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The effect of teaching integers with multiple representations on seventh grade student success and opinions

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ABSTRACT

The aim of this study is to determine the effect of teaching integers with multiple representations on 7th grade student achievement and student opinions. In this study, the method of action research was used. Data related to the study were collected through two types of sources called quantitative and qualitative data collection tools. One of the two classes with equal success was determined as the control group and the other as the experimental group. The control group explained the subject of integers with the traditional method. The subject of integers was explained to the experimental group with multiple representations. The achievement test developed for this research was applied to both groups as a pre-/post-test. Quantitative data were analyzed with SPSS program. Qualitative data were analyzed through descriptive analysis. The study revealed significant differences in favor of the experimental group as a result of the analysis of the post-test scores of the experimental and control groups. It is thought that including activities such as sliding integer ruler, integer animations, number stamps and number line, in the teaching, will contribute to students' learning.

Keywords: integer animations, multiple representations, number line, number stamps, sliding integer ruler

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INTRODUCTION

As children start their school life, they start dealing with numbers. Students get to know the natural numbers in primary school. Integers, which children learn after natural numbers, have a very important place in the world of numbers. It is necessary to learn the subject of integers well in order to make sense of directed numbers, to comprehend the operations with them, and to switch to rational numbers. In the 19th century, with the acceptance of axioms and postulates indicating the position of integers, integers took their place in the number group. In ancient times, it wasn't easy for people to accept negative integers because seeing concepts they'd never encountered before prolonged the acceptance process (Bolyard, 2005). This situation is also evident in today's mathematics education. Students who begin learning mathematics by learning about natural numbers often struggle to understand and accept integers when they encounter negative numbers, which they rarely encounter in daily life. In addition to the need for positive numbers in the teaching process of the subject of integers, not making students aware of the necessity of negative numbers causes students to have difficulties in comprehending integers (Altun, 2006). Although children do not have much trouble determining the place of negative numbers on the number line, they have difficulty comparing the sizes of these numbers. The basis of these difficulties develops the inconsistency of the use of the expressions "quantity" and "size" attributed to numbers in negative numbers in the process of teaching arithmetic. Their tendency to generalize also caused them to have difficulties in comprehending and making sense of these numbers (Işıksal-Bostan, 2009). Even many teacher candidates have misconceptions, especially in the subtraction of integers with the same sign (Rosyidah et al., 2021). Some students make mistakes due to carelessness and lack of understanding of operations on integers (Khalid & Embong, 2020). Students previously learned that the addition operation corresponds to the "+" symbol and the subtraction operation corresponds to the "-" symbol. The fact that these symbols undertake different missions in the integer numbers they encounter in the following years causes a confusion in the minds of the students. For example, referring to the same symbol more than once in the (+2) + (-5) procedure (the + symbol appears twice) makes it difficult for children to understand and learn these operations (Işıksal-Bostan, 2009). Accordingly, the 5th grade students who did not acquire the negative integer knowledge, performed the 4-6 process as 6-4; they tried to solve - 4 + 7 by forming it as 4 + 7 (Bolyard, 2005). The source of the problems

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faced by children is that the "-" sign indicates the direction of the operation and the number has both a quantity and directional meaning, and there is not an adequate representation to define negative numbers (Hativa & Cohen, 1995). For this, Haji Ismail et al. (2023) mention that using multiple representations when learning mathematics in early childhood is important in ensuring meaningful learning. Van de Walle (2012, p. 13) stated that "doing mathematics" in the teaching environment (in schools) is to be able to reveal representations in accordance with mathematical concepts and formulas in order to make sense of mathematics in the world we live in.

The views on multiple representation were introduced as an approach "Lesh multiple representations translations model" (LMRTM) (Lesh et al., 1987). Various means are needed to reflect on the thoughts in mathematics or to transfer these thoughts to others. Among these means are external representations and their special case of multiple representations. Adu-Gyamfi (1993) explained the representations as the graphic model (vertical axis system) that compares multiple quantities, verbal model (written words), painting (drawing, schematic drawings, etc.), and chart model (table, ruler, etc.). There are eight types of structures in the structure of external representation displays. These are concrete representations, graphics, written symbols, tables, algebraic relations (equations), diagrams (diagram-based drawings), experience-based metaphors, and verbal language (Lesh & Doerr, 2003). Nakahara (2008) examined the multiple representations in mathematics education with his studies on Lesh et al.'s (1987) LMRTM under five headings:

1. Symbolic representations: Numbers, letters, or symbols involved in mathematical transformations.
2. Linguistic representations: The concepts are shown in Turkish, or in English
3. Visual representations: The figure, diagram, or graphs shown about a topic.
4. Manipulative representations: Number stamps, fraction bars, and pattern blocks, etc. in the teaching process.
5. Realistic representations: They are classified as materials related to the real situation and concrete tools.

