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The Effects of Balance Model and Algebra Tiles Manipulative in Solving Linear Equations in One Variable

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ABSTRACT

The purpose of the study is to compare the achievements of Basic 9 pupils, taught using balance model and algebra tile, with control group who were taught without any manipulative in linear equations involving one variable. The research design is quasi-experimental methods, specifically the nonequivalent group's pre-test–post-test control or comparison group designs, with all Basic 9 pupils of Bimbilla Municipality forming the population. The sample used was 120 Basic 9 pupils comprising of 70 males and 50 females. Multi-stage cluster sampling procedures was adopted to select the schools in the various circuits in the Municipality and three intact classrooms for the study. The instrument used was achievement test with 10 essay questions on linear equations in one variable. The independent variables were teaching methods using algebra tiles, balance model and traditional method. While the dependent variable is the achievement scores from the pre- and post-test. The results of the ANOVA and independent sample t-test of the study revealed that, there was significant difference between the balance model, algebra tiles groups compared to the control group in post-test analysis, which favored the balance model and algebra tiles groups with large effect size. The study recommends that Basic School mathematics teachers should use the balance model and algebra tiles in the teaching and learning of linear equations in one variable because it will help improve performance in mathematics.

Keywords: balance model, algebra tiles, linear equations, one variable, ANOVA, independent sample t-test

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INTRODUCTION

In Ghana, algebra is studied at all levels of the educational ladder. In the 2018 new curriculum, it is included in the early grade (KG to Basic 3), upper grade (Basic 4-6) and JHS grade (Basic 7-9), with notable courses like (1) *Introduction to Learning and Applying Number and Algebra*, with topics like recognizing and developing patterns, using numbers and number operations, properties of numbers, the concept of sets, number bases and modulo arithmetic, and algebraic expressions. In addition, student teachers will explore operations on algebraic expressions, apply mathematical properties to algebraic equations, and functions. And (2) learning and teaching and applying further algebra topics are binary operations, binomial expansions, quadratics and other polynomials, series and sequences, matrices, simultaneous equations, and introduction to linear programming. The algebra courses at the colleges of education in Ghana are designed to help student teachers to develop demonstrable confidence to explain or justify their thinking based on their observations, the patterns they have observed, or what they know about numbers and algebraic relationships. As they do so, they develop confidence in teaching-related topics in the course *Number and Algebra* to their pupils at the respective grade levels.

Algebra course is grounded on mathematical content on one hand and the strategies and learning experiences in doing mathematics on the other hand. The mode of delivering the *Algebra* course in colleges of education in Ghana is through face to face, practical activity, work-based learning, seminars, independent study and e-learning.

The assessment methods include coursework (assignments, quizzes, project works, and presentation) and end of semester examination to provide a comprehensive outlook of student teachers' competencies and skills. These modes of deliveries seek to achieve the aims of the new curriculum which are to instill in new teachers the nation's core value of honesty, integrity, creativity, and responsible citizenship and to achieve inclusive, equitable, high-quality education for all learners in line with sustainable development goal (SDG) four.

To be able to teach algebra well then, teachers need to use the required manipulatives. Manipulatives such as algebra tiles and balance models have been emphasized in the 2018 mathematics new curriculum of Ghana as far as teaching of linear equations in one variable is concerned. Hence, examining its impact in solving linear equations with one variable is in the right direction in this study.

Figure 1. Balance model displaying x+3=7

LITERATURE REVIEW

According to Dugopolski (2006), linear equation in one variable is defined as an equation that can be written in the form of "ax+b=c", where "a", "b", and "c" are real numbers and "a≠0". Learning and solving linear equations problems is so significant that it has become prerequisite to the study of algebra (Dugopolski, 2002). A good grasp of linear equation help improves pupils' performance in application of sets (two set problems), powers of numbers, numeration systems, plane geometry, polygons, and many more mathematical topics (Mathematical Sciences Education Board, 1998).

Cass et al. (2003) posit that manipulative can be described as concrete objects that learners can physically assemble or use in a way to denote several mathematical relationships. They also see manipulatives as apparatuses that enable learning of a new mathematical skill a real process. Teachers purchase ready-made materials from stores or prepare the manipulatives themselves, or sometimes allow pre-service teachers to prepare them as part of their portfolio building for grades or marks in colleges of education in Ghana. Manipulatives can be both concrete and virtual. Swan and Marshall (2010) have stated that "a mathematics manipulative material is an object that can be handled by an individual in a sensory manner during which conscious and unconscious mathematical thinking will be fostered" (p. 14). Some examples of mathematics manipulatives are as follows: cuisenaire rods, tangrams, geoboards, pattern blocks, algebra tiles, balance model, fraction strips, and base-ten blocks. According to Heddens (1986), Picciotto (1998) and Sebesta and Martin (2004), student benefits from the use of manipulatives, as follows:

1. Verbalizing mathematical thinking,
2. Discussing mathematical ideas and concepts,
3. Relating real-world situations to mathematical symbolism
4. Working collaboratively,
5. Thinking divergently to find a variety of ways to solve problems,
6. Expressing problems and solutions using a variety of mathematical symbols,
7. Making presentations,
8. Taking ownership of their learning experiences, and
9. Gaining confidence in their abilities to find solutions to mathematical problems using methods that they come up with themselves without relying on directions from the teacher.

