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The effect of realistic mathematics education on the problem-solving competency of high school students through learning calculus topics

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ABSTRACT

Fostering and developing students' problem-solving competency is one of the main goals of most mathematics education in the world. Problem-solving competency has been used by the Program for International Student Assessment (PISA) to assess students at the age of 15 since PISA 2003. In Vietnam, new research directions to improve problem-solving competency for students through teaching analytic topics is quite small and not systematic. The purpose of the article is to propose a process of teaching calculus topics in high schools based on the realistic mathematics education approach and problem-solving process according to PISA framework 2021 (OECD, 2018). This design is to support teachers in improving problem-solving competency for students, meeting the requirements of the 2018 general education curriculum in Vietnam today.

Keywords: competency, problem-solving, problem-solving competency, teaching calculus, realistic mathematics education

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INTRODUCTION

Mathematics is a highly abstract science with universal practicality. Mathematics was born and developed from the requirements of practice, in order to come back to solving problems of practice and orienting science and technology. Reality shows that mathematics is not only a pure science of reasoning, but also has an active role in human cognitive activities (Da, 2022). With the characteristics of its object, mathematics increasingly penetrates deeper into different scientific fields, holds a special place in many sciences; and thus, covers a wide range of practical activities. The creative role of mathematical thinking in perception is shown quite clearly that mathematics is seen as an indispensable tool for the sciences in discovering and finding nature of objects and phenomena of objective world.

Mathematics education in high schools aims to equip learners with the knowledge and basic math skills to pursue higher education and solve problems in everyday life. These math skills include the ability to problem solve, reason, communicate, connect, and perform mathematical operations, as well as higher order thinking skills, such as critical thinking and creativity (Fauzan dan Yerizon, 2013). Mathematics education contributes to the promotion and development of innovative thinking for students and cultivates the ability to think and reason mathematically correctly in problem-solving. Mathematics education is an active, dynamic and ongoing process. In addition, through mathematical activities, students have the opportunity to develop generalization, specialization, logical thinking, deep critical thinking and openness when solving problems.

Vietnam's Education Law 2005 has identified "educational activities must be carried out according to the principle of learning with practice, education must combine with production labor, theory must try to be associated with practice ...", from there, the educational method must promote the learners' positivity, self-awareness, initiative and creative thinking; fostering learners' ability to self-study and work in groups; practice skills to apply knowledge into practice. One of the goals of the high school program (Ministry of Education and Training, 2006) also stated: "Helping students to solve mathematical problems and apply mathematical knowledge in learning and life" (p. 92), at the same time Ministry of Education and Training (2010) also emphasizes the requirement for textbooks to ensure interdisciplinarity, linking the content of lessons with practice (p. 6).

Equipping students with mathematical knowledge not only helps them solve pure mathematical tasks, but importantly, students need to know how to apply that knowledge to solve some tasks. actually happen in real life. However, in practice, most students think that this is one of the difficult tasks that cannot be overcome. One of the main reasons leading to this situation is the limited ability of students to apply mathematical knowledge to solve problems. Thus, fostering and improving the problem-solving capacity of students is a necessary and important task of mathematics education. We believe that choosing the

Table 1. Control group and experimental group

Group	Class	Number of student	School
Control	12A3	39	Nong Cong 2 High School
Experiment	12A2	37	Nong Cong 2 High School

approach of realistic mathematics education (RME) will open up a new opportunity for teachers to help students practice and develop problem-solving skills effectively.

The main principle in RME approach is that mathematics is viewed as a human activity and mathematical learning means doing mathematics (Fauzan dan Yerizon, 2013). This means that in learning mathematics, student involvement is expected as well as directed to solve mathematical problems related to real life. Furthermore, Freudenthal (1991) emphasized that in mathematics learning students should be allowed and supported to create their own ideas and use their own strategies. In other words, they must learn mathematics in their own way (Fauzan dan Yerizon, 2013).

RME-based learning that emphasizes the formation of math skills will lead students to make and solve their own problems by leveraging informal knowledge from their own lives. This aligning students' thinking will help students solve mathematics through mathematization process based on RME approach. Through mathematization, students' informal knowledge is linked to formal knowledge to be learned. This will train students in problem-solving skills and have a positive impact on students' problem-solving competency.

RESEARCH METHODS

In this study, we used the following methods:

1. *Analyzing and synthesizing*: Synthetic analysis of documents related to RME theory, teaching principles according to RME to design appropriate teaching situations, contributing to the development of problem-solving competency for high school students.
2. *Select and classify situations* according to each knowledge unit and content topic containing calculus knowledge that students need to acquire.
3. *Pedagogical experiment*:
    a. **Purpose of the experiment**: The pedagogical experiment is to test the effects and influences of RME approach on the development of students' problem-solving competency through teaching and learning activities on some calculus topics at some high schools in Vietnam today.
    b. **Experimental subjects**: The experiment was conducted at Nong Cong 2 High School, Thanh Hoa Province in Vietnam. Information about experimental and control group is given in **Table 1**.
    c. **Experimental time**. We conducted the experiment with control and experiment classes from August 4, 2022, to December 20, 2022, of the school year 2022-2023.

Before and after each experiment, we had a test to assess the learning results of the control and experimental classes. The results of the tests are analyzed, processed and interpreted by using Jamovi 2.3.21 software.

RESULTS

What is Realistic Mathematics Education?

RME is an instructional theory dedicated to the field of mathematics, developed by the Freudenthal Institute in the Netherlands. Characteristic of RME is that rich, "realistic" situations that are given an important place in the learning process. These situations serve as a starting point for the development of mathematical concepts, tools, and processes, and a context in which students can apply their mathematical knowledge at a later stage, which then gradually becoming more formal and general and less context-specific (Khanh, 2015). Currently, RME theory is mainly defined by Freudenthal's view of mathematics (Freudenthal, 1991). In most of his research works, Freudenthal (1991) said that "teaching mathematics needs to be connected with situations related to everyday life, to society in general in order to be of value to learners". His two important views were that mathematics must be connected with reality and mathematics as a human activity. *Firstly*, mathematics must be close to children and relevant to all situations of everyday life. However, the word "reality" refers not only to the connection with the real world, but also to real problem situations in the student's mind. For problems presented to students, this means that the context can be a real world, but this is not always necessary. *Second*, he emphasized mathematics as a human activity. Mathematics education is organized as a guided re-invention (re-creation) process, where students can experience a similar process to the one in which mathematics was invented. Furthermore, the principle of reproducibility can also be inspired by informal processes or solutions. Informal student strategies can often be understood as intended for the formation of more formal processes (Da, 2022).

