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The connections children develop between science and mathematics: An example of temperature measurement in the kindergarten

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ABSTRACT

Children gain experiences that enables them to build concepts through everyday activities, while the use of cultural tools leads them to the development of early mathematical experiences. This paper presents a case study in a public kindergarten classroom in a semi-urban area of Greece (n=21), where discussion and recording of temperature sparked children's interest in using the thermometer as a cultural tool. By observing children engaging in mathematical experiences related to number construction through a lesson plan it was evident that children tried to explain the use and function of the thermometer.

Keywords: mathematics, science education, kindergarten children

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INTRODUCTION

Concept of Temperature in Preschool Age

The role of science education in preschool is considered to be of significant importance and starts off with the experiences children already have from the environment in which they grow up. According to Institute of Educational Policy, Ministry of Education and Religious Affairs (2014), educational interventions in the context of science education are viewed as necessary in the direction of scientific literacy. Whilst their teaching is directly linked to other learning areas such as mathematics, offering a framework for negotiation and joint construction of meanings (Institute of Educational Policy, Ministry of Education and Religious Affairs, 2014, p. 229-234). Although children's interaction with the world around them is an important part of learning (Fleer & Hardy, 2001), it is observed that such opportunities are often not provided to children (Christidou et al., 2013; Lindemann-Matthies, 2006).

Research suggests that preschool children face a difficulty in understanding the concept of temperature (Gerhátová et al., 2021; Havu-Nuutinen, 2007; Kampeza et al., 2016). Although they often use the terms "hot" and "cold" to describe temperature, often their perceptions are not scientific (Qonita et al., 2019) resulting in misconceptions (Lewis & Linn, 2003). In a study by Havu-Nuutinen (2007) with children aged 5-9 an attempt was made to explore children's understanding how to use the thermometer with regards to the different seasons. The study showed that children aged 5-6 years where not in a position to understand the use and function of the thermometer as they could not comprehend the basic thermal concepts that were involved. Contrary to the previous study, another study by Inan et al. (2010) with children aged 3- to 5-year-olds stated that preschool children understand how to measure the temperature by using the appropriate terms that relate to the weather and at the same time are in a capable of using the thermometer in order to measure the temperature. On the other hand, Inan et al. (2010) argue that children aged three-five years were able to measure temperature using measurement tools. Similarly, in another study (Gerhátová et al., 2021) with primary school children, it was observed that children approached the concept of temperature more efficiently through the open inquiry method than the traditional teaching method. However, Kampeza et al. (2016) denote that it is apparent that children 'may be merely reproducing the words of their teachers and imitating their practices' (p. 186) making it clear that more research in this field is needed in order to support early childhood teachers practices regarding the use of the thermometer and the engagement of young children with this tool. Thus, the thought arises whether children's initial perceptions can yield cognitive conflicts and construction of new concepts through an exploratory and discovery approach, which at the same time provides freedom and strategies with a scientific basis.

Thermometer as a Cultural Tool

Despite the fact that the majority of children recognize the thermometer as an object, they seem to have difficulty in distinguishing its function (Kampeza et al., 2016). They often 'read' its readings without, however, corresponding them to the concepts of heat (Havu-Nuutinen, 2007) or associating them with the movement of the liquid. On the other hand, it is argued that the use of this tool with stimuli and an appropriate scientific context in an educational environment can be supported (Inan et al., 2010) through an appropriate teaching approach (Kampeza et al., 2016; Vellopoulou & Ravanis, 2010). Research has shown that temperature is directly related to measurement fundamentals (Helenius et al., 2016; Qonita et al., 2019). The numerical increase that children observe on the thermometer is an indication of temperature variation (Christodoulakis & Karina, 2024). Institute of Educational Policy, Ministry of Education and Religious Affairs (2014) includes among the objectives of science education the familiarization of children with the thermometer as a tool and the measurement of temperature in order for children to make qualitative assessments (Institute of Educational Policy, Ministry of Education and Religious Affairs, 2014, p. 259).

