anísio Teixeira, INEP-acronym in Portuguese, 2019) and in this pandemic context, the difficulties were accentuated.

That said, in the current situation, technology, in addition to helping in the progress of activities during this period of pandemic, showed teachers several possibilities beyond traditional teaching, being a way to rescue the desire of these students to learn mathematics, from methods and methodologies that facilitate their understanding (Maciel, 2018; Oliveira & Pereira, 2021).

One of the topics that students face difficulties in assimilating in basic education is the study of functions, especially the quadratic function (Calil et al., 2010; Vieira et al., 2021), both in the interpretation of problems and in the relationship between the elements of the function and the images generated by its graphic construction.

According to studies by authors such as Brito et al. (2019), Celestino and Pacheco (2010), Nobre (2006), and Prado (2014), some of the most recurrent difficulties that students have regarding quadratic functions are the mathematical writing of a function from a problem, the construction of graphs and sketches, through algebraic functions, position of points and orientation of the axes of the Cartesian plane and orientation of the concavity of the parabola.

To better understand some of these difficulties, we analyze the results of the research presented in this article from the perspective of the theory of figural concepts, conceived by Efraim Fischbein, a Romanian psychologist and student of learning in the field of mathematics education. According to Fischbein (1993), the integration between conceptual and figural properties in mental structures, with the predominance of conceptual restrictions over figural ones, is not a natural process of thinking and cognitive development of the student in mathematics, and this fact should receive constant attention from the teacher.

We bring as a resource the PhET Colorado simulator, making use of a simulation called "graphing quadratics", to understand how students relate the concept of quadratic function, its elements, and the manipulation of its graph, through a dynamic activity. PhET is a platform developed by a team from the University of Colorado (2020) and brings a series of simulations of different phenomena, directed to the area of science and mathematics, in a fun and interactive way, in order to relate the contents of the classroom with real situations.

The objective of this work is to present the results of an investigation in the teaching of quadratic function with the help of PhET, analyzed from the perspective of the theory of figural concepts in the context of blended learning, using the teaching methodology flipped classroom.

The research methodology adopted for the development of this work was of a qualitative nature, of the case study type, because according to Gil (2002), the results resulting from a case study are presented as hypotheses and not as closed conclusions. For the treatment and analysis of the data, we used the content analysis methodology, proposed by Bardin (1977), to analyze the written productions and responses of the participating students. In this way, we observed the receptivity of the students in relation to the proposed activity, as well as their conjectures and manifestations of learning about the subject with the use of PhET.

THEORETICAL FOUNDATION

Learning the topic of quadratic function requires algebraic and graphic knowledge, as well as an ability to interpret its use in problems, which is still a challenge for basic education students, as shown by studies (Bohrer & Tinti, 2021; Feltes & Puhl, 2016; Silva et al., 2021; Vieira et al., 2021).

According to Vieira et al. (2021), the work with quadratic functions in the classroom still occurs, in most cases, with the use of textbooks and through traditional methodologies, in which little is suggested, in the context of activities, the use of technology as a way of facilitating the understanding of this subject.

Ribeiro and Cury (2015) bring in the work algebra for teacher education, some ideas from researchers who are concerned with detecting students' difficulties, more specifically in the contents of equations and functions, so that they can outline teaching strategies that will help them in learning of the concepts in question. In this way, these authors bring that

> Ball et al. (2008) consider it essential that the teacher recognize when students give a wrong answer or when the textbooks used have a wrong definition. The authors point to the importance of looking for patterns in students' mistakes, in order to know the best method to teach a certain topic. These authors also suggest, in their scheme on the domains of knowledge for teaching, what they call horizontal knowledge, which involves knowing what has already been taught at a given level of education and what will be taught later, in order to have the vision, for example, the importance of discussing with students those recurring mistakes that originated in previous years and that will become obstacles to learning future content (Ribeiro & Cury, 2015, p. 83).

Regarding the difficulties in algebraic procedures, Ribeiro and Cury (2015) as well as Mariotti and Cerulli (2001) report the incorrect use of the distributive property of multiplication in relation to addition. Specifically, in working with functions, these authors state that language-related errors are common. Mariotti and Cerulli (2001) mention that the influence of a "mental schema" is present, that is, a "mental image of an operative procedure that the student recorded without having fully understood". Regarding the second, Ribeiro and Cury (2015) point out that students can understand when it is said, for example, that "the image of 5 is 3", but they cannot understand the expression $f(5)=3$.

Bohrer and Tinti (2021, p. 217) add that the use of technology to teach the quadratic function allows a better visualization in relation to the graphic behavior of the functions, given the fact that "the different ways of seeing, analyzing and understanding the graphs of function, in software, cannot be shown so easily in books and on the blackboard".

Faced with the challenges encountered specially in teaching quadratic functions, we used the theory of figural concepts (Fischbein, 1993) as a basis to analyze and understand how students associate the parameters (coefficients) that make up the quadratic function to their graphic representations, as well as understand the figure of the parabola in the context of functions. According to Fischbein (1993), through the dual nature of concept and image, a mental image (figural concept) can be constructed primarily based on previously established and formalized concepts.

Fischbein (1993), in his theory, when dealing with the essential components of geometric objects–the concept and the image–, states that the relationship between them can conceive a significant way of learning in the field of geometry, and can be extended to the relationship between geometry and algebra, in which the passage from the experimentation stage to abstraction requires a balance between such components, which can be provided by the use of mathematical software and applications. Also, according to Fischbein (1993), the particularity of the meaning of a figural concept is that it includes images as an intrinsic property, and these images must be entirely controlled by their definitions.

Alves and Borges Neto (2011) also point out, from Fischbein's (1993) perspective, that images (figures) constitute a mental entity, elaborated from a geometric reasoning, in which a figure is different both from its formal definition and from its image mental and in turn is supported by a sensory perception of a particular representation provided, which requires the student, in this sense, to develop a visual perception.