When the literature is examined, it has been revealed that multiple representations reduce student errors, eliminate misconceptions, have positive effects on attitudes and confidence (Akkoç, 2005; Bruno & Martinon, 1999; Çıkla-Akkuş, 2004; Hayes & Stacey, 1996; Wichaidit & Wichaidit, 2016), contribute to increasing achievement and meaningful learning (Çetin, 2017), and develop critical thinking skills (Abdurrahman et al., 2019). In his study investigating the effects of an enriched learning environment on mathematical thinking and attitudes, Erdem (2015) found that teaching in an enriched learning environment (a learning environment where different models are used) significantly improved the mathematical thinking of the participants. In a study examining students' ability to switch between multiple representations, Gürbüz and Şahin (2015) found that students were more successful with certain representations but experienced significant difficulties with others, particularly transitions to graphs. Özgün-Koca (2004) found that technology-supported multiple representations had positive effects on students' ability to understand linear relationships and switch between representations. From here, we can say that technology support in multiple representations has a significant effect on improving transition skills.

In our study, we believe that technology-supported multiple representations can contribute to the concretization of abstract concepts and the interpretation of operations performed in the teaching of integers. It is thought that learning integers in this way may contribute to overcoming the difficulties encountered in these numbers. It is thought that computer-aided animations will positively affect learning in terms of their vibrant, attractive appearance, and animating abstract concepts. Computer aided animation models in the study can contribute to understanding the difference between number signs and operation signs thanks to animated animation in the computer environment. As a matter of fact, in some studies, it is stated that the use of animation in the teaching increases students' success develop the course and gives students a positive perspective towards the lesson (Çepni et al., 2006; Rowe & Gregor, 1999).

The mathematics curriculum in the Turkish Education system focuses on understanding the various representation models of the concepts in mathematics and understanding the connections established between these representations and making use of data and tools that enable children to comprehend the relationships in mathematics (Ministry of National Education [MEB], 2013). Teaching integers is handled with the representation of the number line and number stamps in the textbooks of the MEB (2013), allowing the same concepts and operations to be seen through different representations. However, unlike these representations, there are no examples of multiple representations created with integer rulers, animations, and number line analogies in the textbooks. Teaching integers with multiple representations will guide teachers. They will have an idea of choosing the materials they will use on integers in their classes. With the help of the "sliding integer ruler" developed for this study, it can help them perceive by touching this material and moving the arrow on it. In addition, with the help of this representation type, it is thought that the contrast concepts and operations in integers can be beneficial for them to make sense by touching them. This study will shed light on new research and researchers in developing multiple representations. In addition, the use of the sliding integer ruler and the computer animation in classrooms may become widespread. This will help both teachers and students in learning integers.

Purpose of the Study

The aim of this study is to investigate the effect of instruction created by using multiple representations (sliding integer rulers, number line animations, analogies and other representations) on integers in the learning environment. The problem question of the study was determined as "What is the effect of teaching created by using multiple representations on integers on the learning environment?" Sub-problems of the study:

1. Does teaching with multiple representations of integers have an effect on the 7^{th} grade student achievement?
2. What are the opinions of the 7^{th} grade students on teaching integers with multiple representations?

METHODOLOGY

Research Design

The design of this research was determined as "action research". Action research involves a planned and programmed study process, with the aim of identifying and changing forward-looking practices, in

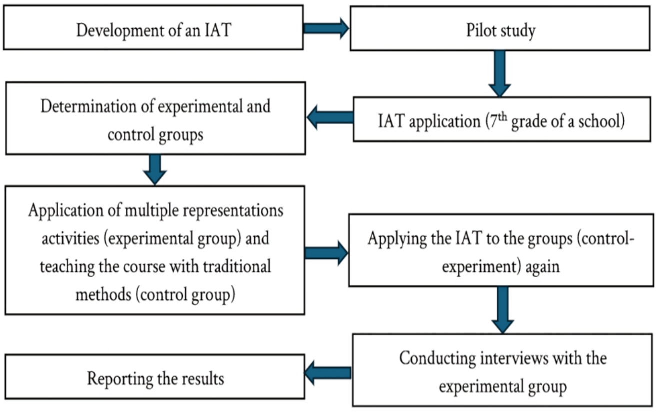

Figure 1. Action plan for research (Source: Akyüz, 2019)

which the teacher is the leader (Ferrance, 2000). Both qualitative research methods and quantitative research methods can be used in action research method (Aksoy, 2003). If we look at this study, which includes both qualitative and quantitative data, from another perspective, we can also evaluate this study as a quasi-experimental design with paired control groups including pre- and post-test repeated measures (pre- and post-test). Qualitative data consists of observations and interviews, while quantitative data consists of pre- and post-tests of the experimental and control groups. During the study, the researcher conducted the study in both the experimental and control groups. The action plan of the research is given in **Figure 1**.