Structuring of Number

For children number is conceptualized as the skill of managing numerical judgments and/or developing strategies for managing them (Perry & Dockett, 2002). Understanding the concept of number is not achieved as such through the projection of numerical symbols. Rather, it is structured as a result of children's conceptualization and engagement in activities that combine number operations (Zacharos et al., 2013). The development of the concept of number needs elaboration and development by combining all numerical approaches that constitute number in an organized educational context (Mullis et al., 2012).

In particular, the structuring of number goes through quantification, measurement and estimation of quantity (literature). Still an understanding of the sequence of numbers and ways of representing them, along with an understanding of numerical relationships and ways of structuring number systems is included. In this way the understanding of operations the connections between them and the solution of numerical problems is sought (Martín-Díaz et al., 2020). Finally, the individual's practice of written and mental calculations and estimations is also encouraged (National Council of Teachers of Mathematics [NCTM], 2000a, 2010b).

Similarly, the structuring of number units relates to the child's acquired knowledge and conceptual understanding of number (LeFevre et al., 2009). It is evident that the combination of knowledge and the conceptual frame of reference are able to structure number, promoting understanding and the ability to apply knowledge (National Research Council [NRC], 2009). Problem-solving is an element of mathematical thinking and runs through the curricula at all school levels (Evangelou et al., 2008). Given the possible non-holistic view of the problem, the teacher must utilize a variety of pedagogical practices (Kaldrimidou, 2002).

From the above, it emerges the need for targeted activities that develop reasoning and are linked to children's everyday life (Cheeseman et al., 2012). Through these, representational thinking in mathematical problem situations is developed by utilizing visual representations (Mason, 2008).

Role of the Play

In the context of developing mathematical thinking, play is the unintentional or intentional context (Montague-Smith & Price, 2012) in which children recall previous experiences, make connections, represent them creatively, explore possibilities and make sense of them (Perry & Dockett, 2007).

Creating mathematically oriented experiences and engaging children in interactions that support and extend mathematical thinking are structured within children's sociocultural context (Dunphy, 2006), providing rich learning points for the development of mathematical thinking (Wood & Attfield, 2005).

However, these points cannot be supported without teachers incorporating them into educational practice (Anthony & Walshaw, 2009, p. 110). In this context, the pedagogical interaction of children, both with each other and with adults through activities that promote reflection, hypotheses, predictions and verification is highlighted (NRC, 2009).

METHODOLOGY, ETHICS, & DATA COLLECTION

This paper presents a teaching proposal as it emerged from the observations of children's emergent actions. The study was implemented in a public Greek kindergarten class with children aged four-six years (n=21) and the recording of children's observations lasted four weeks. The appropriate ethical guidelines were taken into consideration for this study. The presented teaching proposal was part of an educational program. Parental written consent as well as children's oral assent was obtained for the use of children's photographs, note and drawings. The use of these photographs and notes applied to the protection of children's identity by using pseudonyms and blaring their faces in all instances.

The focus of the study was on how children approached temperature through linking the learning areas of science and mathematics in their attempt to explain the measurable characteristics of temperature. The activity evolved according to the principles of the project method and the nodal phases that determined its development are analyzed below. Participatory observation with field notes was used for data collection.

A photographic record and evidence from the children themselves (drawings, notes, etc.) was also kept. It should be noted that the above teaching strategy is part of the classroom philosophy and therefore students and teacher were familiar with its use. As it is usually the case in research with young children the researcher has to carefully plan the 'most appropriate solution to recording events as close to reality as possible' (Stamatoglou, 2023, p. 34). Through the presence of focal points, the development of children's thinking as it this was shaped through cognitive conflicts, testing and exploration is presented. The data that emerged from the daily routine of weather observation were analyzed through a qualitative perspective.

FINDINGS

In this part the findings of the study will be presented based on the participatory observations, children's discussions, photographs of the children and children's drawings and notes of their own observations.