With regard to the use of technology for teaching functions, the document that establishes the Brazilian curriculum matrices–the National Common Curricular Base (BNCC, abbreviation in Portuguese)–reinforces the importance of using different digital resources to promote the learning of this subject, bringing into their specific skills the importance of diversifying the records of mathematical representation (Ministry of Education of Brazil, 2018), such as in the same activity proposal using manual notes on paper and the help of a software, platform or digital application.

With this, we bring in this work an activity aimed at teaching the quadratic function with the use of technology through PhET, which is a platform that provides interactive simulations aimed at the teaching of science and mathematics. Reis and Rehfeldt (2019) point out some principles used by simulations available in PhET, to stimulate students in their involvement with science, which are

> encourage scientific research, provide interactivity, making visible what is invisible, show visual mental models, include multiple representations (e.g., motion object, graphics, numbers, etc.), establish connections with the real world, give users implicit guidance (e.g., through boundary controls) in productive exploration, and create a simulation that can be used flexibly in many educational situations (Reis & Rehfeldt, 2019, p. 198).

Thus, PhET is a resource with great potential to help the student in his learning about the quadratic function, in which he can explore the simulations in a visual and interactive way, relating his concepts to the graphic images generated from the manipulation of its parameters, being a potential facilitator of learning.

Given the pandemic scenario, we reiterate that the activity described in this work took place in the blended learning modality. According to Moran (2015), this modality seeks to think of education as a continuous learning process, using not only the classroom environment, but the numerous possibilities and environments outside it, combined with the use of technologies, as a way of encouraging the students' learning in a more flexible and dynamic way.

Regarding the inverted classroom or flipped classroom methodology, according to Gomes (2020), the student develops the role, being the main actor in the construction of his knowledge, since the content/material to be taught, before possession only of the teacher, becomes available to the student through different channels–online or offline platforms, printed material, social networks, etc.–can be explored inside or outside the classroom.

With the adoption of the flipped classroom methodology, what used to be a home activity is now performed in the classroom domain, bringing dynamism to the face-to-face environment, given the fact that more complex definitions and concepts can be explored by the teacher during the class, since it is assumed that the student comes to the classroom with previous knowledge about the subject in question.

METHODOLOGY

The methodology adopted in this work is of a qualitative nature, of the case study type, in which we observe the students' resourcefulness in the face of the proposed activity through the PhET, associated with the flipped classroom teaching model. Thus, according to Gil (2002), we bring a description of the situation from the context in which the investigation is being carried out. We also present the content analysis methodology, proposed by Bardin (1977), to analyze the written productions and responses of the students surveyed, in which we analyze the activities produced by the students and the difficulties presented by them.

Data analysis had a descriptive character, as this type of analysis aims to describe and understand events as they happen. In this way, we made a description of the moments of the classes. We seek to perform an interpretation of the data, which according to Gil (2002) basically consists of establishing a connection between the results obtained and others already known, whether the results of theories or studies carried out previously. In this case, we relate our interpretation of the data with the theory of figural concepts and learning difficulties on the theme of quadratic functions already reported in the research.

The research was carried out with a group of 45 students from the 1st year of high school (age group between 15 and 16 years old) from a public school of vocational education in the city of Sobral, Ceara, Brazil, in the second semester of 2021. The activity developed addresses the subject of quadratic functions, with the general objective of analyzing the graph of this type of function. As specific objectives, we have:

1. understand what happens with the graph of a quadratic function from the manipulation of parameters *a*, *b*, and *c* in the PhET Colorado simulator;
2. Identify the coordinates of the vertex of the quadratic function and the behavior of the vertex when the parameters *a*, *b*, and *c* of the function are manipulated; and
3. relate the 2nd degree function discriminant with its roots from a graphical perspective.

The inverted class was divided into two moments, one remotely and the other in person. It is worth mentioning that due to the COVID-19 pandemic and social distancing measures, among the 45 students that make up the class, all participated in the socialization moment, but only 21 of them were in person at school, while the others participated in the broadcast discussion via Google Meet, interacting through the platform's chat and using the microphone. However, all performed and sent the proposed activity.

In the virtual moment, the students accessed the proposed activity entitled "graphing quadratics [1]" in the PhET and manipulated the elements of the presented graph. After the manipulation, they were instructed to solve an activity directed in the Google Classroom platform, about the movement of the coefficients *a*, *b*, and *c* of the quadratic function, from their observations and understanding of the behavior of the graph. The questions proposed in the activity are listed in **Table 1**.