Based on their research, Van den Heuvel-Panhuizen and Drijvers (2014) gave six core principles of RME theory:

(1) *The activity principle*: Students learn math by doing math, students are awarded the opportunity to perform horizontal mathematization and vertical mathematization.

(2) *The reality principle*: Real-life contexts and situations should be the starting point of the learning process.

(3) *The level principle*: This principle emphasizes that, in mathematics learning, students pass through various levels of understanding: from solutions involving informal contexts, through to making operations mathematics such as symbols, diagrams, and mathematical representations to gain insight into related concepts and strategies. Models are important for bridging the gap between "informal mathematics", in relation to context, and "formal mathematics".

(4) *The intertwinement principle*: According to this principle, areas with mathematical content such as arithmetic, geometry, measurement, and data processing are not considered separate curriculum chapters but are integrated with each other, so students need to mobilize their combined knowledge and diverse mathematical tools.

(5) *The interactivity principle*: learning mathematics is not only an activity of each individual learner but also a social activity. Therefore, RME encourages whole class discussions or group work, providing opportunities for students to share their strategies and inventions with others.

(6) *The guidance principle*: In RME, this principle refers to Freudenthal's (1991) idea of "guided reinvention" of mathematics. Specifically, teachers must take an active role in student learning, and educational programs must contain situations that are able to act as a lever to achieve change in students' understanding.

Competency

According to Phe's (2002) Vietnamese dictionary, "competency is a psychological quality that enables people to complete a certain type of activity with high quality". In Vietnam, there are also many different views on capacity. Nhat (1996) said that "competencies are unique attributes of an individual that are suitable for the requirements of a certain activity, ensuring that that activity has results". In terms of the purpose and personality of the competence, Hac (2001) defines:

> Competence is a combination of psychological characteristics of a person, this combination of characteristics operates according to a certain purpose to produce the results of a certain activity.

From a practical perspective, Khanh (2015) believes that competence is the ability to apply knowledge, experience, skills, attitudes and interests to act appropriately and effectively in diverse situations of life. The concept of competence in the general education program (Ministry of Education and Training, 2010, 2018) is defined as an individual attribute formed and developed by the inherent qualities and the learning and training process, allowing people to exercise the synthesis different knowledge, skills and personal attributes such as interest, belief, will, etc., to successfully perform a certain activity, achieve the desired results under specific conditions.

Problem-Solving Competency

Problem-solving is a process that requires the problem solver to find the connection between the experience (plan) they have with the problem they are facing and then find a solution to solve it. Problem-solving is emphasized in many mathematics programs and has recently become one of the most researched areas of mathematics education. The general mathematical curriculum that emphasizes the development of problem-solving abilities for students is considered one of the important goals of teaching mathematics in the 21st century.

There are different definitions of problem-solving. Problem-solving is an important part of learning mathematics (Ball et al., 2005; Davis, 1992). Furthermore, Soedjadi (1994) argues that mathematical problem-solving competence is the ability of students themselves to use mathematical activities to solve problems in mathematics or other sciences, even in everyday life.

Beigie (2008) also believes that through problem-solving, students can learn to improve their understanding of mathematical concepts by solving carefully selected problems, using the application of mathematics in the real-life problems. Developing mathematical problem-solving competency can equip students to think logically, analytically, systematically, critically and creatively (Surya et al., 2017). Therefore, problem-solving competence is an important ability that should be equipped for students to help them learn mathematic as well as solve real-life problems.

PISA's Problem-Solving Process and Students' Problem-Solving Competency

Problem-solving competency has been included in the assessment of students at the age of 15 by the Program for International Student Assessment (PISA) since the 2003 PISA period. Through PISA evaluation periods 2003, 2012, 2021, the problem-solving process was adjusted to suit the actual situation (Seifi et al., 2012). According to PISA framework 2021 (OECD, 2018), the problem-solving process consists of three steps:

1. **Step 1**: *Set up situations by mathematical method.*
2. **Step 2**. *Apply mathematical concepts, data, processes and inferences.*
3. **Step 3**. *Interpret, apply, and evaluate the obtained mathematical results.*

In Vietnam, the 2018 general education program uses the term: "mathematical problem-solving competence", which is an important component of students' mathematical competence. Four components of mathematical problem-solving competence are emphasized by the General Education Program, including:

(1) recognizing and detecting problems that need to be solved by mathematics,

(2) select and propose ways and solutions to solve problems,

(3) use relevant mathematical knowledge and skills (including tools and algorithms) to solve the problem, and

(4) evaluate the proposed solution and generalize to a similar situation.

Phuong (2001) believes that the component competencies of mathematical problem-solving competence have many similarities with the three steps in the problem-solving process outlined in PISA 2021 evaluation framework.

Accordingly, components (1) and (2) belong to **step 1**; component (3) belongs to **step 2**; component 4 belongs to **step 3**. Accordingly, this author believes that "mathematical problem-solving competence is the ability of an individual to establish, apply, and interpret (evaluate) mathematics in the process of solving a mathematics problem".

Scale Component of Problem-Solving Competency

Combining the competencies of the problem-solving process according to PISA framework 2021 (OECD, 2018) and some RME-based teaching principles, we have designed the problem-solving competency scale framework (**Table 2**, **Table 3**, and **Table 4**).

The score corresponding to each level of the scale is evaluated in detail according to **Table 5**.