Vlassis (2002) explained that balance model is a balance comprising of a lever with two equal arms and a pan suspended from each other. A balance model is also known as balance scale or beam balance. Its operation depends on pull of gravity used to compare weights, not masses. The unknown mass is put in one pan and standard masses are then added to the other pan until the beam is as close to equilibrium as possible. Again, Vlassis (2002) posits that a linear equation is comparable to a balance model in which the left-hand side is equal in value to the right-hand side. **Figure 1** depicts a picture of balance model displaying x+3=7.

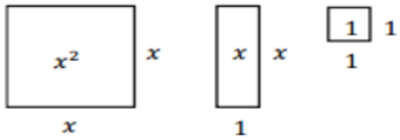

Figure 2. Algebra tiles

For equality to be ensured, whatever is done to one side must be replicated to the other side. Vlassis (2002) further indicated that balance model has proven to be very effective in helping pupils comprehend the equality between both sides of an equation.

Algebra tiles are two-dimensional shapes that are used to represent constants and variables. Algebra tiles are made up of four different colors with yellow, blue, green and red. Yellow, blue, and green represents positives tiles, while red only represents negative tiles. "Algebra tiles usually come with a small square, an oblong rectangular strip, and a larger square. The tiles are purposely designed so that the side length of the larger square is not an integral multiple of the side length of the smaller square" (Chappell & Strutchens, 2001, p. 20). The various algebra tiles are shown above in **Figure 2**.

This concrete representation uses an area model. The area of the small square with dimensions 1 by 1 is 1 square unit; the rectangle has dimensions 1 by x and an area of x square unit; and the large square has dimensions x by x and an area of x^2 square unit. Several mathematical processes can be understood and also help students visualize (Brahier, 2016), integer operations and evaluating equations and expressions (Chappell & Strutchens, 2001) with algebra tiles. It also assists teachers to guide their students to make connections between the manipulative and abstract idea it is representing (Chappell & Strutchens, 2001). Algebra tiles can be used throughout algebra instruction to teach students the following concepts; adding, subtracting, multiplying and dividing integers, completing the square, factoring, and distributive property can be taught by using algebra tiles (Leitze & Kitt, 2000).

In a study by White (2012), it was found that there were no significant difference post-test scores of the two sub groups (low-achieving control versus low-achieving experimental, high-achieving control versus high achieving experimental) with 145 7th grade students using hands-on learning and manipulatives. The study used a quasi-experimental non-equivalent control-group as the design. Similarly, a study that was conducted by Palabiyik and Akkus (2011) also revealed no significant difference between the groups in terms of procedural algebra achievement and attitudes towards mathematics, when they conducted that study to determine the effects of pattern based and non-pattern based algebra instruction on 7th grade students' algebraic thinking and attitude towards mathematics. The experimental group used algebra tiles, matchstick and pattern blocks as manipulatives while the control group had the regular instruction without any manipulative.

However, there are other studies that favored students without the use of manupulatives. For instance, Magruder (2012) study that used embedded quasi-experimental mixed methods research to figure out differences between control group and two experimental groups that used concrete and virtual manipulatives in solving simple linear equations. Also, the study examined unique benefits and drawbacks associated with each manipulative. The finding indicates a statistically significant difference in favor of the control group because the experiment used a lot of time to operate the manipulative. The sample used was 60 students: 20 in the control group, 20 in the virtual group, and 20 in the concrete group.

On the other hand, there are also studies that favored the experimental groups using manipulatives. Aburime (2007) studied for instance that lasted for 10 weeks and used 287 high school students as the sample size revealed that there was a significant difference in students using mathematics manipulatives. Aburime (2007) used six experimental groups (manipulatives) and six control groups (no manipulatives). The study used mathematics achievement pre- and post-tests to collect the data from the groups. Also, Battle's (2007) quantitative research study used a sample of 16 low-achieving students; eight each for control group and experimental group. The purpose was to determine if manipulatives would increase math grades of the students. The study found that students taught addition and subtraction with counters performed better than those taught without manipulatives in the pre-test–post-test design. In another study by Ogg (2010) that used 12 5^{th} grade students as participants undertook a pretests and posttests with and without the use of manipulatives and the results favored the manipulative use. Similarly, Gurbuz's (2010) analysis revealed that activity-based instruction was more effective than traditional method in students learning about probability, when a sample of 25 each for the treatment and control were used. The study used quasi-experimental to examine the effects of activity-based instruction and traditional based instruction on fifth-grade students. Furthermore, Doias (2013) used 44 students as the sample: 22 each in the experimental group and control groups with mixed methods approach to explore the effects of manipulatives (concrete and virtual) with a 7^{th} grade on addition and subtraction of fractions. She found that the blend of concrete manipulatives with virtual manipulatives enhances students' achievements over the control group without manipulatives.