Because 7^{th} grade students who were ready to learn integers were selected for the study, the "purposive sampling" method, a non-random sampling method, was used. Because two classes with equal academic achievement were selected from these classes, the "similar (paired and homogeneous) sampling" method, a purposive sampling method, was used (Patton, 2014). Furthermore, this study employed "convenience sampling," an economical, easy, and practical method (Golzar et al., 2022). A small sample size was formed because the schools and students in the study were selected from individuals who were easily accessible to the researcher and who volunteered to participate in the data collection process. Given the limited time, accessibility, and application conditions, these methods were considered convenient sampling methods.

Participants

The pilot study was applied to 14 students, who were randomly selected from 7^{th} grades with medium academic achievements and socio-economic levels. Main application was made with the 7^{th} grade students at a different school. Before the application, an evaluation was made to determine the two classes whose academic levels were close to each other. In order for the students to participate in the study to be close to each other in terms of cognitive structure, two branches with close academic achievement levels were selected in line with the opinions of the school administrators, the teachers who taught the 7^{th} grade classes, and the school counsellor. After that, by drawing lots from these two classes, a total of 23 students, 14 girls, and 9 boys, studying in 7/E class, were determined as the experimental group and a total of 20 students, 11 boys, and 9 girls studying in 7/D class were determined, as the control group. Data of mainstreaming students (2) in the experimental and control groups in the study were not included in the evaluation. Main study was conducted with 41 students.

1) [0-4]=? 2) [3-8]=?
3) [-3+-8]=? 4) [-3+3]=?
5) [-8+3]=? 6) [-2+5]-[-3]=?
7) [3-7]+[-5+-3]=? 8) [7-(-3)]-[6-(-2)]=?
9) [-3+(2-+4)]+[-1-(-6)]=? 10) [(-2)-(-5)+4]-(-2)=?

Figure 2. Questions of the achievement test (Source: Akyüz, 2019)

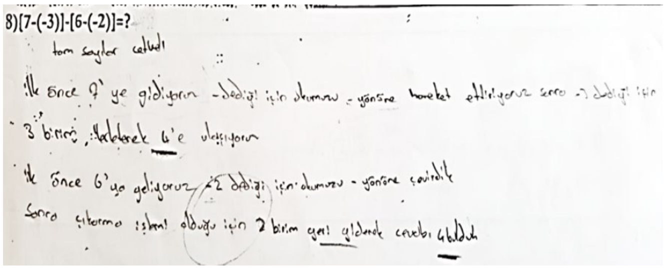

Figure 3. Score 0 (Source: Akyüz, 2019)

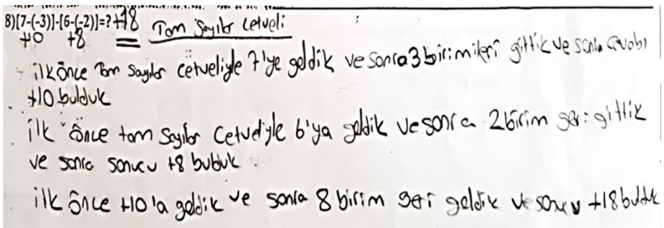

Figure 4. Score 1 (Source: Akyüz, 2019)

Data Collection Tool

Data related to the study were collected through two types of sources called quantitative and qualitative data collection tools.

Interviews were conducted face-to-face with students for qualitative data. Immediately after the interview, the students' opinions were recorded on the interview forms.

Sample interview questions are given below:

1. In which of the multi-representation activities used in the implementation process did you have difficulty?
2. In which multiple representation activities or activities do you think you learned better about integers? Why?

Integers achievement test (IAT) was used as a quantitative data collection tool unit, which was created by the researcher and applied to the participants as a pre-test at the beginning of the application process and as a post-test at the end of the application process (**Figure 2**).

The IAT, scoring criterion created by Çetin (2017) was used. The scoring criteria used on the test are given in **Figure 3**, where 0 points means o solution or wrong or no results or wrong.

1 point means solution is partially correct, result is wrong. Modelling or math sentences are partially correct, result is incorrect (**Figure 4**).

2 points means solution is correct, result is wrong. No modelling or math sentence is set, the result is correct (**Figure 5**).

3 points means the solution is correct, the result is correct. The modelling or math sentence is established correctly, the result is correct (**Figure 6**).