Table 2. Scale component of problem-solving competency (set up situation models by mathematical methods)

EC	Understand task to be solved	Accurately identify relevant mathematical knowledge	Identify appropriate mathematical method to solve problem	Accurately identify concepts, theorems, rules, & algorithms to analyze & set up problems to be	Convert from reality tasks to mathematical tasks	Identify position, role, & relationship between actual factors in problem to
L 1	Completely unable to understand task to be solved	It is not possible to determine relevant mathematical knowledge at all	Completely unable to identify right mathematical method to solve problem	It is completely impossible to identify exactly right concepts, theorems, rules, &algorithms to solve problem	Completely unable to convert reality tasks into mathematical tasks	It is completely impossible to determine position, role, & relationship between reality factors in problem to be solved

Table 2 (Continued). Scale component of problem-solving competency (set up situation models by mathematical methods)

EC		Understand task to be solved	Accurately identify relevant mathematical knowledge	Identify appropriate mathematical method to solve problem	Accurately identify concepts, theorems, rules, & algorithms to analyze & set up problems to be	Convert from reality tasks to mathematical tasks	Identify position, role, & relationship between actual factors in problem to
L	2	Understand part of task to be solved but not exactly	Identified a part of relevant but incorrect mathematical knowledge	Identify part of mathematical method to solve problem, but it is not really reasonable	Identify part of a concept (theorem, rule, & algorithm) but not exactly	Converting part of reality task into a corresponding part of mathematical task, but it is not accurate & has many shortcomings	Partially identified relationship between reality factors, but there are many shortcomings
	3	Understand task to be solved but not really sure & clear	Identify relevant mathematical knowledge but not really sure	Identify mathematical methods to solve problems but are not really clear	Indicate one (several) concept, (theorem, rule, & algorithm) but it is uncertain & unclear	Converts real-life tasks into mathematical tasks, but not exhaustively & rigorously	Relationship between reality factors is partially determined, but it is not clear
	4	Recreate the task to be solved in a relatively complete and clear way	Relatively fully identified relevant mathematical knowledge	Relatively clear identification of a mathematical method to solve a problem	Indicate a relative concept, (theorem, rule, & algorithm) suitable to solve problem	Can convert reality tasks into mathematical tasks in a relatively complete but not rigorous way (such as lack of data & constraints	Relatively fully defined about role as well as relationship/binding between actual factors in problem to be solved, but still lack of rigor
	5	Fully understand task to be solved fully & deeply	Define fully & clearly the system of relevant mathematical knowledge to solve problem	Fully & unambiguously define system of mathematical equations involved to solve problem	Identify fully, clearly, & systematically concepts, (theorems, rules, & algorithms) suitable to establish problem to be solved	Fully & rigorously convert reality tasks into mathematical tasks	Fully, clearly, & accurately define role as well as relationship/binding between reality factors in problem to be solved

Note. L: Level & EC: Evaluation criteria

Table 3. Scale component of problem-solving competency (applying concepts, data, processes, & deduction of mathematics)

EC		Accurately apply concepts (or theorems, rules, signs, & algorithms), data, processes, & mathematical reasoning to solve	Exact mathematical operations including use of processes, procedures, & algorithms to provide appropriate mathematical solutions & methods	Understand process of case model; accurately identify appropriate mathematical knowledge; make rigorous & precise mathematical arguments	Generalizing from a situational model to a general model for similar situations
L	1	Completely unable to correctly manipulate concepts, data, & make incorrect inferences to solve problem	It is absolutely impossible to perform any mathematical operations exactly to solve mathematical task	Completely unaware of situational modeling process; it is not possible to accurately determine appropriate mathematical knowledge to give a suitable mathematical solution	It is not possible to suggest a mathematical procedure for similar situations at all
	2	Partially apply concepts or theorems, rules, algorithmic signs, data, but presentation & reasoning still have many errors	Participating in mathematical operation but not being able to do it correctly	Understand part of process of situation model; partially determined mathematical knowledge but failed to propose an appropriate mathematical method to give an accurate mathematical solution	Propose a partial mathematical procedure for similar but not exact situations
	3	Apply concepts or theorems; know how to use rules, signs, algorithms, data, but argument lacks rigor & logic	Can perform some mathematical operations correctly but still lacks rigor	Understand process of situational modeling & suggest appropriate mathematical methods but perform incorrect mathematical solutions	Propose exactly a partial mathematical procedure for similar situations
	4	Accurately apply concepts or theorems, rules, algorithmic signs, data; argument is relatively rigorous but incomplete	Accurately perform most mathematical operations such as identification, comparison, evaluation, inference, transformation, using rules, algorithms, etc.	Understand process of situation model & propose an appropriate mathematical method, but only partially perform mathematical solution	Proposing a relatively complete mathematical procedure for similar situations, but it is not rigorous & lacks logic
	5	Accurately apply concepts or theorems, proficiently use rules, symbols, algorithms, & data; provide accurate, complete & coherent arguments	Accurately perform all mathematical operations such as identification, comparison, analysis, evaluation, inference, transformation, use of rules, algorithms, etc. to give accurate mathematical solutions	Understand process of situation model & propose appropriate mathematical method & perform correct mathematical solution	Proposing a complete, accurate, & strict system of rules, procedures, algorithms for solving a class of similar problems

Note. L: Level & EC: Evaluation criteria

Table 4. Scale component of problem-solving competency (interpret, evaluate, & apply mathematical results)