In another development, Larbi (2011) study with algebra tiles as the manipulatives for the experiment group, when she sought to investigate the effect of algebra tiles on students' performance in algebra. Her study used two intact classes from two different schools for the research which lasted three weeks. The analysis revealed that there was significant difference between the two groups. The group that received instruction using algebra tile outperformed their counterparts without the tiles. It was concluded that the use of manipulative promote student understanding in the learning of mathematics. Also, Akkus (2004) conducted a study to determine the effects of multiple representations-based instructions using experimental research design study with 131, seventh grade students as the sample. The study focused on algebra performance, attitudes toward mathematics, and representation preference. The study used two experimental groups and two control groups who received multiple representations-based instruction and regular instruction respectively. The experimental groups used algebra tiles, balance, pattern blocks, marbles, cartoons, cotton buds, and activity sheets as the manipulative. The results from the study showed that students in the experimental group achieved higher algebra performance than students who took traditional instruction. It also revealed that visualization approach not only affected the students' attitudes towards mathematics but also affects their mathematics achievement positively in Kog and Baser (2012) study. The purpose of the study was to examine the effects of visualization approach on the 8^{th} grade students in a pretest-posttest experimental. The experimental group was taught with the help of visualization approach used algebra tiles, computer-assisted visual materials, concept cartoons, metaphors, and activity sheets while the control group took traditional instruction approach.

Also, Gurbuz and Toprak (2014) concluded that activity-based instruction was more effective than regular instruction when 58 7^{th} grade students were used as the sample when the students wrote both pre and posttest. The experimental group used activity-based instruction using balance, counters and algebra tiles as the manipulatives while control group had regular. The purpose of their study was to design, implement and evaluate activities that enable 7^{th} grade students to make transition from arithmetic to algebra.

Atteh et al.'s (2017) study used a sample of 30 junior high school pupils whose purpose was to use the balance model approach as intervention to enhance students' conceptual understanding in the principles of solving linear equations in one variable. The descriptive analysis of the tests' results showed that the use of the balance model in linear equations improves students' understanding of the principles of solving linear equations in one variable. The results also showed that there was improvement in the students' academic achievements.

Likewise, Amoako's (2013) study examined whether the application of the balance model could improve students' conceptual understanding in the principles of solving linear equations in one variable. His study used 30 pupils of Buasi Roman Catholic Junior High School as the sample. The instruments used were pretest, posttest and interview to collect the data. From the descriptive statistics in his study, it revealed that the use of the balance model in teaching and learning of linear equations improves pupils' understanding of the principles of solving linear equations in one variable in post-test.

Fiakumah's (2012) study investigated whether the beam balance model could assist diploma in basic education students of St. Francis College of Education, Hohoe to improve on their conceptual knowledge of linear inequalities in one variable with sample size of 40. The instrument used in his research was achievement pre and post- test. The findings of his study revealed that there has been a drastic improvement in the students' achievement in post-test as compared to pre-test in linear inequalities in one variable using the beam balance model.

Larbi and Okyere (2014, 2016) examined the efficacy of using algebra tile manipulatives in junior high school students with a sample of 56 students from two schools purposely selected divided into experimental and the control group. The experimental group was taught using algebra tiles whilst the control group was taught using the traditional approach over a period of four weeks. The results indicate that the experimental group out- performed their counterparts in the control group significantly and also improved students thinking process as they solved problems in algebra. Similarly, Saraswati et al.' (2016) study entitled "supporting students' understanding of linear equations with one variable using algebra tiles" investigated one variable using algebra tiles combined with balancing method, which consists of three phases, namely preliminary design, teaching experiment and retrospective analysis. The approach used was design research. The findings are that algebra tiles could enhance students' understanding in solving linear equation with one variable. The study concluded that the use of algebra tiles can minimize the common mistakes.