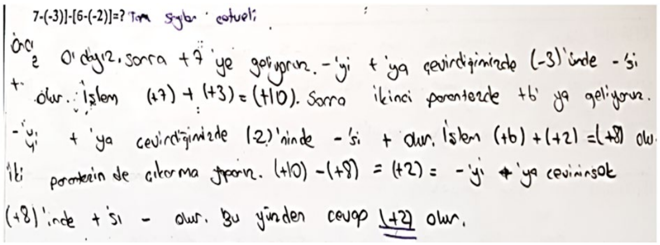

Figure 5. Score 2 (Source: Akyüz, 2019)

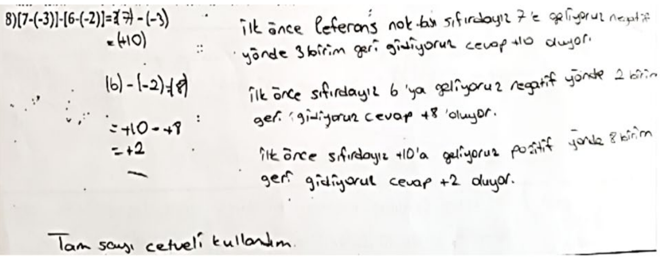

Figure 6. Score 3 (Source: Akyüz, 2019)

Process

Before starting the main study, pre-test was applied to the experimental and control groups in order to determine the success of the participants about integers in the second term of the 2017-2018 academic year. The lessons in the control group were taught with traditional teaching method (direct instruction) adhering to the MEB (2013) textbook. In the experimental group, lessons were taught by making use of multi-representations and related worksheets. The learning contents of the control and experimental groups are the same and are limited to the acquisition of "performing addition and subtraction of integers".

The multiple representations were created by examining the relevant literature and considering the verbal symbols, manipulatives, written symbols, diagram-based visuals, and real-life simulations, which are present in Lesh et al.'s (1987) LMRTM model. In addition, each multiple representation contains objects (tools), verbal expressions and formulation representations in Janvier's (1987) simulation model represented as a star. Equality-quantitative representations and directed models, which are used in the subject of directed numbers and which are in the literature, are mentioned.

The animation was developed based on inspiration from the work of Güveli and Güveli (2015). It took about 3 months to develop the animation and adapt it to the technology environment. Animation is suitable for all representation classifications (input-output representations, symbolic-visual representations, and internal-external representations)

During the application process, the application was made with number stamp, number line, sliding integer ruler, and animation multiple representation activities, respectively. Before the application was made, the participants were informed about the multi-representation activity to be held in a certain period of the first lesson, and they were informed about how the activities work. In the following process, the multi-representation activity was applied to the participants. During the course of the activity, attention was paid to choosing questions from daily life in order to direct the curiosity of the participants to the event. Collaborative groups in the experimental group were asked to create similar examples and solve them cooperatively.

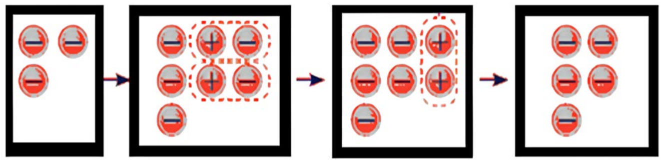

Figure 7. Modeling the operation (-3) - (+2) with number stamps (Source: Akyüz, 2019)

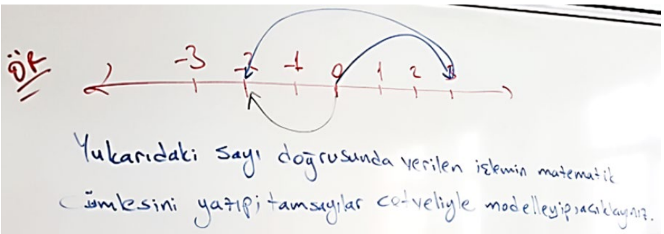

Figure 8. A frame from the number line activity in the implementation process (Source: Akyüz, 2019)

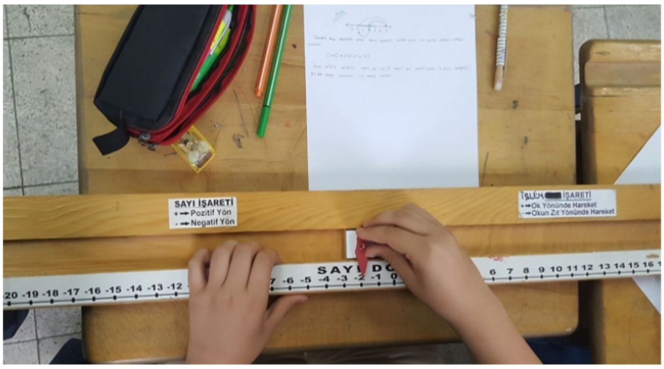

Figure 9. A section from the implementation process of the sliding integer ruler activity (Source: Akyüz, 2019)

Counting stamps are stamps that are made up of negative stamps with "-" signs and positive stamps with "+" signs, which are also found in MEB (2013) textbooks, and are used to create a neutral state by combining negative and positive stamps. For example, (-3) - (+2): models to remove +2 stamps from -3 in a box. Since we cannot subtract +2 stamps from -3 stamps, we add 2 neutral (+ 2, -2) stamps and remove +2 stamps, -5 stamps remain in the box (**Figure 7**).

The number line model used in the application was drawn on the board and the students were asked to practice on it (**Figure 8**).