EC		Evaluate, check, & reflect accuracy of mathematical solutions & mathematical results; check relevance of mathematical results in context of real world	Evaluate significance of mathematical results in actual context; interpret & interpret mathematical results into real-world	Evaluate optimality of mathematical solution, point out opportunity to improve mathematical solution	Point out mistakes in math solutions & difficulties that other students may encounter when solving situations	Point out typical signs for appearance of mathematical knowledge that will (may) appear in situation after solving problem
L	1	Absolutely do not test & reflect accuracy of mathematical results; knowledge cannot be applied to a similar situation; it is absolutely impossible to test relevance of mathematical results in context of real-world	Completely fail to appreciate meaning of mathematical results in a real-world context & fail to translate interpretation of mathematical results into results in real situations	Completely fail to evaluate optimality of mathematical solution, do not indicate an opportunity to improve mathematical solution, or suggest an incorrect solution	Completely fail to point out mistakes & difficulties that other students may encounter when solving situation	There is absolutely no indication that mathematical knowledge will (probably) appear in a similar real-life situation (sign of mathematization)
	2	Tests & accurately reflects part of mathematical results	Failure to assess significance of mathematical results in a real-world context; partially interpret mathematical results to results in real situations, but not exactly	Evaluates a small part of optimization of mathematical solution, indicating an opportunity to improve mathematical method but is not accurate	Point out a few mistakes & difficulties of other students but not related to mathematical problem just solved	Indicate some typical signs that mathematical knowledge will (may) appear in similar but it is not exactly
	3	Check & reflect accuracy of mathematical results but have not determined its conformity with reality	Evaluate part of significance of mathematical results in real context; accurately interpret part of a mathematical result into a real-world result	Fully evaluate optimality of mathematical solution, but have not shown opportunity to improve mathematical method	Point out a few mistakes & difficulties of other students related to problem just solved	Indicate exactly an indication that mathematical knowledge will (probably) appear in a similar situation but do not show a specific example
	4	Check & reflect accuracy of mathematical results, but only partially evaluate its conformity with reality	Fully & accurately interpret mathematical results into results in real situations, but only partially appreciate meaning of mathematical results in real contexts	Fully evaluate optimality of mathematical solution, indicating opportunity to improve mathematical solution, but not yet closely	Can fairly fully point out mistakes & difficulties other students have made	Point out some typical signs that mathematical knowledge will (may) appear, point out corresponding but not exact examples
	5	Check & reflect accuracy of mathematical results; fully assess its conformity with reality	Fully & accurately interpret mathematical results into results in real situations; accurately assess meaning of a mathematical result in a real-world context	Fully evaluate optimality of mathematical solution, point out the opportunity/potential to be able to expand development of problem	Point out all mistakes & difficulties related to mathematical problem that has just been solved or that other students have done	Fully indicate differences that characterize mathematical knowledge that will (probably) appear in a similar situation, show an illustrative example

Note. L: Level & EC: Evaluation criteria

Table 5. Score for assessing problem-solving ability

Components of competence	Evaluation criteria	Level				
		1	2	3	4	5
		Score				
1. Set up situation models by mathematical methods	1. Understand the task to be solved	0.00	0.15	0.25	0.40	0.50
	2. Accurately identify relevant mathematical knowledge	0.00	0.15	0.25	0.40	0.50
	3. Identify the appropriate mathematical method to solve the problem	0.00	0.15	0.25	0.40	0.50
	4. Accurately identify concepts, theorems, rules, algorithms to analyze and set up problems to be solved	0.00	0.15	0.25	0.40	0.50
	5. Convert from reality tasks to mathematical tasks	0.00	0.15	0.25	0.40	0.50
	6. Identify the position, role and relationship between the actual factors in the problem to be solved	0.00	0.15	0.25	0.40	0.50
2. Applying concepts, data, processes, & deduction of mathematics	1. Accurately apply concepts (theorems, rules, signs, algorithms), data, processes, & mathematical reasoning to solve problems	0.00	0.15	0.50	0.75	1.00
	2. Exact mathematical operations including the use of processes, procedures, and algorithms to provide appropriate mathematical solutions and methods	0.00	0.15	0.50	0.75	1.00
	3. Understand the process of the case model; accurately identify appropriate mathematical knowledge; make rigorous and precise mathematical arguments	0.00	0.15	0.50	0.75	1.00
	4. Generalizing from a situational model to a general model for similar situations	0.00	0.15	0.50	0.75	1.00
3. Interpret, evaluate, & apply mathematical results	1. Evaluate, check and reflect the accuracy of mathematical solutions and mathematical results; Check the relevance of mathematical results in the context of the real world	0.00	0.15	0.40	0.50	0.60
	2. Evaluate significance of mathematical results in actual context; interpret mathematical results into real-world results	0.00	0.15	0.40	0.50	0.60
	3. Evaluate the optimality of the mathematical solution, point out the opportunity to improve the mathematical solution	0.00	0.15	0.40	0.50	0.60
	4. Point out mistakes in math solutions and difficulties that other students may encounter when solving situations	0.00	0.15	0.40	0.50	0.60
	5. Point out typical signs for appearance of mathematical knowledge that will (may) appear in situation after solving problem	0.00	0.15	0.40	0.50	0.60

Developing Problem-Solving Competency Through Teaching Some Contents of Calculus for High School Students According to the Approach to Realistic Mathematics Education

Based on the combination of the competency components of problem-solving competency from the point of view of the 2018 general education curriculum in mathematics with the problem-solving process according to PISA framework 2021 (OECD, 2018), in this section we propose the teaching calculus topics for high school students according to RME approach.

Teaching process is designed according to a specific 6-step process:

1. **Step 1**. *The teacher selects and introduces the situation.*

a. Selected situations should be close and familiar to students' life, possibly happening or real in their mind. This represents second characteristic of RME-***reality principle***.

b. Teacher introduces some relevant context, inform about the situation in which that context occurs.

c. Teachers should know how to exploit situations where solving it requires a combination of much different knowledge, not only knowledge within mathematics and requires the existence of knowledge outside of mathematics, such as physics, chemistry, biology, economics, finance, environmental science, life science, etc. This represents the 4th characteristic of RME-***intertwinement principle***.

2. **Step 2**. *Students understand the situation, the context.*

a. Teachers let students discuss in groups to understand the contextual problem, students can support each other in determining the task to be solved, then set up a plan to solve the mathematical problem.

b. Students discuss to determine the exact mathematical method, clarify the relationship between the real situation and the relevant calculus knowledge, thereby accurately determining the mathematical knowledge that can be used to solve problems.