Statement of the Problem

The performance of pupils in basic education certificate examination in the Nanumba North Municipality in mathematics is very poor. The pupils' performance at external examination is not encouraging. This is supported by WAEC (2011), which indicated that majority of the pupils could not solve questions on linear equation in one variable and those who tried exhibited little or no understanding of the principles of solving linear equations. Also, Brizuela and Schliemann's (2004) study revealed that pupils were not able to solve linear equation problems in one variable. Johnson (1993) and Mereku (2001) concluded that the poor performance in linear equation in one variable is as a result of the less usage of the manipulative materials to teach linear equation in one variable. This poor performance might be attributed to pupils not taught linear equations in one variable by using a manipulative material. Also, as link tutor of E.P. College of Education, Bimbilla my regular visit to schools for pre-service teachers' supervision revealed that some of regular teachers do not use manipulatives in teaching mathematics at the basic level.

To address this poor performance, previous studies (Aburime, 2007; Akkus, 2004; Atteh et al., 2017; Battle, 2007; Doias, 2013; Kog & Baser; 2012; Larbi, 2011; Larbi & Okyere, 2016; Magruder, 2012; Ogg, 2010; Saraswati et al., 2016) suggested the use of concrete manipulative in teaching mathematics, because those studies have advance the argument that the use of manipulative enhance students' performance and also increase pupils' positive attitude to mathematics. With these benefits some teachers do not still use manipulative with the view that manipulative waste time and hence resort to teaching without manipulative. The gap identified is that there is no single study in the Nanumba North Municipality comparing students with manipulative use and those who do not use manipulative in teaching linear equation with one variable especially using balance model and algebra tiles as manipulative. Therefore, this present study sought to investigate Basic 9 pupils who use balance model and algebra tiles manipulative compared to those who do not use any manipulative in linear equations in one variable with three basic schools in the Nanumba North Municipality.

Theoretical Framework: Constructivism

This study is based on the constructivist learning theory, which traces its origin to cognitive scientists like Jean Piaget, John Dewey, Jerome Bruner, and Vygotsky among others. Constructivist teaching is based on the belief that learning occurs as learners are actively engage in task rather than being passively receiving information. Learning is facilitated by social interaction between teacher and learners in a purely democratic environment. With such an environment learners learning depends on learner's previous experiences as well as knowledge, thus learning is viewed as reorganization of prior conceptual schemas. Through interaction with the physical situations, or concrete objects, a child's physical experience accumulates and he is able to conceptualize, think creatively and logically. In constructivist approach activities are interactive and student centered, learners are also encouraged to be responsible and autonomous. Constructivists' teaching promotes independent learners who are critical thinking and are motivated. In Constructivism, mathematics learning is viewed as the development of ideas, processes, and understanding in a social setting. This suggest that mathematics teachers who are teaching at the basic level are required to incorporate in their teaching any concrete manipulative that will help learners relate mathematics to the real-life situations and give pupils appropriate hands-on activities that will discourage rote-learning, purely declarative knowledge, and memorizing facts, theorems, formulas, and algorithms.

Purpose of the Study

The purpose of this study is to compare the achievements of Basic 9 pupils taught using balance model and algebra tile with control group who did not use any manipulative in linear equations involving one variable.

Objectives of the Study

This study aims at examining the effectiveness of using algebra tiles and balance model in teaching linear equations in one variable by the research assistants.

Research Questions

1. What difference exists if any, in Basic 9 pupils' pre-test achievements among the balance model, algebra tiles, and the control groups?
2. What difference exists if any, in Basic 9 pupils' post-test achievements among the balance model, algebra tiles, and the control groups?
3. What difference exists if any, in Basic 9 pupils' post-test achievements between the balance model and the control groups?
4. What difference exists if any, in Basic 9 pupils' post-test achievements between the algebra tiles and the control groups?
5. What difference exists if any, in Basic 9 pupils' post-test achievements between the algebra tiles and the balance model groups?
6. What difference exists if any, in Basic 9 pupils' pre-test and post-test achievements of the balance model group?
7. What difference exists if any, in Basic 9 pupils' pre-test and post-test achievements of the algebra tiles group?

Research Hypotheses

1. There is no significance difference in Basic 9 pupils' pre-test achievements among the balance model, algebra tiles, and the control groups?
2. There is no significance difference in Basic 9 pupils' post-test achievements among the balance model, algebra tiles, and the control groups?
3. There is no significance difference in Basic 9 pupils' post-test achievements between the balance model and the control groups?
4. There is no significance difference in Basic 9 pupils' post-test achievements between the algebra tiles and the control groups?
5. There is no significance difference in Basic 9 pupils' post-test achievements between the algebra tiles and the balance model groups?
6. There is no significance difference in Basic 9 pupils' pre-test and post-test achievements of the balance model group?
7. There is no significance difference in Basic 9 pupils' pre-test and post-test achievements of the algebra tiles group?