The integers sliding ruler prepared by the researcher provided the students with the opportunity to create the concept of "+", "-" direction, and "forward" and "backward" movement that the students can move by touching. For example, for (+3) + (-5): we move the slider from the starting point (0) to +3, then we move it 5 units in the - direction (**Figure 9**).

Integer animation was designed as an animation that allows students to interact, move the animation character by giving direction, and receive feedback on their answers (**Figure 10**). In this animation, the functions of the number sign and the operation signs are explained "motion" and "direction" analogies. At the last stage in animation, modeling a process was given and the following questions were asked:

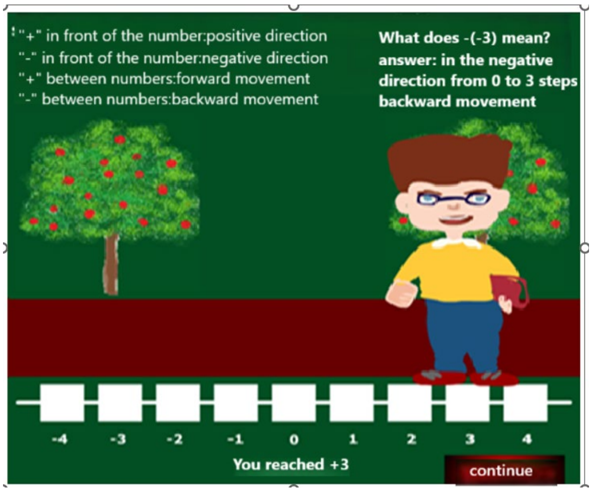

Figure 10. A section from the multiple representation of integers' animation (Source: Akyüz, 2019)

1. Where was he who moved in the beginning?
2. Which way did he turn his direction?
3. How many steps did he go?
4. Did they go forward or backward?
5. Now write the arithmetic operation.

In this process, worksheets were prepared using the mathematics textbooks of MEB (2013). These worksheets were used as an additional tool to assist with multiple representations and as a knowledge gathering tool during the implementation phase.

At the end of applications, interviews related to the activities were conducted for the experimental group students. After, a post-test (IAT) was applied to both groups in order to compare the academic achievements of the experimental and control groups.

All applications in the control and experimental groups were completed in 16 hours, which is the time specified for the subject in the secondary school mathematics curriculum program. Application steps and duration are given in **Table 1**.

Data Analysis

In the process of the IAT and interview items, a field educator, an assessment specialist, and five mathematics teachers experienced in their profession (one with 10 years, one with 12 years, two with six years, one with 15 years of experience) were consulted. Ideas were also obtained from these people about whether the items in the test and interview require reasoning, whether they are suitable the level of the students to be applied, and whether they are related to the issue of integers. Some changes were made to the test and interview items in line with the suggestions of an expert Turkish language teacher in terms of the compatibility of the draft questions in IAT with Turkish language rules.

The answers given to all items in the IAT were scored independently by two math teachers. As a result of scoring, Spearman correlation coefficient ® was determined to conclude the consistency between the two raters. The consistency between the scores made by the two raters independently was found to be 99% ($p = .000$, $r = .944$).

Item analysis was carried out in order to determine the items to be included in the main study (**Table 2**). While Bayrakçeken's (2008) item analysis is used in item analysis of mostly objective tests and especially multiple-choice tests, it is also used in item analysis of written exams. The following procedure was carried out item analysis of the written exam. For each item, M is item score, A is total score of the upper group, B is total score of the subgroup, N_1 is number of upper groups, N_2 is number of subgroups, $P_{difficulty}$ is item difficulty index, $D_{distinctiveness}$ is item distinctiveness, $P_{difficulty} = (A + B)/([N_1 + N_2] \times M)$, and $D_{distinctiveness} = (A - B)/([(N_1 + N_2)/2] \times M)$.

The Cronbach's alpha coefficient of IAT was determined as 0.84. It is taken into consideration that the reliability level of the measurement units used in research in education should be at least 0.70 (Tezbaşaran, 2008). Therefore, it is seen that the reliability level of the prepared achievement test is high.

The data obtained from the participants' responses to the IAT in the pre- and post-tests were analyzed using the SPSS-20 package program in a computer environment using appropriate statistical analysis techniques in the light of the characteristics of the data.

The non-parametric Mann Whitney-U test was conducted to determine whether there was a significant difference between the scores of the experimental and control group participants in their pre- and post-tests. The non-parametric Wilcoxon paired pairs signed ordinal numbers Test was conducted to determine whether the post-test scores of the experimental and control groups differed significantly according to the pre-test scores.