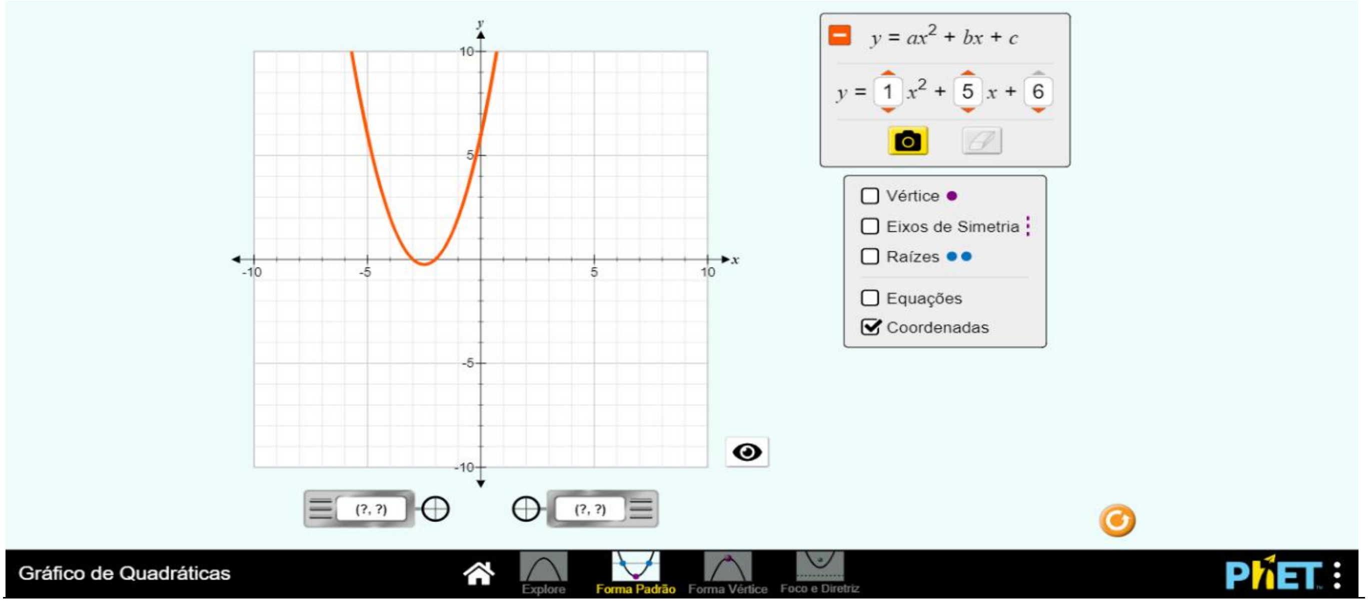
In the face-to-face moment, the teacher mediated and socialized the students' responses, presenting the simulator and verifying the veracity of the assertions presented. From there, each student was asked to mathematically schematize the behavior of a quadratic function based on the manipulation of its coefficients, using the activity proposed in PhET, the discussion based on Google Classroom responses and the textbook as support.

The research instruments used in data collection were a questionnaire designed to collect notes and answers from students and accurate observation of the two professors who conducted the research and collected the data. So, for data collection, records of student responses on the Google Classroom platform and photographs of the activity performed in the classroom were used. To preserve the identity of research participants, students will have their names represented by S1 (student 1), S2 (student 2), and so on.

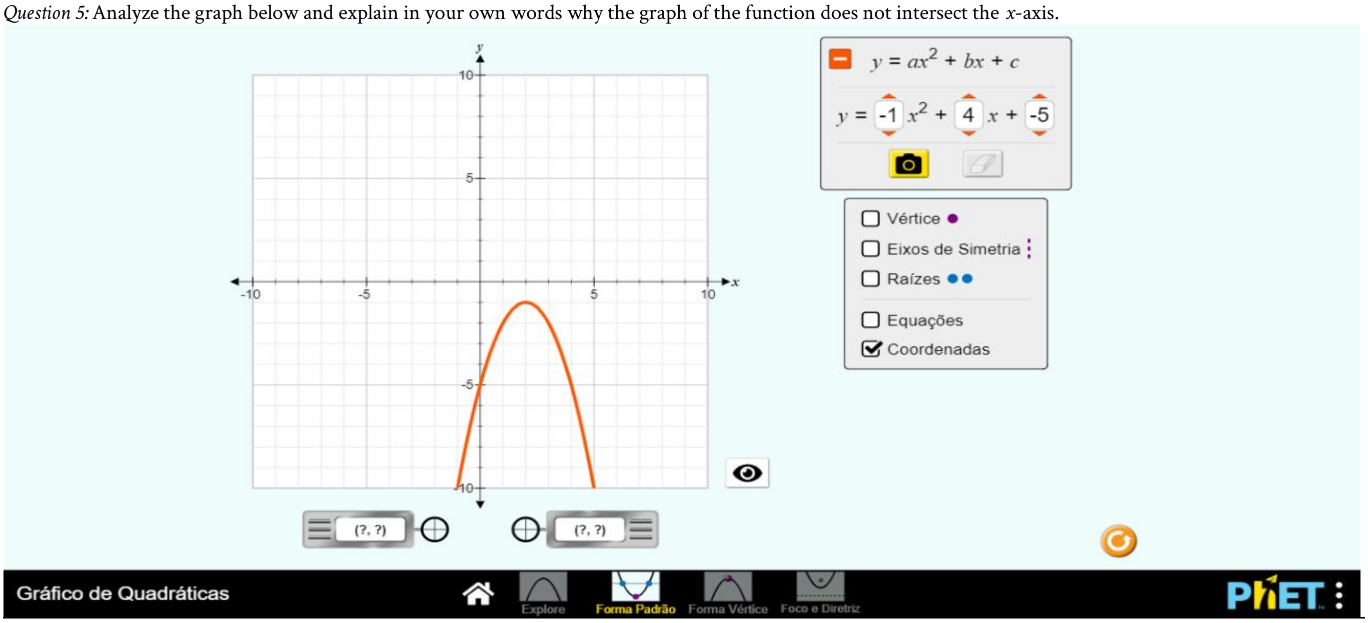
[1]The simulation is available on the electronic address: https://phet.colorado.edu/sims/html/graphing-quadratics/latest/graphing-quadratics_pt_BR.html

Table 1. Questions proposed in the activity

Questions
Question 1: From the law of formation of the quadratic function $f(x)=ax^2+bx+c$, from parameter movement a of the function, describe what happens to the graph when $a>0$, $a=0$, and $a<0$.
Question 2: Using the same logical idea, describe what happens to the graph when $b>0$, $b=0$, and $b<0$.
Question 3: Finally, analyze what happens to the graph when $c>0$, $c=0$, and $c<0$.
Question 4: Given the graph illustrated in the figure, identify their coefficients a, b, and c, their roots x' and x'', and the vertex coordinates $V(x_v, y_v)$ according to the situation indicated in the diagram below:



Question 5: Analyze the graph below and explain in your own words why the graph of the function does not intersect the x-axis.