METHODOLOGY OF THE STUDY

Research Design

The research design was quasi-experimental methods, specifically the nonequivalent groups pre-test–post-test control or comparison group designs. The study was carried out using balance model and algebra tiles activities to improve Basic 9 pupils' performance in linear equations with one variable.

Population, Sample and Sampling Procedure

The population was all Basic 9 pupils of all junior high schools in Bimbilla Municipality. The sample was 120 Basic 9 pupils comprising of 70 males and 50 females from three junior high schools. Multi-stage cluster sampling procedures was adopted to select the schools in the municipality and three intact classrooms for the study. The average age of Basic 9 pupils was 15 years.

Instruments and Pilot Study

The instrument used in this study was achievement test on linear equations with one variable designed by the researcher. Both the pre- and post-test were ten essay type questions measuring Basic 9 pupils' competencies on solving linear equations in one variable. Specifically solving linear equations of the form ax=b, ax+b=c, and ax+b=cx+d. Each question was marked out of five marks with a maximum score of 50 marks. The instrument was piloted in a junior high school in the Tamale Metropolis to 30 pupils.

Validity and Reliability

The test items were validated by an experienced mathematics senior lecturer and two colleagues of the researcher's department. The validation of the test was based on the concepts, skills, difficulty level and clarity of the questions as well as the language. The experts' comments and recommendations were used to improve the test items. Split-half test method was used to calculate the reliability coefficient which was 0.81. This value indicates a high degree of reliability of the instrument which is acceptable according to Salifu (2018).

Independent and Dependent Variable

The independent variables in this study were teaching methods using algebra tiles, balance model and traditional method. The dependent variable is the scores from the pre and post-test from the various groups in linear equations involving one variable.

Procedure and Data Collection

Before the start of the study, three mathematics pre-service teachers of E.P. College of Education, Bimbilla were trained at a workshop to teach the three groups namely the control group, balance model group and algebra tiles group for one week, in junior high schools A, B, and C respectively. These trained pre-service teachers served as research assistants for the study and were teaching in those junior high schools. **Table 1** shows the summary of the procedure.

Phase 1

After permission was granted to the research assistants by the selected basic schools head teachers, pre-test on linear equations in one variable was administered to Basic 9 pupils of school A (control group), school B (balance model group), and school C (algebra tiles group) on 21st January, 2020 a week before the intervention. The pre-test consisted of 10 essay type questions for 1 hour 30-minute duration. Each pupil was given a printed question paper and answer sheets. The questions covered the following linear equations of the form ax=b, ax+b=c, ax+b=cx+d concepts. Random numbers 01-40 were generated from Microsoft Excel to identify the Basic 9 pupils on the pre-test and post-test instead of participants' names. The researcher and the research assistants invigilated the pre-test together.

Phase 2

The treatment duration for all the groups was from 27th January, 2020 to 7th February, 2020. The schools' regular timetable was used for the interventions. There were three meetings per week in a school. The control group without any manipulative was taught by one of the pre-service teachers of E.P. College of Education, Bimbilla with Basic 9 pupils of junior high school A. The concepts treated on solving linear equations in one variable were on ax=b, ax+b=c, ax+b=cx+d.

The experimental group1 (balance model group) had their lessons on solving linear equations in one variable with concepts on ax=b, ax+b=c, ax+b=cx+d by using balance model approach taught by one of the pre-service teachers of E. P. College of Education, Bimbilla with Basic 9 pupils of junior high school B. During this intervention, the square box made from cardboard was used to represent the x-variable and squeezed A4 papers sellotape into circular shapes were also used to represent constant values. The strategy used was (1) use balance model to model linear equations in one variable, (2) draw pictures representing the balance model, and (3) working the linear equations using algebraic notation.

The last pre-service teacher of E.P. College of Education, Bimbilla taught experimental group 2 with algebra tiles with Basic 9 pupils of junior high school C. The concept treated were on ax=b, ax+b=c, ax+b=cx+d on solving linear equations in one variable. During the intervention of this study, the algebra tiles that were used were the small squares representing the constant values while the oblong rectangles represented the x-variables. The bigger algebra tile was however not used because this study focused on linear equations with one variable concepts and not quadratic equations that makes use of the bigger algebra tiles. The strategy used was (1) use algebra tiles to model linear equations in one variable, (2) draw pictures representing the tiles, and (3) working the linear equations using algebraic notation.

Phase 3

After treatment on 7th February, 2020, the pupils were asked to use the weekend for revision and comeback to write the post-test on 11th February, 2020. The post-test conducted to all the groups with 1 hour 30-minute duration for 10 essay type questions similar to the pre-test, it was supervised by the researcher and research assistants.