Figure 1. Children's initial observations (Source: Field study)

Initial Ideas That Emerged on a Weather Report

As the children observed the weather and commented regarding cold and heat, one child referred to the temperature and the process of measuring it. One group of children focused further and began to look for information and evidence regarding the process of measuring temperature. Gradually, this engagement led them to contact with the cultural tool of the thermometer, which in turn led to their engagement with numbers.

Observing the weather is a daily classroom routine. In this process the children observe the weather on their way from home to school. They are encouraged to discuss their observations, what they see, feel, hear and sense. They are then encouraged to record their observations in a multimodal way (notes, drawing activity, symbols, letters, etc.). One day, a child brought a drawing (left part in **Figure 1**) from home, where he had taken notes from the afternoon weather report on TV. He presented it to the plenary and a discussion started:

> Mickle: We watched the weather report, and it showed the map of Greece. It had numbers, arrows, clouds, rain and sun.
>
> Thekla: If it's sunny it means we'll have good weather, if it's raining it means it's going to rain.
>
> Mickle: My mum says that if numbers are small, it will be cold!
>
> Sera: What does that mean?

Numbers on Thermometer

The teacher took the opportunity to encourage the children to express their ideas through brainstorming practice. The children's views were varied "numbers are for counting", "big numbers are better than small numbers", and "weather does not have numbers, mathematics does". All ideas were recorded and posted on the documentation board for exploration. Over the next few days the children collected information from various sources in their family and wider community. One child brought a digital thermometer to school "this is a thermometer, we press the button, it shows numbers, and the numbers tell us if it's cold or hot" and "so, what do they call it?" The thermometer remained in the school for investigation. The children worked on this cultural tool in order to find out how it works and process the information. In the meantime, the information the children found was transferred from one learning environment (school) to another (family) and vice versa in a process of continuous communication and feedback.

> Thekla: At home we have a thermometer ally. It has many numbers written together. Some are red and some are blue. (middle part in **Figure 1**).
>
> Zoe: We have a thermometer at home too. At noon when I went to lunch it was 11 and at night when I went to bed it was seven (right part in **Figure 1**).
>
> Kindergarten teacher: What can these numbers mean?

Linking Abundance of Numbers to Temperature Characteristics

The kindergarten teacher constantly provided motivation for exploration and reflection. In this direction, she urged the children to take notes (left part in **Figure 2**).

The children measured the temperature daily and posted their findings on the temperature observation board (middle part in **Figure 2**). On this board, the children collected all their observation materials and were able to discuss, post their questions, and monitor the progress of their investigations at the time and pace they wished. During their measurements they identified the number on the digital thermometer and tried to match it to the number on the traditional thermometer. There they faced a challenge "it has zero and 10 and 20 but not 17 …" (right part in **Figure 2**). The kindergarten teacher encouraged the children to observe the special features of the thermometer and transfer them to the meter paper. Children identify "each number is on a long line", "from one number to the other has short lines", and "yes, but where are the other numbers?"

This question seems to have occupied the children for several days. In this respect, the kindergarten teacher encouraged the children to visualize their findings.