The students' responses were observed, analyzed, and discussed by two mathematics teachers from the school where the research was carried out, so that the writing of this article was carried out. This research involves human beings, so we declare that all participants' identities were preserved, as well as reinforce that the ethical issues that correspond to the analysis and interpretation of the data collected in this research were met.

For data processing and analysis, a posteriori epistemological reasoning was used, in which a deduction was made from the collected data, being interpreted based on the theoretical reference pointed out in this research and on readings of results of research related to this topic. When we refer to the subjectivity and reflexivity of the analysis of the students' answers and the moment of the class, we are not only evaluating their successes or errors, but also the different ways in which they expose their knowledge, as a way of observing the students' learning difficulties.

The reliability of this study is based on the dialogue between the research of other authors related to the topic, as well as the experience of the professors who carried out the research. Recurring situations of learning difficulties related to the subject have already been reported, however we bring as a differential a search for the interpretation of these difficulties based on the theory of figural concepts, as there is a scarce amount of work that uses this theory together with the study of the quadratic function (Parameswaran, 2007; Sousa et al. 2022).

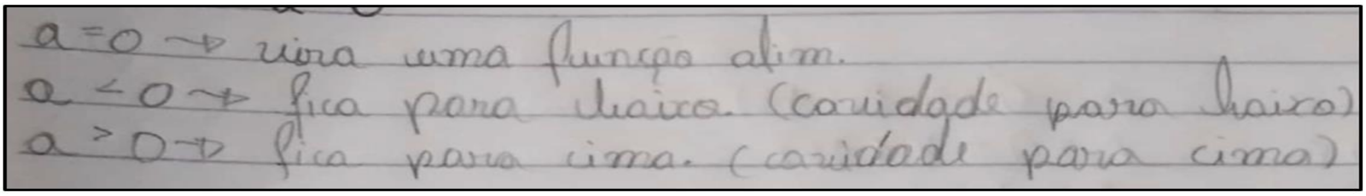


Figure 1. S3's response

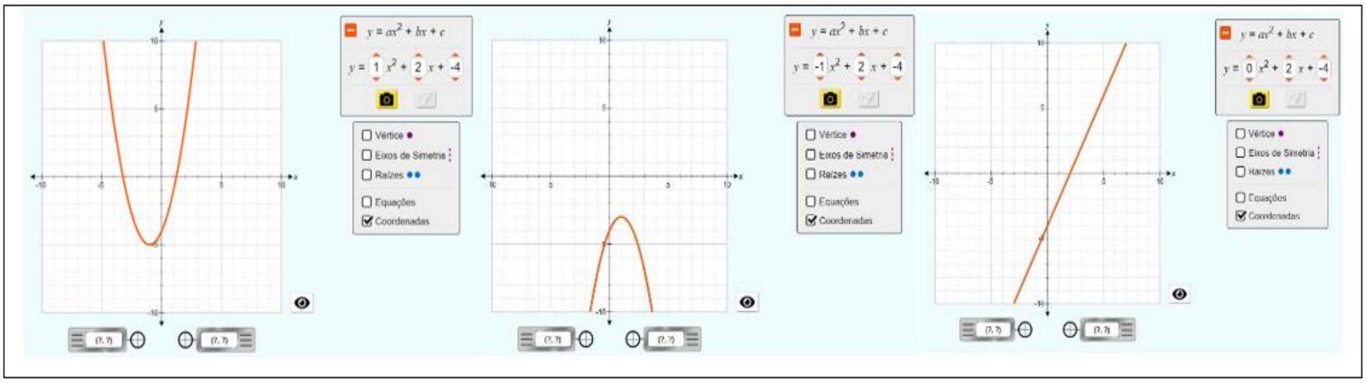


Figure 2. Movement of parameter a in PhET

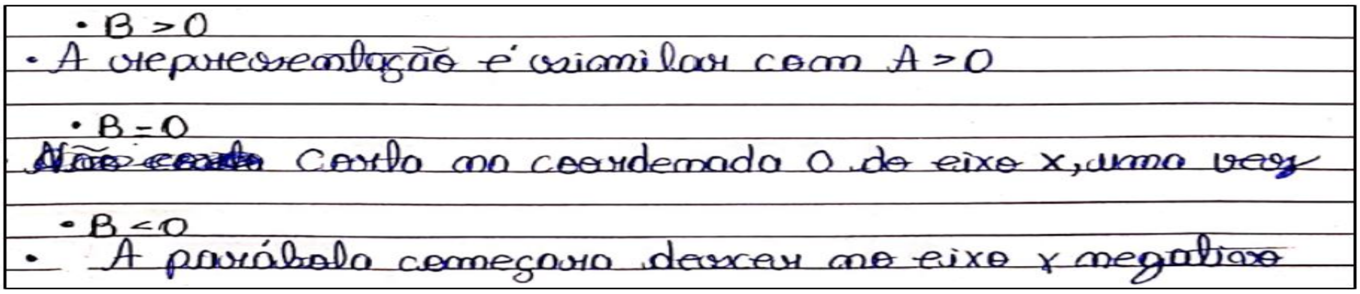


Figure 3. S5's response

RESULTS AND DATA ANALYSIS

In this section, we bring a discussion about the answers presented by the students to the proposed activity, supported by the theory of figural concepts. In addition, we point out some photographs as a record and reflections on the use of PhET in blended learning and the experience with the use of the flipped classroom methodology.

From the simulation provided and directed to this activity, we started the discussion by the first question, which asks the student to describe the behavior of the graph of the function from the movement of parameter *a*, as illustrated in **Figure 1**.

The notes presented by student S3 in **Figure 1** represent what was pointed out as a response by the majority of the class to the movement of parameter *a*, showing that the concavity (many students used the term "cavity") is turned upwards, if $a>0$ and facing downwards if $a<0$. It is also worth mentioning the fact that the students noticed that when $a=0$ the function is no longer quadratic.