Table 1. Summary of the procedure

Groups	Phase 1	Phase 2	Phase 3
Control group without manipulative	Pre-test	Treatment without manipulative	Post-test
Experimental group 1 (balance model group)	Pre-test	Treatment with balance model	Post-test
Experimental group 2 (algebra tiles group)	Pre-test	Treatment with algebra tiles	Post-test

Table 2. Descriptive statistics of pre-test scores of control group, balance model, and algebra tiles group

Group	N	Mean	SD	SE	Minimum	Maximum
Control group pre-test	40	32.08	5.95	.941	20.00	40.00
Balance model group pre-test	40	32.05	5.56	.878	25.00	40.00
Algebra tiles group pre-test	40	31.18	6.08	.961	25.00	40.00
Total	**120**					

Note. SD: Standard deviation; SE: Standard error

Table 3. One-way ANOVA pre-test scores of control group, balance model, and algebra tiles group

Group	Sum of squares	df	Mean square	F	Sig.
Between groups	21.017	2	10.508	.305	.737
Within groups	4,026.450	117	34.414		
Total	**4,047.467**	**119**			

Table 4. Descriptive statistics of post-test scores of control group, balance model, and algebra tiles group

Group	N	Mean	SD	SE	Minimum	Maximum
Control group post-test	40	41.53	4.13	.653	32.00	45.00
Balance model group post-test	40	44.55	4.34	.687	38.00	48.00
Algebra tiles group post-test	40	45.58	2.93	.463	37.00	48.00
Total	**120**					

Note. SD: Standard deviation; SE: Standard error

Table 5. One-way ANOVA post-test scores of control group, balance model, and algebra tiles group

Group	Sum of squares	df	Mean square	F	Sig.
Between groups	354.717	2	177.358	11.956	.000
Within groups	1,735.650	117	14.835		
Total	**2,090.367**	**119**			

Data Analysis

The data that was collected from the Basic 9 pupils on the various groups were analyzed using statistical package for social science (SPSS) software version 16.0 and Microsoft Excel 2010. Descriptive statistics, paired sample t-test, independent sample t-test, and one-way analysis (ANOVA) were used to analyze the data. The Cohen's "d" effect size was also calculated to determine the level of the impact of the interventions.

RESULTS AND DISCUSSION

The findings of this study are discussed according to research questions.

Research Questions 1

From **Table 2**, the results of the Basic 9 pupils' pre-test among the control group, balance model, and algebra tiles group are M=32.08, SD=5.95; M=32.05, SD=5.56; and M=31.18, SD=6.08, respectively. The minimum and maximum scores among the group of Basic 9 pupils are Min=20.00, Max=40.00; Min=25.00, Max=40.00; Min=25.00, Max=40.00 are for control group, balance model, and algebra tiles group, respectively. The spread of majority of the score for the control group was within 26.13-38.03, while that of the balance model and algebra tiles group are 26.49-37.61 and 25.10-37.26, respectively. By this result it was difficult to figure out whether there was significant difference among the groups, so the one-way ANOVA was employed to test whether or not differences exist between control group, balance model, and algebra tiles group. Some of the Basic 9 pupils encountered difficulties in dividing both side by a coefficient, grouping like terms and removal of brackets in their attempt to solve some of the linear equations in one variable.

The results from **Table 3**, using the one-way ANOVA pre-test scores of control group, balance model, and algebra tiles group show that there is no statistically significant difference between the groups at $F(2, 117)=.305$, $p=.737>.005$; hence, the Basic 9 pre-test achievements of control group, balance model, and algebra tiles group are the same. This implies that the null hypothesis is upheld. This finding totally corroborates with findings of Aburime (2007) whose study groups in pre-test did not reveal significant difference.

Research Questions 2

From **Table 4**, the results of the Basic 9 pupils' post-test results revealed that the control group, balance model, and algebra tiles group are M=41.53, SD=4.13 with majority Basic 9 pupils score spread between 37.40-45.66, M=44.55, SD=4.34 with majority score range of 40.21-48.89, M=45.58, SD=2.93, and finally with majority score range of 42.65-48.51, respectively.

The minimum and maximum scores among the group of Basic 9 pupils are Min=32.00, Max=45.00; Min=38.00, Max=48.00; and Min=37.00, Max=48.00 are for control group, balance model, and algebra tiles group, respectively. With this result it was obvious that difference exist among the groups, so the one-way ANOVA was used to detect the difference between control group, balance model, and algebra tiles group.

The results from **Table 5**, the one-way ANOVA post-test scores of control group, balance model, and algebra tiles group shows that there is statistically significant difference between the groups at $F(2, 117)=11.956$, $p=.000<.005$; hence, at least two groups of the Basic 9 pupils post-test achievements of control group, balance model, and algebra tiles group differs. In order to detect the differences between the groups that differs the post-hoc analysis was used as in **Table 6**.