This step represents 1st and 5th feature of RME (***activity principle and interactivity principle)***–using student contribution and interaction between students and teachers and between students.

3. **Step 3**. *Explain the problem in context.*

The second step is taken if a student does not understand the problem given. If all students have understood, this step is not necessary. In this step, the teacher explains the situation and the condition of the problem by giving necessary instructions for some problems in the situation that have not been clarified by the students. This step is the 5th characteristic of RME, which is interactive activities, interaction between students and teachers and between students.

4. **Step 4**: *Solve the problem contextually.*

Solving contextual problems involves several activities:

a. Student's activities:

– Students solve problems in groups or individually. Depending on the level of the situation, the teacher allows students to work individually, then group activities. When solving problems, students are allowed to use different ways.

– Set up situation models by mathematical method.

– Apply mathematical concepts, facts, processes, and inferences.

– Explain, apply and evaluate the obtained mathematical results.

– Explain, apply and evaluate the obtained mathematical results.

– Check the correctness and suitability of mathematical results with practical situations.

b. Teacher's activities:

– Teachers use suggested questions to support students when they have difficulty.

– Teachers encourage students to solve problems in their own way by providing guidance in the form of prompting questions.

– Teachers monitor and observe students' activities when solving problems/performing proposed tasks in the situation.

– Teachers encourage students to use models in the problem-solving process.

– This step represents 5th and 6th feature of RME (***guidance principle***).

5. **Step 5:** *Compare and discuss answers.*

The teacher facilitates discussion and provides time for groups to compare and discuss problem-solving options. This step represents 1st and 5th feature of RME-using student contribution and interaction between students and teachers and between students. This phase includes the following activities:

a. Students present the results of working in the mathematical environment of each individual or group. The remaining students monitor, to give comments or suggestions (if necessary), in addition, they can ask questions to clarify the problem and add ideas to improve the solution.

b. The teacher acts as moderator and facilitator in the whole class discussion.

c. The teacher gives comments on the presentation of the student's mathematical results through problem-solving. Accordingly, teachers need to point out the difficulties and challenges of finding the solution to the problem; point out errors or mistakes that students may encounter in the process of solving mathematical problems.

6. **Step 6.** *Draw conclusions.*

From the results of class discussion, the teacher asks students to draw conclusions about a mathematical knowledge including concepts, system of rules, and methods, then summarize or complete the concepts, rules, and methods in the explanation. This phase includes several activities:

a. Students can point out signs to identify mathematical knowledge that will appear or be related (both directly and indirectly).

b. Students analyze specific and detailed steps and processes to solve problems.

c. Students arrange the operations to be performed through the situation.

d. Students can point out the difficulties or errors that may be encountered in the process of solving the model and in other but similar situations.

e. Students point out opportunities for developing mathematical competencies through problem-solving.

f. Students provide feedback on the situation designed in the lesson.

This step represents 3rd and 6th feature of RME (***level principle-guidance principle***), whereby students can perfect mathematical knowledge, adjust from "situational model" to "formal model", which requires more abstraction, more generalization.

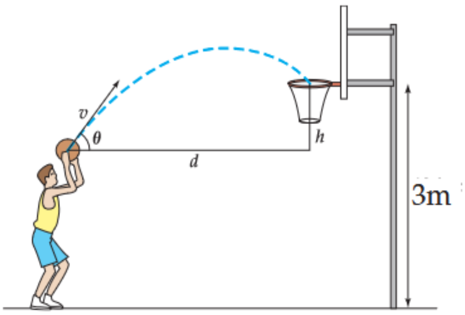

Figure 1. Shoot a basketball (Berresford & Rockett, 2014, p. 558)

Example of a Situation We Designed When Teaching Lesson "Maximum Value and Minimum Value of a Function"

Situation

To shoot a basketball, player must launch the ball with the right angle and initial velocity for it to pass over the rim (**Figure 1**). It can be proved from Newton's laws of motion that the initial velocity v and the angle θ need to satisfy the equation: $v = \sqrt{\frac{16d}{cos^2\theta\left(\frac{sin\theta}{cos\theta}-\frac{h}{d}\right)}}$,

where the constants d and h are shown in the simulation on the right. To minimize the chance of the ball bouncing off the rim, coaches recommend that athletes make the shooting with the least speed possible, known as the "soft shoot". Assume that for a free throw the constants are d=4.5 meters and h=0.9 meters. Find the "soft shoot" angle θ so that the initial velocity function reaches the minimum value.

The teaching process based on the RME approach is conducted by us, as follows:

1. **Step 1.** *Teacher states the situation.*
   a. Teachers can let students watch a short video about the situation where the athlete is throwing the ball into the basket in a standing position.
   b. The teacher distributes a study sheet to each individual on the sheet with the content of the situation that students need to solve.
   c. **The reality principle** of the situation we chose is reflected in the words/phrases: basketball; initial velocity; coach; athletes.
2. **Step 2**. *Students understand the problem, the context.*
   a. Teachers let students discuss in groups to understand the contextual problem, students can support each other in determining the task to be solved, then set up a plan to solve the mathematical problem.
   b. Students discuss to determine the exact mathematical method, clarify the relationship between the real situation and the relevant calculus knowledge, thereby accurately determining the mathematical knowledge that can be used to solve problems.
3. **Step 3**. *Explain the problem in context.*
   a. The teacher describes the situation in words or replays the video about "shooting basketball problem" so that students can clearly understand the problem to be solved.
   b. Students can ask questions to the teacher if they do not really understand the task to be solved.
4. **Step 4.** *Solving contextual problems involves several activities*:
   a. Students' activities:
   - Students discuss in groups to choose and propose ways and solutions to solve problems.
   - Students work individually and discuss in groups, identify fixed factors and variable factors in the model.
   - After students have identified the relationships between the data in real situations, the teacher suggests so that students can accurately determine the mathematical method and relevant knowledge to solve the problem.

The problem involves finding the minimum value of an algebraic expression.