Table 1. Application steps of the control and experimental groups

Dates	Learning content	Control group		Experimental group	
		Source and method	Duration	Source and method	Duration
1st week (02.04-06.04)	Pre-test		40 minutes	Pre-test	40 minutes
2nd week (09.04-13.04)	Recognizes whole numbers and represents them on the number line	MEB textbook, presentation	4 hours	Worksheets, group work	4 hours
3rd week (16.04-20.04)	Compares and orders integers	MEB textbook, presentation	3 hours	Worksheets, group work with the number line model	3 hours
4th week (23.04-27.04)	Performs addition and subtraction of integers and solves related problems	MEB textbook, problem-solving	3 hours	Group work with worksheets and counting stamps	3 hours
5th week (30.04-04.05)	Understands that subtraction of integers means adding with the opposite sign of the minuend	MEB textbook, question-answer, presentation	3 hours	Group work with worksheets, number lines, sliding integer ruler	3 hours
6th week (07.05-11.05)	Uses the properties of addition as strategies for fluent processing	MEB textbook, presentation, question-answer, problem-solving	3 hours	Worksheets, multiple representation model with animation support	3 hours
7th week (14.05-18.05)		Post-test	40 minutes	Post-test	40 minutes
8th week (21.05-25.05)				Interview	+30 minutes

Table 2. Item analysis

Items	Groups (n = 14)	Total score	Item difficulty	Interpretation	Item discrimination	Action
1	Upper group	21	0.71	Very easy	0.57	Very good
	Sub-group	9				
2	Upper group	20	0.71	Very easy	0.48	Very good
	Sub-group	10				
3	Upper group	18	0.71	Very easy	0.29	To be revised
	Sub-group	12				
4	Upper group	21	0.71	Very easy	0.58	Very good
	Sub-group	9				
5	Upper group	21	0.79	Very easy	0.48	Very good
	Sub-group	11				
6	Upper group	17	0.60	Easy	0.43	Very good
	Sub-group	8				
7	Upper group	19	0.79	Very easy	0.24	To be revised
	Sub-group	14				
8	Upper group	13	0.31	Moderate	0.62	Very good
	Sub-group	0				
9	Upper group	17	0.50	Very easy	0.71	Very good
	Sub-group	4				
10	Upper group	15	0.48	Moderate	0.48	Very good
	Sub-group	5				

Table 3. Comparison of experimental and control group pre-test scores

Application	Group	N	X	U	z	p
Pre-test	Experimental	22	1.30	168.5	-1.06	0.289*
	Control	19	1.07			

* $p > 0.05$

Table 4. Comparison of pre- and post-test scores of the control group

Group	Application	N	X	Standard deviation	z	p
Control	Pre-test	19	1.07	0.69	-2.30	0.02*
	Post-test		1.08	0.70		

* $p > 0.05$

At the end of the multi-representation activities, interviews were conducted with the participants in the experimental group to reveal the students' ideas about the multi-representation-supported learning environment. The interviews with the participants were conducted in the education room. The answers given by the students to the questions were noted in the form by the researcher during the interview with the students. The obtained data were analyzed based on the descriptive analysis method (Yıldırım & Şimşek, 2011). The categories for each question in the interview form were determined one by one, and an "x" symbol was put in order to represent each repeating category in the answers given to the same item. The resulting categories were collected and transferred to scoreboards, and frequencies were formed.

Validity and Credibility

To increase the validity and reliability of this study:

- A pilot study was conducted.
- Expert opinions were obtained to ensure content validity during the test development and action plan preparation.
- Item analysis of the test items was conducted, and the item difficulty index and item discrimination index for each were calculated. Reliability was found 0.84.
- The time period specified in the mathematics curriculum was adhered to for the administrations in both the control and experimental groups.
- The objectives and learning contents specified in the mathematics curriculum were the same for the administrations in both the control and experimental groups.
- Observations were made, notes were taken, and records were kept at every stage of the administration.
- The sample, data collection tools, and administration stages are described in detail in the study.

FINDINGS

Findings on the Effect of Teaching with Multiple Representations of Integers on 7th Grade Student Achievement

According to the IAT applied to these two groups, it was determined that there was no significant difference between these two groups in terms of success. The analysis results of the pre-test results of the experimental and control groups are given in **Table 3**.

As seen in **Table 3**, there is no significant difference between the control and experimental group pre-test scores according to the results of the Mann Whitney U test ($U = 168.5$, $p > 0.05$). Wilcoxon test analysis was performed to determine whether the change in the scores obtained from the participants' success test before and after the experiment would make a significant difference, and it is given in **Table 4**.