Table 6. Post-hoc one-way ANOVA post-test scores of control group, balance model, and algebra tiles group

					95% confidence interval	
Groups for intervention (I)	**Groups for intervention (J)**	**Mean difference (I-J)**	**SE**	**Sig.**	**Lower bound**	**Upper bound**
Control group post-test	Balance model group	-3.025*	.861	.002	-5.0695	-.9805
	Algebra tiles group	-4.050*	.861	.000	-6.0945	-2.0055
Balance model group post-test	Control group	3.025*	.861	.002	.9805	5.0695
	Algebra tiles group	-1.025	.861	.461	-3.0695	1.0195
Algebra tiles group post-test	Control group	4.050*	.861	.000	2.0055	6.0945
	Balance model group	1.025	.861	.461	-1.0195	3.0695

Note. *The mean difference is significant at the 0.05 level; SE: Standard error

Table 7. Independent sample t-test of post-test scores of control group and balance model group

Group	N	Mean	SD	SE mean	df	t	Sig. (2-tailed)
Control group post-test	40	41.53	4.132	.653	78	-3.191	.002
Balance model group post- test	40	44.55	4.344	.687			

Note. SD: Standard deviation; SE: Standard error

Table 8. Independent sample t-test of post-test scores of control group and algebra tiles group

Group	N	Mean	SD	SE mean	df	t	Sig. (2-tailed)
Control group post-test	40	41.53	4.132	.653	70.244	-5.059	.000
Algebra tiles group post- test	40	45.58	2.925	.463			

Note. SD: Standard deviation; SE: Standard error

From **Table 6**, there was significant difference between the balance model group and control group with a mean difference of 3.025 and statistically significant at .002. Also, statistically significant difference was noticed between algebra tiles group and control group with a mean difference 4.050 and p-value of .000. Finally, the result showed that there was no significant difference between algebra tiles group and balance model group, because mean difference was 1.025 and significant value of .461. This implies that the null hypothesis is rejected.

This result is similar to the studies of Aburime (2007), Akkus (2004), Battle (2007), Doias (2013), Gurbuz (2010), Gurbuz and Toprak (2014), and Ogg (2010) whose findings indicated that students who used manipulatives performed better than their counterparts who were taught without any manipulative. The differences existed because of the good intervention adopted by the research assistants in the study. The improvement of achievement for the balance model and algebra tiles groups in the treatments could be attributed to the Basic 9 pupils grasped of applying principles such as addition property of equality, subtraction property of equality, multiplication property of equality and division property of equality. The improved performance could also be attributed to the Basic 9 pupils been able to divide both sides by the coefficient of the variable in the given equations and group like terms on both sides of the equation before simplifying with very few mistakes.

Research Question 3

The results of independent samples t-test in **Table 7** showed a statistically significant difference in posttest mean scores at t(78)=-3.191, p=.002<.05 between control group and balance model group. The balance model group recorded high mean performance (M=44.55, SD=4.344) than the control group taught without any manipulative (M=41.53, SD=4.132). This signifies the rejection of the null hypothesis. The difference between their means was 3.02. This finding indicates that Basic 9 pupils' performance in linear equation in one variable with balance model approach is better than the control group without the usage of any manipulative. This supports the findings of Akkus (2004) and Gurbuz and Toprak (2014) who stated that students who used the balance model approach in their studies performed significantly better than their colleagues in control group. The spread of majority of the score for the control group was 37.40-45.66 while that of the balance model group was within 40.21-48.89. The balance model group performance could be due to the constructivist approach adopted by the research assistants where they ensure that the pupils worked collaboratively and related real-world situations to mathematical symbolism and verbalized mathematical thinking. The research assistant also allowed pupils to discussed mathematical ideas and concepts in a democratic environment. The balance model group pupils improved in their skills of dividing both side by a coefficient, grouping like terms correctly and removal of brackets in their attempt to solve some of the linear equations in one variable was excellent. The effect size d=0.71 which is large effect size. Detail calculation is shown below:

$$Cohen's\ d = \frac{M_2 - M_1}{\sqrt{\frac{(SD_1)^2+(SD_2)^2}{2}}} = \frac{44.55 - 41.53}{\sqrt{\frac{(4.344)^2+(4.132)^2}{2}}} = \frac{3.02}{\sqrt{\frac{18.87+17.07}{2}}} = \frac{3.02}{4.239} = 0.71$$

The Cohen's d value of 0.71 indicates a large effect size. This means that 71% of the variance in the scores of the post-test of the balance model group revealed that the teaching method was good. Also, it means that there was massive improvement in balance model group achievement in linear equation with one variable.