***b.* Teachers guide students to set up situational models by mathematical methods (Table 6).**

***c.* Teachers guide students to use relevant mathematical knowledge and skills (including tools and algorithms) to solve problems:**

- Mathematical knowledge, formulas and methods that students use in this situation include:

About trigonometric: The relationship between functions tan*x*; sin*x;* cos*x*; trigonometric transformation formulas.

About calculus: Convert real problems into mathematical problems: "find the maximum and minimum value of a function". This function will depend on the result of the student's answer number three.

- Students use correct arguments to solve established mathematical problems.

Substitute h=0.9 and d=4.5 into the expression:

$$v = \sqrt{\frac{16d}{cos^2\theta\left(\frac{sin\theta}{cos\theta}-\frac{h}{d}\right)}}, \text{ we get}$$

$$v = \sqrt{\frac{16d}{cos^2\theta\left(\frac{sin\theta}{cos\theta}-\frac{h}{d}\right)}} = \sqrt{\frac{72}{cos^2\theta(\tan\theta-0.2)}} => v^2 = \frac{72}{cos^2\theta(\tan\theta-0.2)}.$$

Since 72 is a positive constant, therefore, in order to express the expression v^2 to reach its minimum value, the expression $cos^2(\theta)(tan\theta-0.2)$ must have its maximum value. The problem becomes finding the maximum value of function $f(\theta)=cos^2\theta(tan\theta-0.2)$ for all $\theta \in [0; \frac{\pi}{2})$.

To find the maximum value of a function $f(\theta)=cos^2\theta(tan\theta-0.2)$ for all $\theta \in [0; \frac{\pi}{2})$, students can use the following ways:

Method 1: Use the rule to find the maximum value of a function over half an interval.

- The teacher asked a student to repeat the formal mathematical process to find the maximum and minimum value of a function

Table 6. Suggested questionnaire

No	Hints	Object
1	Determine the fixed factor in the expression of v.	Define the correct mathematical object to be optimized
2	How is the expression in square root and v related?	Object transformation and expression simplification v.
3	What is a necessary and sufficient condition for the expression v to reach the minimum value?	Move mathematical tasks to solve.

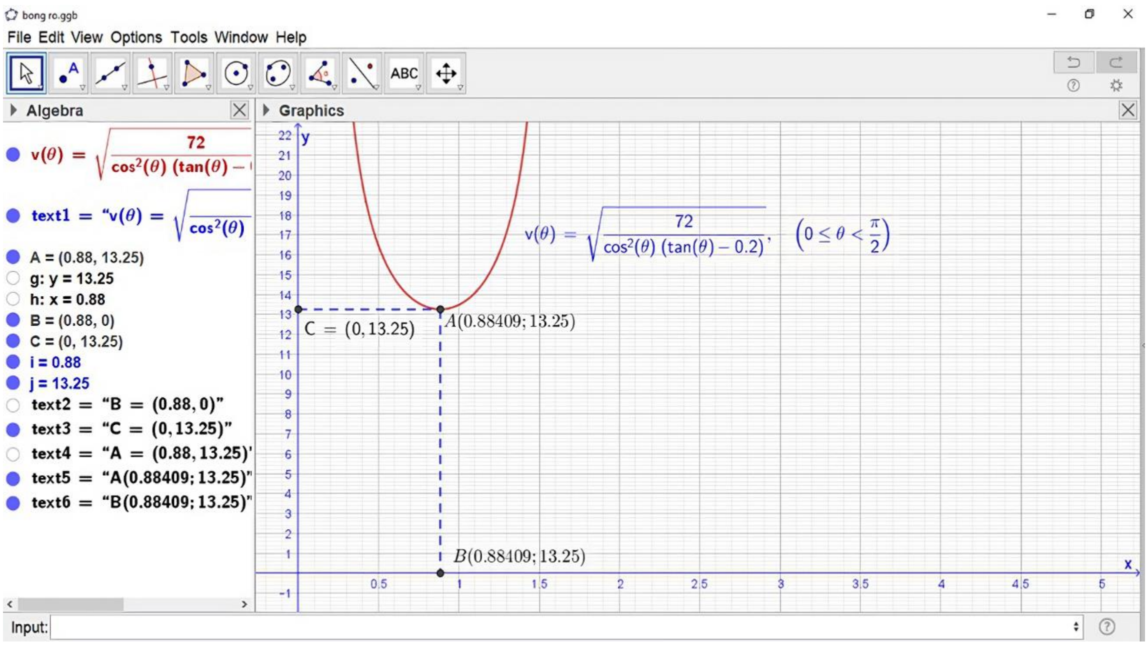

Figure 2. Example of GeoGebra (Source: Author, using GeoGebra software)

over half an interval, teachers can still encourage students to solve in many different ways, possibly informally.

– Teacher allows students to work individually, then work in team to give answers.

Here is an expected response:

Consider the function: $f(\theta)=\cos^2\theta(\tan\theta-0.2)$ with $\theta \in [0; \frac{\pi}{2})$. We have $y=f(\theta)=\cos^2\theta(\tan\theta-0.2)=\sin\theta\cos\theta-0.2\cos^2\theta=0.5\sin2\theta-0.1(1+\cos2\theta)$.

y has derivative $y'=\cos2\theta+0.2\sin2\theta$, let $y'=0 \Rightarrow \tan2\theta=-5 \Rightarrow \theta=0.5(\arctan(-5)+k180^o)$. Because $\theta \in [0; \frac{\pi}{2})$ so $k=1 \Leftrightarrow \theta \approx \theta_0=50.65^o$. Then $f(0)=-0.2$; $f(\theta_0) \approx 0.41$; $\lim_{\theta\to\pi/2} f(\theta)=\lim_{\theta\to\pi/2}\cos^2\theta(\tan\theta-0.2)=0$. On the other hand $0.41>0>-0.2$, therefore

$$max_{\theta\in[0;\frac{\pi}{2})} f(\theta) \approx 0.41 => min_{\theta\in[0;\frac{\pi}{2})} v(\theta) = v(\theta_0) \approx 13.25.$$

Answer: When angle $\theta=50.65^o$, the initial velocity v reaches the minimum value is 13.25 (m/s).