In this question, all students were able to identify and make associations about the behavior of the function based on intrinsically conceptual properties, although the graph of the quadratic function within the PhET is not a mere concept, but a visual image (Fischbein, 1993). These representations in the simulator are exemplified in **Figure 2**.

Regarding the second question, about the behavior of the graph from the movement of parameter *b* of the function, there was greater difficulty in identifying what happens with the parabola, as illustrated in **Figure 3** and **Figure 4**.

Student 5 associates the behavior of the graph of the function from moving parameter *b* as "similar to what happens when moving parameter *a*". Student 6, on the other hand, states that the sign of parameter *b* has a direct influence on the quadrant where the graph is located. We can note in **Figure 3** and **Figure 4** that the students presented answers based only on the image viewed in PhET, but that the position/behavior of the graph when moving parameter *b* somehow depends on the values they left selected for parameters *a* and *c*. "There is certainly a conflict generated here by the fact that the two systems, the figural and the conceptual, have not yet blended into genuine figural concepts" (Fischbein, 1993, p. 148).

Smole and Diniz (2013, p. 115) provide a definition for a quadratic function as "a function *f*, from ℝ in ℝ, which associates every number *x* to the number ax^2+bx+c, with *a*, *b*, and *c* real and $a\neq 0$,", being a common definition in textbooks. The point is that, for an understanding, in fact, of the meaning of the coefficients *a*, *b*, and *c*, previous knowledge is necessary such as the relationship between domain and image of a function, numerical sets, notions of algebra, and arithmetic, in addition to demanding a level of abstraction for which the student is often unprepared.

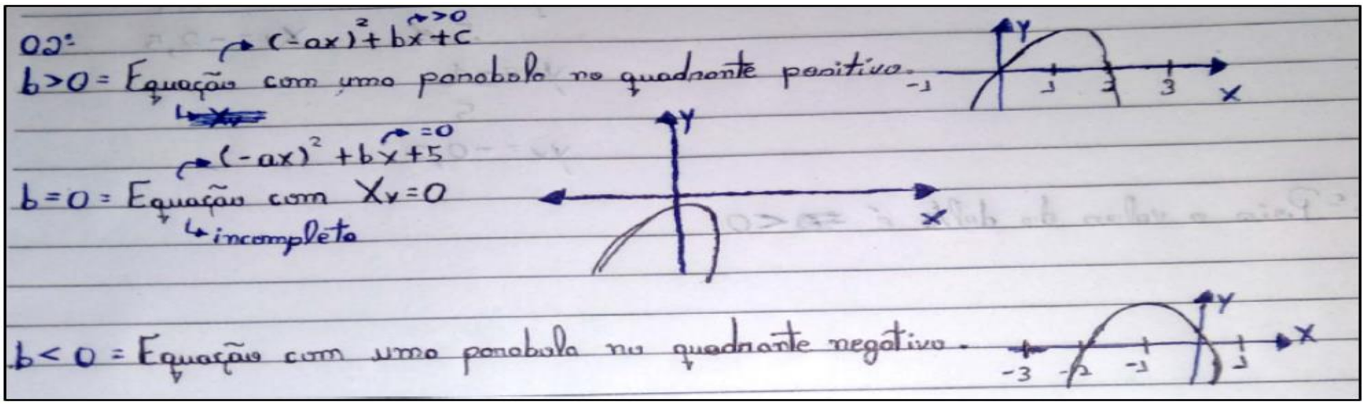


Figure 4. S6's response

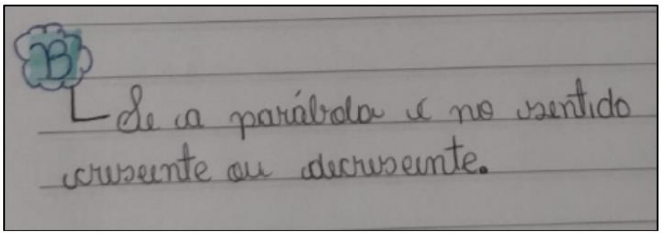


Figure 5. S10's response

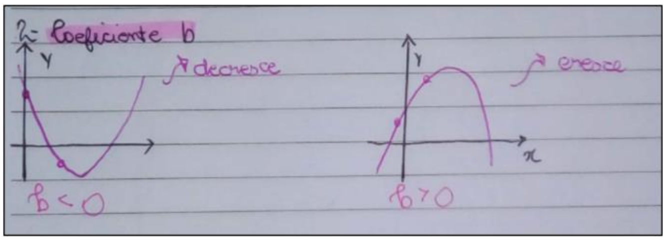


Figure 6. S12's response

Thus, the notes of students S5 and S6 stem from what Fischbein (1993) points out about mathematical reasoning: material objects–solids or drawings–are just materialized models of the mental entities with which the mathematician deals. In this way, the behavior of *b* in the graph, while *a* and *c* do not vary, shows the construction of a second parabola, concavity inversely to the original parabola. However, the fact that this behavior does not appear explicitly in the graph, requiring a greater abstract and sensorial refinement, may have caused the misunderstandings of students S5 and S6. The way these students express themselves shows a gap in understanding the meaning of this parameter. Student S10, on the other hand, presented a different answer from the others, as shown in **Figure 5**.