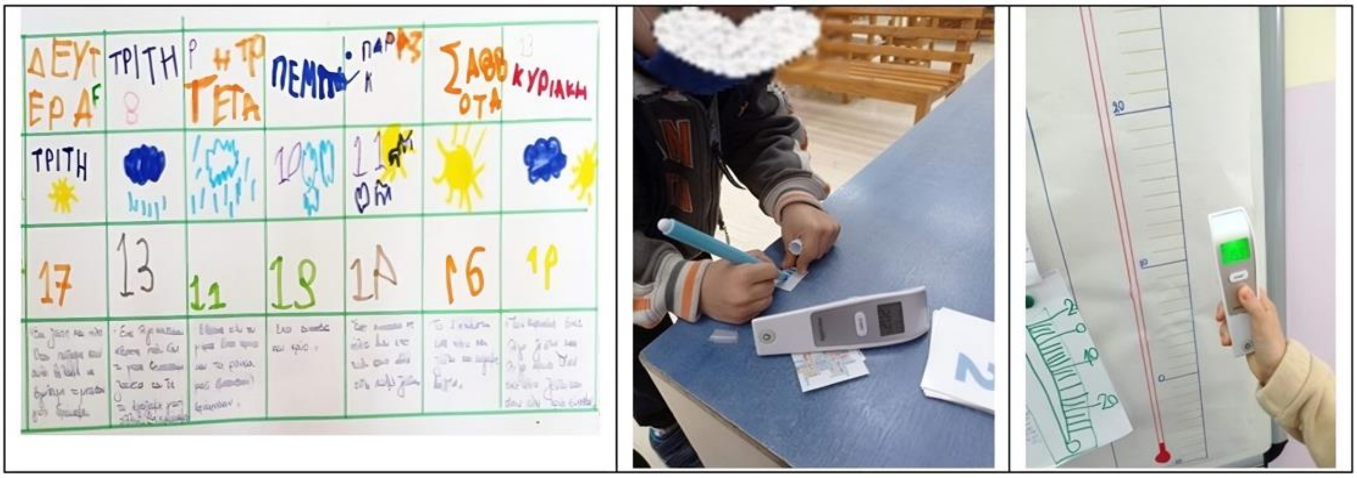

Figure 2. Children's personal notes (Source: Field study)

Figure 3. Graphical representations of temperature (Source: Field study)

Each day they recorded the temperature from the digital thermometer and compared the numbers with each other. They then quantified with building materials, where each unit and a block (left part in **Figure 3**), in order to observe the relationships between them and comment "15 is the biggest of all because its tower is the tallest", "13 and 14 are almost the same, but 14 is a little bigger because it is the tallest", and "11 is the smallest because it is the shortest and 15 is the biggest because it is the tallest".

They then graphically depict this relationship and connect the top of the columns with a line (right part in **Figure 3**) and point out "the line at 11 goes very low and at 15 very high", "every day we have a different number (temperature) and the line goes elsewhere (to another point), up-down, up-down", and "yesterday it was 11 and it was cold and the line is down. The colder it is, the lower the line goes".

These observations motivated children to make further connections about the relationships between numbers and their regularity "when we count five is later than three so 15 is bigger than 13 because one is the same and five is bigger than three" and "yes, and when we count we say 13 first and then 15". Next the children made connections between numbers, their position and temperature "today it is 20, which is very big, and it is very hot, so when the number is big it is hot" and "yes, and if it is small (the number) it is cold".

Linking Arrangement of Numbers to Characteristics of Temperature

These connections led the children to further reasoning and observations. In her attempt to find a way to discover the "missing" numbers on the thermometer, one girl elaborated on the classroom number line. At first, she places blue stickers on zero and 10 and then places blue markers on the numbers in between (**Figure 4**) and counts "1, 2, 3, ..., 9. (thinks and counts again) Nine ... It's nine! Nine are the little lines on the thermometer!".

The kindergarten teacher brought the critical incident to the plenary and encouraged the children to connect the two information environments (number line - thermometer lines). They analyzed the girl's idea and try to integrate the numbers on the number line into the corresponding thermometer lines (zero-10) and check "0, 1, 2, 3, ..., 10" (left part in **Figure 5**).

The kindergarten teacher asked them "can you think about what will happen to the next little lines?" The children tried to find a solution based on the new information "it is like we did from zero to 10 but now we will put other numbers", "yes, let's see in the diary, which numbers

Figure 4. Connection between numbers on thermometer & numbers on number line (Source: Field study)

Figure 5. Arrangement of numbers on thermometer & value of position (Source: Field study)

are from 10 to 20 and put them", and "it is easy, we put 1, 2 ... again. Up to nine but in front we put one" (middle part in **Figure 5**). "Yes, that's how it's done, and then we'll put again (1, 2, 3 up to nine) but in front we put two because we say 21, 22, so ..." (right part in **Figure 5**).