Table 5. Comparison of pre- and post-test results of experimental group

Group	Application	N	X	Standard deviation	z	p
Experimental	Pre-test	22	1.30	0.73	-4.11	0.00*
	Post-test		2.38	0.58		

* $p < 0.05$

Table 6. Comparison of post-test scores of experimental and control groups

Application	Group	N	X	U	z	p
Post-test	Experimental	22	2.38	74	-3.54	0.00*
	Control	19	1.08			

* $p < 0.05$

Table 7. Representations that students have difficulty with and reasons

Rank	Students' views	Frequency
1	It's hard to subtract number stamps.	3
2	It feels difficult to add the zero pair in number stamps.	3
3	The stamps of the number stamps get confusing.	12
4	I am having difficulties because I cannot perform the operation modelled on the number stamps.	1
5	I have difficulty because there are long operations on the number line.	1
6	I am having difficulties because I cannot grasp the number line.	1
7	I had no difficulty.	3
8	I'm having a hard time understanding the movement on the number line.	1
9	I had difficulties with directions in animation.	1

Table 8. Representations that students found more effective in learning integers and reasons

Rank	Students' views	Frequency
1	I learned it easily as I could do the integer ruler by touching it.	4
2	I learned the integer ruler easily because everything was clear.	6
3	I learned it easily because of the arrow functions on the integer ruler.	2
4	I learned it easily because the integer ruler was fun.	6
5	I learned it easily as the integer ruler was interesting.	2
6	I learned it easily because the integer ruler was easy to understand.	4
7	The number line was easy to understand.	1
8	I learned it easily because the number line was fun.	1
9	I learned it easily because the integer animation was easy to understand.	1

As seen in **Table 4**, there is no significant difference between the pre-/post-test scores of the control group according to the Wilcoxon test performed ($z = -2.10$, $p > 0.05$).

As can be seen in **Table 5**, according to the results of the Wilcoxon test, there is a significant difference between the pre- and post-test scores of the experimental group ($z = -4.11$, $p < 0.05$). Looking at the averages, it can be said that this difference is in favor of the post-test.

As seen in **Table 6**, the average of the measured post-test scores of the experimental group students ($X = 2.38$) is higher than the average of the measured post-test scores ($X = 1.08$) of the students in the control group. In addition, as can be seen from **Table 6**, there is a significant difference between the control and experimental group post-test scores according to the results of the Mann Whitney U test ($U = 74$, $p < 0.05$). When the average values are examined, it is seen that this difference is in favor of the experimental group students. In other words, the teaching of multi-supported integers with the students in the experimental group had a positive effect on the success of the participants in the experimental group in integers.

Findings of 7th Grade Students' Views on Teaching Integers with Multiple Representations

In the interviews with the students, responses to the questions were converted into frequencies and are given in **Table 7**.

Within the framework of the above related question, there were no students who stated that they had difficulty with the multiple representation of the sliding integer ruler. Students having difficulty in the integer animation activity correspond to 3.84% of the experimental group. It is seen that this ratio is 11.57% on the number line and 73.05% on the number stamps. Examples from student(s) opinions:

> "I had the most trouble with counting stamps. Because adding and subtracting zero pairs was a very difficult way" (S1).

> "I had difficulties in all models except the sliding integer ruler and animation …" (S8).

Table 8 shows that integer ruler, the number line, and the animation of integers made it easy to learn according to 24 views, 2 views and 1 view, respectively. Based on these results, it can be stated that, according to the experimental group, the most convenient activity in learning during the application process is the integer ruler, then the number line, and then the integer animation. Examples from student opinions:

> "The scale of integers. Because everything is clear on that board (representation). I can do the operations there more easily and properly, so I can do this activity more easily" (S14).

Table 9. Students' opinions for each multiple representation activity used in teaching the subject of integers

Activities	Students' views	Frequency
Number line	It provides convenience.	3
	The subject is better comprehended.	2
	I find it unnecessary.	1
	That's sounds tiresome.	1
	I find it ideal to be modeled.	1
	Sounds fun.	1
	It was difficult to show.	1
	It sounds difficult.	3
	It sounds confusing.	1
	It was extensive.	2
	It shows the process flow.	2
Integer ruler	Looks like the number line.	1
	It was not complicated.	2
	It provides permanence.	2
	It provides effective learning.	1
	I can touch it.	2
	It was fun.	6
	Multirole.	2
	Provides good understanding.	3
	Makes us understand easily and comfortably.	13
Number stamp	It was difficult.	13
	It was useful.	2
	It was confusing.	4
	It was very fun and enjoyable.	1
	It was not permanent.	1
	It was boring.	1
	It was unnecessary.	1
Integer animation	It looks like real life.	1
	It looks like an integer ruler.	2
	Sounds fun.	3
	It makes us understand well.	2
	It was unnecessary.	4
	It makes us understand well.	2
	Interesting.	2

> "I learned better about the number line and the sliding integer ruler. Because it's a simple subject it is not confusing. That is why, thanks to its simplicity, I learned this subject better" (S9).

> "Animation model was good. it looks like real life" (S3).

Considering the students' opinions about the activities in **Table 9**, all of the class mentioned the positive features of the integer ruler, the positive features of the animation for integer except four, the positive features of the number line, and almost all of the class mentioned the negative features of the number stamps. Among these, it can be said that the most preferred, benefited, and adopted activity by the students during the application process is the integer ruler. It can be stated that the least preferred non-retention activity is number stamps.