Method 2: Use the support of software GeoGebra.

Option 2: Teachers guide students to use GeoGebra software, the results are described, as shown below.

Based on **Figure 2**, we see that the lowest point of the graph over half the interval $[0; \frac{\pi}{2})$ is $A(0.88409; 13.25)$, this means the minimum value of the function $v(\theta) = \sqrt{\frac{72}{cos^2\theta(\tan\theta-0.2)}}$ over half the interval $[0; \frac{\pi}{2})$ is 13.25 when the angle $\theta \approx 0.88409$ (i.e., about 50.65^o).

Method 3: Continue to transform the equality: $y=0.5\sin2\theta-0.1(1+\cos2\theta)$ to $\sin2\theta-0.2\cos2\theta=2y+0.1$ (Eq. 1)

Use the solution condition for the Eq. (1), we obtain, $1+(0.2)^2 \geq (2y+0.1)^2 => |2y+0.1| \leq \sqrt{1.04} => \frac{-0.1-\sqrt{1.04}}{2} \leq y \leq \frac{-0.1+\sqrt{1.04}}{2}$.

Considering $y=\frac{-0.1+\sqrt{1.04}}{2}$, we have $\sin2\theta-0.2\cos2\theta=0.47 \Leftrightarrow \sin(2\theta-\beta)=1$, where β is the angle defined by: $\begin{cases} cos\beta = \frac{1}{\sqrt{1.04}} \\ sin\beta = \frac{0.2}{\sqrt{1.04}} \end{cases}$.

We have $\sin(2\theta-\beta)=1 \Leftrightarrow 2\theta-\beta=(\pi/2)+k2\pi \Leftrightarrow 0.5(\beta+(\pi/2)+k2\pi)$. On the other hand, $\theta \in [0; \frac{\pi}{2})$, we choose $\beta=\tan^{-1}0.2$; $k=0 \Rightarrow \theta \approx 0.88409$ rad($\approx 50.65^o$). So $v(\theta)$ reaches its minimum value when the angle $\theta \approx 50.65^o$.

5. **Step 5.** *Discuss and compare the results.*
   - The teacher guides the students to evaluate the proposed solution and generalize it to a similar situation (this is the final component of mathematical problem-solving competence).
   - Teachers give students the opportunity to compare and discuss answers to problems in groups, and then class discussions are held.
   - Students correctly interpret the mathematical solution back to the context of the real situation, making the mathematical results meaningful in practical terms.
   - Some activities that students can do in this step:
     - Present a summary of diagrams in the form of models and mathematical inferences from contextual problems to mathematical problems, using mathematical language instead of actual data.
     - The process of converting mathematical results to actual results.
     - Re-evaluate the whole process of giving mathematical solutions.
     - Check the accuracy of mathematical results by comparing mathematical products between individuals in a group and between groups.
     - Express mathematical results back to real context.

Switch from the mathematical result "the minimum value of $v(\theta)$ is 13.25" to the solution for the situation: Minimum pitching speed when pitch angle is $\approx 50.65^o$.

– The teacher asks the representative of the group of students to present a summary of the correct mathematical solution

Table 7. Information about control and experimental groups

Group	Subject	Number of students	High school	Teacher's name
Experiment	12 A2	37	Nong Cong 2	Nguyen Thi Trang
Control	12A3	39	Nong Cong 2	Nguyen Thi Thanh Ngoan

on the board, the remaining individuals observe and comment on the solution.

- The teacher comments and evaluates the solution and points out the errors or difficulties of students when implementing the solution.

The difficulty of students when finding solutions to solve problems lies in a number of problems:

- *Firstly*, students have difficulty in identifying suitable minimization objects.
- *Second*, students have difficulty when moving from the relationship between the facts or data in the situation to the relationship between mathematical objects.
- *Third*, students have difficulty in setting up accurate mathematical models because they have to mobilize and synthesize much different knowledge from trigonometric, algebra to calculus, which is not the strength of most students.
- *Fourthly*, another difficulty of students is finding the conditions for equality $v(\theta)=13.25$ when students choose way 1 or way 3. Because doing this to a trigonometric equation requires ingenuity in choosing the right solution in accordance with the request $\theta \in [0; \frac{\pi}{2})$.

6. **Step 6.** *Draw conclusions from the situation.*
   - Teachers guide students to draw lessons and experiences from solving situations, the mathematical knowledge they have acquired.
   - Students pointed out the note of applying knowledge about the maximum and minimum value of a function to solve real-life situations.
   - Teacher asks students to point out the difficulties or mistakes of students when performing math tasks in the situation of "shooting the basketball".
   - Students make their own formal process to find the solution for the solved situation and for similar situations.
   - The teacher makes the final conclusion about the mathematical knowledge that students need to acquire.

Results of Pedagogical Experiments

The problem-solving process according to the approach to RME was pedagogical experimented by us through 15 teaching periods with topics related to derivatives and integrals in the 12th calculus program at two high schools of Nong Cong 2 of Thanh Hoa Province in Vietnam.

Table 7 shows the information of the control class and the experimental class.

Pre-experiment results

Through the preliminary investigation, the experimental and control classes have the same learning results, below are the results of the test before the pedagogical experiment.

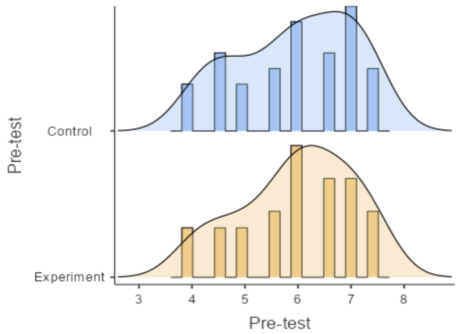

Figure 3. Representation of pre-test scores of experimental & control classes (Source: Author, using Jamovi 2.3.21 software)

Table 8. Descriptive analysis before experiment

	12A2	12A3
n	37	39
Mean	5.99	5.95
Median	6.00	6.00
Standard deviation	1.04	1.09
Minimum	4.00	4.00
Maximum	7.50	7.50
Shapiro-Wilk W	0.936	0.925
Shapiro-Wilk P	0.034	0.013

Table 9. Mann-Whitney U test before experiment

	Statistic	p-value
Mann-Whitney U	713	0.929

Comparison of pre-test results of two experimental classes and control classes are shown in **Figure 3** and **Table 8**.