The reasoning continued and the children commented on the value of the position of the number on the thermometer "zero is down at the bottom, everything near there is cold, the higher it is the hotter it is" and "yes, because 20 is a very big number and zero is the smallest of all!"

DISCUSSION

This small-scale study aimed to present children's experiences and views during a specific lesson plan on the use of the thermometer. Once the children chose to explore the thermometer's features, they were exposed to new challenges and concepts that were constantly emerging. The biggest challenge we faced was on the one hand managing the concept of temperature as something not visible and tangible and on the other hand connecting it to another concept, that of number, from the field of mathematics. In this effort, the children approached the learning areas of science education and mathematics through multimodal means (symbols, representations, etc.), observations and recordings from the cultural and scientific context. The choice of these diverse representational media corresponds to the children's understanding, interpretation and utilization of their ideas, serving their functional management (Elia & Philippou, 2004). Similarly, the visualization of the problem supported their understanding by organizing, presenting and updating the data (Elia & Gagatsis, 2006).

Their emergent actions in trying to solve the original problem often led to solutions that generated new questions. This process motivated their interest and gradually led them to solve the original problem autonomously. This strategy led to a continuous interaction of the children through verbal reasoning and documentation (Papandreou & Tsiouli, 2020).

An important outgrowth of the process was the encouragement of problem creation and the understanding of their elements, structure and solution (Luwel et al., 2001). During problem-solving, children engaged in self-regulation processes by internally monitoring and strategically controlling their mathematical choices (Berk & Winsler, 1995). In the present study, daily feedback and mathematical reasoning around children's findings structured metacognitive strategies of independent learning that beyond mathematical reasoning provided strategies for managing similar mathematical issues (Whitebread, 2007). Reflective pedagogical dialogue was particularly conducive to the above, as combined with explanations it encouraged children to structure their ideas descriptively and check their evidence (Coltman, 2006; Fragkiadaki & Ravanis, 2016).

Methodologically, the activity went through all of the key stages of problem-solving (Polya, 1967). In particular, the children understood the problem by representing and articulating it in a variety of ways. They then devised, organized and implemented action plans to manage it inside and outside the classroom, checked their findings in the whole group and finally proceeded to evaluate the problem itself (DeCorte et al, 2000). Through this activity, the children were able to discuss around the problem, contrasting ways of solving, comparing and improving their methodology (Verschaffel, 2002). It is clear that the evolution of the process rejects the linear path, following a cyclical one with returns and re-examinations (Greens et al., 2004).

The role of the kindergarten teacher was crucial, as she constantly encouraged the children's spontaneous explorations and brought the focal points to the plenary with purpose. In order to capture the children's perspective, the teacher left room for the children's free exploration for a long period of time without interference and guidance (Papandreou & Konstantinidou, 2020). This resulted in a careful focus on the children's prior knowledge and perceptions around the concepts of temperature and number and to convey more accurately and consistently to the whole class the key points. Utilizing exploratory situations with free handling of materials (thermometer, blocks, documentation board, etc.) provided motivation for problem-solving, use of symbolic systems and emergence of learning experiences to develop children's thinking. The children's spontaneous ideas were each time the basis for new activities in the plenary and vice versa (Konstantinidou, 2018). This strategy helped to 'share' the findings of individual or small group explorations in the plenary, to bring about discussion and argumentation.

CONCLUSIONS

This small-scale research study shows that children have developed representations that are compatible with scientifically accepted knowledge, but it is left open that a systematic didactic exploratory approach over time can help children co-construct concepts related to temperature (Ioannou et al., 2023; Kampeza et al., 2016). In the present study, it was observed that children, through free exploration, developed reasoning in their attempt to approach the concept of temperature. These reflections led to connections with the field of mathematics, specifically with the concept of number as far as their ordering and the value of the position of numbers are concerned. Through these connections the children were encouraged to observe the fluctuations in temperature and focus on the importance/meaning that numbers represent.

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Declaration of interest: Authors declare no competing interest.

Data availability: Data generated or analyzed during this study are available from the authors on request.

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