RESULTS, DISCUSSION, AND RECOMMENDATIONS

The Effect of Teaching Integers with Multiple Representations Student Success

In the study, the scores that the participants got from the integer achievement test (pre-test applied before the application and the post-test performed at the end of the application) were compared. As a result of the comparison, a significant difference ($p < 0.05$) emerged in the comparison of pre- and post-test scores. It was determined that the activities consisting of multiple representations done with students studying in the secondary school grade 7 increased the academic success of the participants (experimental group) in the subject of integers. In parallel with this result, there are various studies conducted internationally and locally (Çetin, 2017; Çıkla-Akkuş, 2004; Erdem, 2015; Hayes & Stacey, 1996; Marrades & Gutierrez, 2000; Özgün-Koca, 2004; Palmiter, 1991).

Regarding Teaching Integers Using Multiple Representations According to Student Opinions

Many students stated that they had difficulty in number stamps activity, very few students reported that they had difficulty in the number line activity, and one student mentioned having difficulty in the animation of integers. Based on this, it is seen that the most difficult activity in the application process compared to the experimental group is number stamps. When the difficulties experienced in the number stamps are examined within the framework of the expressions of the students, it is seen that the number stamps are confusing, and they had difficulty in creating zero pairs (neutral stamps) and subtracting. The difficulties faced by students in number line, integer animation, and sliding integer ruler activities are minimal compared to number stamps. Those who had difficulties in integer ruler and integer animation activities during the application process correspond to approximately 4% of the experimental group. It is seen that this ratio is about 12% on the number line and about 73% on the number stamps. When the results of the related study are compared with the study of Bozkurt and Polat (2011), it is seen that similar and different results have emerged. In the conclusion of the study conducted by Bozkurt and Polat (2011), it was determined that there was a difference in the opinions of the teachers on the question of whether number stamps are effective and that teachers did not support the use of number stamps in modelling some operations. Therefore, the inference to be obtained from the students' opinions in the relevant study shows similarities that about 73% of the students have difficulties in the number stamps. Contrary to this result, in a study, 30 7th grade students were more successful in adding integers by modeling them with number stamps (Demir & Tekin, 2023). Opinions of most students show that sliding integer ruler ensures good learning. In this context, based on these results, it is seen that integer ruler made from the board supports students' learning at a good level. The opinions expressed about integer rulers are as follows: It is similar to the number line, it is not complicated, it provides permanent learning, it provides effective learning, it is tactile, it is fun, it is multi-purpose, and it provides easy and comfortable understanding. Considering all these features, it can be concluded that all of them are positive, and the presence of such positive features in a model is a very useful model. it can be concluded that this activity, which has such beautiful features, will positively affect the students' perspective on integers and contribute to their learning. The findings of the quantitative data also support this. It is because, when looking at the answers of the students in the experimental group, it is seen that most of the questions were solved by the multi representation of the integer ruler. The fact that the final success of the students in the experimental group was about 79% also supports this idea. it is similar to the view that it is indispensable to make use of the number line representation in increasing the understanding of the concept of operations and number in mathematics (Bolyard, 2005). Children see the number line model starting from the first grade of primary school. This makes them think that they grasp the number line model more quickly and understand it better, and therefore it can be said that the integer ruler representation is an important factor in students' learning well. According to the results of the data in the study, the students mostly expressed positive opinions such as the animation of integers resembling real life, a scale of integers, fun, and providing good understanding. Based on this, it can be said that integer animation can be considered as an alternative teaching model in teaching the subject of integers. This result shows similarities with the result of Güveli and Atalay (2017) studies that students are willing to learn with animations, it is beneficial to learn with animations, especially in abstract lessons, and animations improve their problem-posing skills. According to the experimental group, the best activity that ensures good learning in the application process is the integer ruler activity, the animation of the integers and the number line. Studies show that concrete materials and virtual learning objects are needed in lessons (Bohan & Shawakwer, 1994; Bolyard, 2005; Ross & Kurtz, 1993). It would be beneficial to include a concrete material such as integer ruler and a virtual learning tool such as integer animation in the teaching process as a result of teaching with activities based on multi-representation activities. We think that this study will contribute to mathematics for educators.

Suggestions

It was found that most of the students had difficulty in understanding the multiple representation of the number stamps. Therefore, it is thought that including activities such as sliding integer ruler integer animation, and number line, which can be alternative to number stamps in the teaching process of teachers, will contribute to students' learning. It is seen that the lesson process with multi-representation-supported activities is a very positive process compared to traditional lessons. Teachers should use multi-representation in their classrooms. This study is limited to the learning outcome of addition and subtraction in integers. The sample size is limited to 41 students. However, this number is believed to provide sufficient data to obtain and evaluate findings. Sliding integer ruler and integer animation used for this study can be developed and how to use in other acquisitions (for example, multiplying and dividing integers) can be investigated.

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