Student S1, about the movement of parameter *b*, only wrote "if the parabola is in the ascending or descending direction". S10's response shows us that (possibly) the student understood that when $b<0$, the direction of the parabola is decreasing, from the point of intersection of the parabola with the y-axis. Similarly, if $b>0$, the direction of the parabola is increasing from the point where the parabola crosses the y-axis. However, S10 could not explain this clearly, with adequate mathematical language. Furthermore, he did not present in his answer a formal argument about what happens when $b=0$. So, they have difficult in "to translate between representations and to interpret symbols related to functions" (Borke, 2021, p. 676). The graphic image and its conceptual relationship with parameter *b* are categories of distinct mental entities and the figural concept referring to this coefficient is apparently not clear for S10. The most reasonable logic for your answer is related to what Fischbein (1993, p. 144) says that

> the most plausible hypothesis seems to be that we are in fact dealing with a game in which active conceptual networks interact with imaginative sources. Furthermore, we have reason to admit that, in the course of this interaction, meanings shift from one category to another, images gain more generalized meaning, and concepts greatly enrich their connotations and their combinatory power.

See **Figure 6**, from another perspective: student S12, due to the difficulty of expressing himself mathematically, chose to sketch what happens with parameter *b* when moving it.

In **Figure 6**, student S12 was able to express visually what happens with the graph of the function if parameter *b* is positive or negative, however, he also did not make it clear what happens when $b=0$. This can tell us that, possibly, the students study and internalize this subject in a mechanized way, with little theoretical exploration and little understanding of the meaning of mathematical terms properly, which reinforces the importance of working from this perspective, seeking to give meaning to the functions. Regarding the third question, about the parameter *c* of the function, many of the answers were correct, as exemplified in **Figure 7**.

Student S20, as well as many others, concluded that the parameter *c* represents the point in the Cartesian plane where the function *f* cuts the y-axis, analyzing that if $c>0$, then the point of intersection of the graph of the function with the y-axis it will be above the origin, otherwise, if $c>0$, this point will be below the origin. If $c=0$, the point will pass through the origin of the coordinate system. Note that in **Figure 7**, student S20 felt the need to express himself through a graphic sketch and to write his visual perception as simply as possible from the exploration of the simulation, managing to adequately synthesize the behavior of the parameter *c*.

We perceive the beginning of a fusion between concept and figure, culminating in the understanding of the behavior of the graph, however, we must remember that "it is the conceptual organization that must completely dictate the figural properties and relationships" (Fischbein, 1993, p. 150). So, we draw attention to the difficulties of theoretical understanding of coefficients of the function and a possible disconnection between the formal definition and the graphic image.

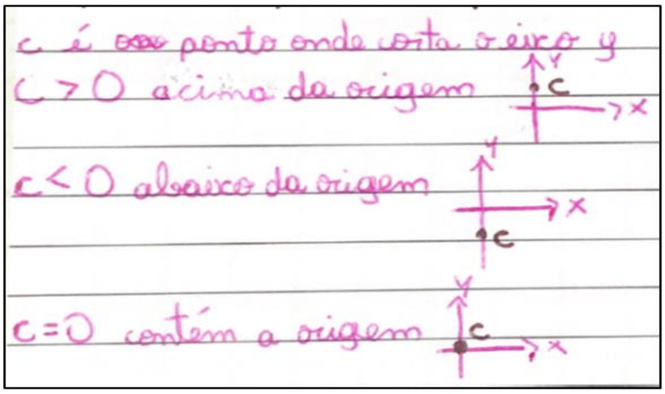


Figure 7. S20's response

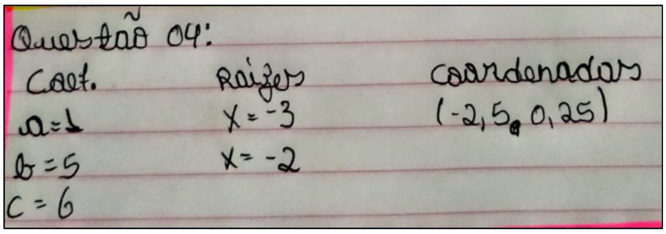


Figure 8. S34's response

In question 4, a graph was presented, where we asked the students to identify the coefficients *a*, *b*, and *c*, the roots x' and x'' and the coordinates of the vertex $V(x_v, y_v)$. In this question, some students used only the arrows of the simulator and the image presented in the question, manipulating the parameters in PhET to meet what was requested in the problem and visualizing this information in a practical way and taking notes, as shown in **Figure 8**.

However, other students did it in a demonstrative way, using formulas to prove the value of the coordinates of the vertex and roots, as shown in **Figure 9**. Note in **Figure 9** that the need to demonstrate the calculations, even with the PhET providing all this information, is intrinsic to the mathematical culture of traditional teaching, where the solution of a problem must always be demonstrated through formal calculations. According to Fischbein (1999), this creates a difficulty, in the field of mathematics in particular, in internalizing and genuinely accepting some concepts.

In this case, the student is "in the phase of concrete application of strategies, use of formulas, elaboration of drawings that effectively help

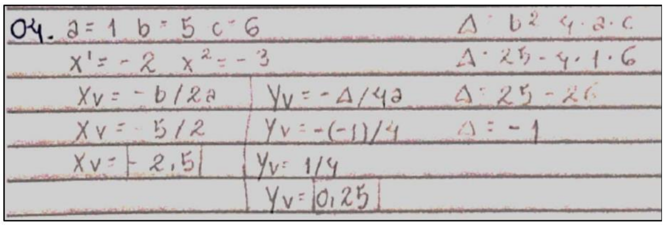


Figure 9. Demonstrative calculation of question 4 performed by S31

the identification of a solution" (Alves & Borges Neto, 2011, p. 44). However, sometimes there are other methods/paths to reach the solution, being the calculation, in this particular case, something important for the student, but irrelevant to the solution of the question in the proposed format.

Finally, in the last question, students were asked to observe a pre-established graph and explain why the graph does not intersect the x-axis. In this question, the students presented coherent and very similar answers, possibly because the textbook (Leonardo, 2016, p. 116) presented this information explicitly, which facilitated the understanding and use of this information in association with the PhET simulator. The illustration from the textbook can be seen in **Figure 10**. The information available in the textbook (**Figure 10**) possibly enabled the answers, satisfying what was requested. However, some interpretations caught our attention:

> "Because *c* is negative, that is, $c<0$, it is below *x*" (S20).
>
> "Because $a<0$ makes the graph be a function of the 1st degree" (S30).
>
> "Because the coefficients *a* and *c* are negative" (S42).