The results of descriptive statistics of the scores of the control and experimental classes (**Table 8** show that the mean scores of the two classes are 5.99 and 5.95, respectively. Using the normality test of the Shapiro-Wilk distribution, both have $p<0.05$, so the distribution of scores of both experimental and control classes is not normally distributed. Using the Mann-Whitney U test, we have $p=0.929>0.05$ (**Table 9**), so we accept the hypothesis H_0: the learning results of the two experimental and control classes are equivalent (the median of both classes is 6.0) with a level meaning. In other words, before the experiment (with the influence of the researcher), the learning performance of both experimental and control classes was almost the same.

Post-experiment results

Qualitative results: At the end of the experiment, students have a more positive attitude in learning, they are more aware of the important role of mathematics in life.

Compared with the normal class, through the lesson designed according to RME, we observed:

- Students get to work more, they show excitement when participating in solving situations.

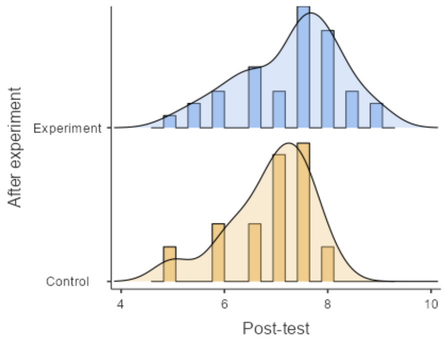

Figure 4. Representation of post-test scores of experimental & control classes (Source: Author, using Jamovi 2.3.21 software)

Table 10. Descriptive analysis after experiment

	12A2	12A3
n	37	39
Mean	7.20	6.78
Median	7.50	7.00
Standard deviation	0.961	0.849
Minimum	5.00	5.00
Maximum	8.50	8.00
Shapiro-Wilk W	0.928	0.857
Shapiro-Wilk P	0.020	<.001

Table 11. Mann-Whitney U test after experiment

	Statistic	p-value
Mann-Whitney U	519	0.032

- They become more confident and have a more positive attitude in solving learning tasks.
- Most of the students actively participated in the group discussion and were excited to be given the opportunity to present their ideas and opinions.
- Students actively support each other in learning, creating a friendly learning environment and lively atmosphere of the classroom.
- The interaction between teachers and students as well as between students takes place regularly with a "thick" frequency. Students who are weak in math also feel that they are shared, supported and helped when they are in trouble, thereby reducing their fear of learning math and being afraid to study math.

Quantitative results: At the end of the experiment, we let the control class and the experimental class conduct a posttest to assess the progress of students as well as test the effectiveness of fostering and improving problem-solving for high school students through teaching the topic of application of derivatives and the topic of integral primitives according to RME approach.

The results of the post-experiment test of the two pairs of control and experimental classes are shown in the following **Figure 4** and **Table 10**.

From the results of descriptive statistical analysis of Post-test scores of the two classes after the experiment, we see that the mean test scores of the control class and the experimental class are 6.78 and 7.2, respectively, the median of the control class was 7.0 while the median of the experimental class was 7.5. Using the test for normality of the Shapiro-Wilk distribution, in the experimental class, we see that $p=0.02<0.05$ (**Table 10**), so the score distribution of this class is not a normal distribution. Using the Mann-Whitney U test, we have $p=0.032<0.05$ (**Table 11**), so the hypothesis H_0: The learning results of the experimental and control classes are the same, rejected at the significance level. In other words, the learning achievement of the experimental class was better than the control class after the experiment.

DISCUSSION AND RECOMMENDATIONS

Fostering and improving problem-solving capacity for high school students through teaching some content of the topic calculus according to the real RME is our research direction and has been experimented with pre-test and post-test model. The teaching process is designed based on the RME approach including six steps (adjusted), specifically:

1. **Step 1.** *Teacher states the situation or context.*
2. **Step 2.** *Students understand the problem context/situation.*
3. **Step** 3. *Explain the problem/context.*
4. **Step** 4. *Solve the problem contextually.*
5. **Step** 5. *Discuss and compare the results.*
6. **Step** 6. *Draw a conclusion.*

Through teaching practice, the problem-solving step is still a difficult step for many students. Teachers spend a lot of time on this activity. One of the main reasons for that is that students' ability to detect and reflect on the existence of mathematical knowledge hidden through real-life situations is still limited and lacking. In the process of working individually and in groups, teachers had to spend a lot of time to guide students how to find and apply existing knowledge, even to receive and absorb new knowledge.

To do this well, there needs to be a synchronous cooperation between teachers and students. In addition, teachers should prepare a set of questions to give suggestions and guide students in necessary cases and at appropriate times to lead students to approach appropriate mathematical methods. During the process of students' group work, teachers need to constantly observe, monitor activities, interact and exchange information between students in each group, so that they can give timely suggestions or instructions to students to have a reasonable approach to solving the problem. In addition, teachers must be proactive about the teaching process and need timely interventions to avoid rambling and out-of-focus teaching.

The results of empirical research have shown that developing problem-solving competency for students through teaching calculus topics according to RME approach is feasible and initially brought about good results.

CONCLUSION

The content of the article has obtained a number of results:

1. Clarifing some issues about problem-solving competency and its components.
2. Propose a 6-step process of teaching calculus topics according to RME approach and illustrative examples.

3. Organize a pedagogical experiment to consider the feasibility and effectiveness of the designed teaching process, the experimental results have shown that the learning results of the experimental class students are higher than that of the control class students.

The results of the research contribute to supporting teachers in applying them to teaching practice in order to contribute to fostering and improving problem-solving skills of high school students.

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