Students S20, S30, and S42 presented answers that disagreed with the observation of the simulator, being inadequate to solve the question. S20's response shows that he related the position of the graph given only to the c parameter of the function. Already S30 mentioned that the negative discriminant ($\Delta<0$) makes the graph no longer a quadratic function, but an affine function, however, the graph that was presented in the question is a parabola and not a line. Finally, S42 presented a response that only relates the signs of parameters a and c as determinants for the graph not to touch the *x*-axis, without considering the sign of the discriminant in fact.

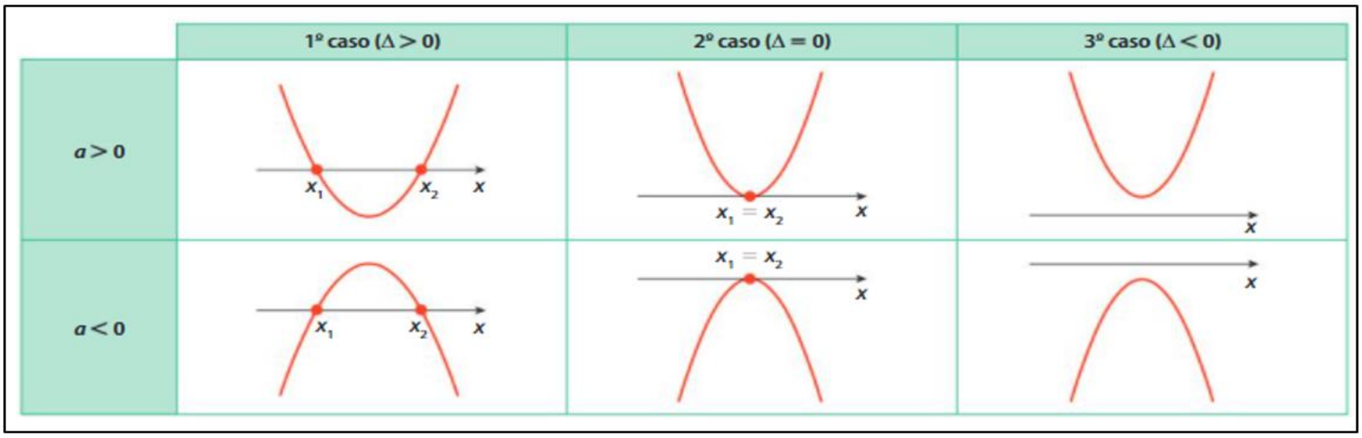


Figure 10. Graph behavior based on the discriminant value of a quadratic function

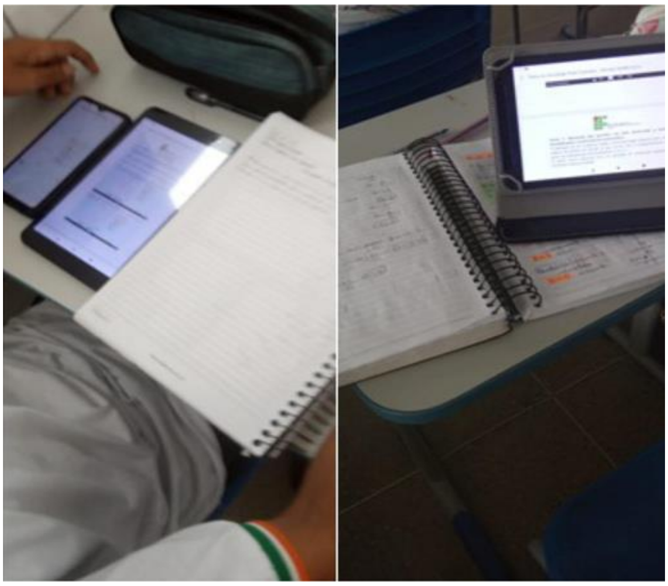


Figure 11. Records of the in-person meeting

Figure 12. Records of the in-person meeting

We understand that there was a mistake in the interpretation of this question by these students in a specific way, possibly because they did not understand the theoretical part, which was clarified by the teacher at the time of the discussion in the classroom. According to Fischbein (1993) the integration between conceptual and figural properties, always considering formal restrictions, does not occur naturally in the student's mind, which must be a continuous and systematic concern of the teacher in the classroom.

Figure 11, **Figure 12**, and **Figure 13** show records of the second moment of the activity, in person, where the issues explored in the PhET were discussed and doubts about the proposed activity were clarified.

When the answers were shared, based on the students' comments in the classroom and their resolutions, it was noticeable that the activity was well accepted by them. However, some of them reported difficulties in understanding and properly interpreting all the questions using the PhET and explaining them using appropriate mathematical language. Another difficulty pointed out by a small group was the visualization of the simulator on the cell phone, both because the screen is very small and because some students have older cell phones, in which there was sometimes a delay in opening the PhET and executing the commands.

The exploration of the simulator in the classroom by the teacher after carrying out the activity at home, following the flipped classroom methodology, provided a more fluid learning, in which the students' responses to the proposed activity occurred organically.

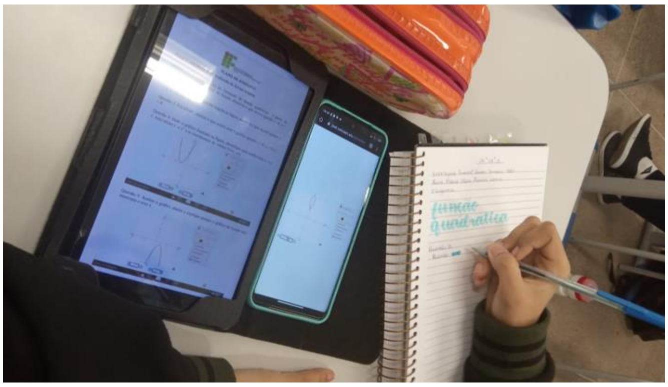


Figure 13. Records of the in-person meeting

The main differentiating points of this research in relation to the other researches that were read and presented in the theoretical framework of this work are: the fact that this classroom experience was carried out in hybrid teaching; the use of PhET as a pedagogical resource together with the flipped classroom methodology; and the analysis carried out from the theory of figural concepts.

However, we can say that the research by Vieira et al. (2021) inspired us to use PhET with quadratic functions, as well as research by Borke (2021), Brito et al. (2019), Nobre (2006), and other cited works helped us to understand the main difficulties of students in understanding the quadratic function. Furthermore, the works of Fischbein (1993, 1999) gave us the means to understand these difficulties and interpret them based on his theory.

DISCUSSION

In the search for the causes of the mistakes pointed out by the students and based on the theoretical reference raised, we can see that the way in which the mathematics textbooks approach the subject in a very abbreviated way, which generates gaps in learning, as shown by Vieira et al. (2021).

Thus, as shown in the works of Brito et al. (2019), Celestino and Pacheco (2010), Nobre (2006), and Prado (2014), the study of the quadratic function can be motivated via application problems, in which it is necessary to find a certain point maximum, for example in problems of determining maximum area. According to these authors and based on the results collected, we can understand that the study of this type of function must be carried out in such a way that the student can establish the relationships between the aspect of the graph and the coefficients of its algebraic expression, avoiding the rules memorization. We also consider it pertinent to deduce the formula that calculates zeros of the quadratic function and the identification of its graph with the parabola, so that there is an explicit relationship between image and concept.

In addition, as the research by Feltes and Puhl (2016) and Ribeiro and Cury (2015) show, the methodology of teachers in the Brazilian context brings with it gaps, with traditional classes and little use of technological or recreational resources and thus, students develop little and present a narrow and reductive view of quadratic functions. Furthermore, there is a lack of a deeper discussion on the concept of quadratic function in the classroom, as well as the exploration of the multiple representations of functions, considering the relationship between the mathematical concept and the mental image, as defended by Fischbein (1993), explained in theoretical foundation of this work.

The observations and answers presented by the students allow us, as teachers, to reflect on our practice, in order to bring to the classroom materials and methods that work with mathematics (especially algebra) in a clearer and more concrete way for the student, reinforcing the importance of theoretical concepts for solving problems that demand calculations in a practical way.

The result of the study suggests that mathematics teachers should diversify the choice of teaching strategies for quadratic function in basic education, as only the problem-solving method and use of the textbook are not enough for students to learn algebraic calculations.

Finally, we emphasize the importance of complying with the BNCC's suggestion, adopting the use of technological resources as a way of gathering prior knowledge and leveraging students' potential, given that we have a generation of connected students, who demand teaching methodologies that explore their potential.

FINAL CONSIDERATIONS

This work presents the description of an activity carried out in the hybrid teaching model with students from the 1st year of high school in the study of the quadratic function, analyzed from the perspective of theory of figural concepts, in which we seek to analyze the relationship between concept and image demonstrated by students through a simulation on the PhET.

The study of the quadratic function with the support of PhET can provide the teacher with a better view of the student's development. The student shows more interest and better resourcefulness in carrying out activities with the support of PhET and with the use of the inverted classroom methodology.

The Theory of Figural Concepts supported the analysis of the results, showing that there is a certain fragility in the association between the elements of a conceptual order of a quadratic function and its graphic representation, emphasizing the importance of working on this subject in order to encourage the student to establish hypotheses and see the meaning in real situations involving this subject, which became more evident due to the use of PhET.

The PhET platform and its simulations have enormous potential for the development of the student's mathematical reasoning. In this work, we observed that the moment of socialization of the answers was productive, in which the students were able to visualize their mistakes and reflect on them, as well as internalize their successes and demonstrate an understanding of the quadratic function spontaneously.

The changes in the educational scenario generated from the new Coronavirus pandemic demand new working methods, different from the traditional expository class. From this experience, we reflect on the need for a new attitude towards the classroom, in which we consider the role of technology for the evolution and learning of students and optimization of pedagogical time undeniable.

In carrying out this work, we learned that one of the ways to make the study of quadratic functions more exciting for the student is to use technological resources as an aid. In this experience with PhET, we learned that with the use of this resource we can develop not only activities about the functions discussed in this work, but also other themes and activities that allow the student to research, observe, reason, and mainly develop their own methods to solve problem situations.

Among the mishaps of this study, we have the difficulty of accessing the internet for some students, as well as the lack of adequate technological equipment for the development of home activities. Unfortunately, this is something common, intrinsic to the reality of many Brazilian public schools. In addition, we also have the fact that part of the class participated in both moments of the class in a virtual way, in view of the health restrictions and social distance imposed by the COVID-19 pandemic, as we have students with comorbidities and belonging to the risk group.

We recommend the reading and reproduction of this research, as well as the use of the PhET simulator and the flipped classroom methodology for high school mathematics teachers. Future developments in this line of research may include a comparative analysis of the use of PhET with other digital resources or even expanding the use of PhET to other subjects in mathematics and in other high school grades.

In addition, as a future perspective, it is important to encourage other mathematics teachers in Brazil and in other countries to teach with technologies, as well as to motivate them to use the flipped classroom method, which yielded a good experience. Finally, we hope that this study can encourage other teachers to work with active methodologies, using technology beyond the classroom environment and using platforms such as PhET, exploring other methodological possibilities for teaching mathematics.

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