Research Question 4

From **Table 8**, the results showed significant difference between mean performances of Basic 9 pupils' scores in post-test. Control group post-test mean score (M=41.53, SD=4.132) compared to algebra tiles group (M=45.58, SD=2.925) was statistically different [t(70.244)=-5.059, p=.000<.05]. Therefore, the null hypothesis is rejected. The mean difference was 4.05 in favor of algebra tiles group. The spread of majority of the score for the control group was within 37.40-45.66, while that of the algebra tiles group are 42.66-48.51. This result is related to findings of Gurbuz and Toprak (2014), Kog and Baser (2012), Larbi (2011), Larbi and Okyere (2016), and Saraswati et al. (2016) whose various studies indicated significant difference in favor of experimental group using algebra tiles as compared to control group.

Table 9. Independent samples t-test of post-test scores of balance model group and algebra tiles group

Group	N	Mean	SD	SE mean	df	t	Sig. (2-tailed)
Balance model group post-test	40	44.55	4.344	.687	68.342	-1.238	.220
Algebra tiles group post-test	40	45.58	2.925	.463			

Note. SD: Standard deviation; SE: Standard error

Table 10. Paired sample t-test of pre-test and post-test scores of balance model group

Group	N	Mean	SD	SE mean	df	t	Sig. (2-tailed)
Balance model group post-test	40	44.55	4.344	.687	39	10.753	.000
Balance model group pre-test	40	32.05	5.556	.879			

Note. SD: Standard deviation; SE: Standard error

Table 11. Paired sample t-test of pre-test and post-test scores of algebra tiles group

Group	N	Mean	SD	SE mean	df	t	Sig. (2-tailed)
Algebra tiles group post-test	40	45.58	2.926	.4626	39	15.042	.000
Algebra tiles group pre-test	40	31.18	6.076	.9607			

Note. SD: Standard deviation; SE: Standard error

The differences in this study could be as a result of the exhibition of the constructivist theory employed by the research assistants where they ensured that Basic 9 pupils worked collaboratively and allowed them to think divergently to find a variety of ways to solve the problems in linear equations in one variable. The algebra tiles group pupils' performance was due to minimal mistakes in dividing both side by a coefficient, grouping like terms correctly and removal of brackets in their attempt to solve some of the linear equations in one variable was excellent. In the intervention, the Basic 9 pupils took ownership of their learning experiences and developed a lot of confidence in the intervention process. The effect size of the treatment was calculated to determine the extent of the intervention. The effect size d=1.13, which is large effect size. Detail calculation is shown below:

$$Cohen's\ d = \frac{M_2 - M_1}{\sqrt{\frac{(SD_1)^2+(SD_2)^2}{2}}} = \frac{45.58 - 41.53}{\sqrt{\frac{(2.925)^2+(4.132)^2}{2}}} = \frac{4.05}{\sqrt{\frac{8.56+17.07}{2}}} = \frac{4.05}{3.58} = 1.13$$

The Cohen's d of 1.13 indicates a large effect size, signifying that 113% of the variance in the scores of algebra tiles group was a massive intervention in the post-test. It implies that algebra tiles as an instructional tool had a positive impact on the Basic 9 pupils' achievement in linear equations in one variable.

Research Question 5

From **Table 9**, the results of post-test of balance model group (M=44.55, SD=4.344) is compared to algebra tiles group (M=45.58, SD=2.925) with t(68.342)=-1.238, p=.220>.05 using independent sample t-test indicate no significant difference between the balance model and algebra tiles group; hence, the groups were at same level after the treatment. Since there is no significance difference hence no effect size for the two groups. This result agrees with the findings of Akkus (2004) and Kog and Baser (2012) who also indicated that there was no significant difference between student who use balance model and algebra tiles because both groups were taught the same constructivist theorem. The spread of majority of the score for the balance model and algebra tiles group are 40.21-48.89 and 42.67-48.51, respectively. Therefore, the null hypothesis is upheld.

Research Question 6

The results of paired samples t-test in **Table 10** showed a statistically significant difference in pre-test and posttest mean scores at t(39)=10.753, p=.000<.05 between pre-test and post-test of the balance model group. The post-test recorded high mean performance (M=44.55, SD=4.344) better than the pre-test (M=32.95, SD=5.556). A mean difference of 12.5 was revealed in favor of the post-test. This finding indicates that Basic 9 pupils' performance in linear equation in one variable in post-test is better than pre-test. The pre-test scores of the balance model were within the range 26.49-37.61, while that of post-test of the same group was in the range 40.21-48.89. This implies that the null hypothesis is rejected.

This study also corroborates with that of Amoako (2013), Atteh et al. (2017), and Fiakumah (2012), where students' performance in post-test was better than their pre-test in a study that focused on balance model method in solving linear equation and inequalities in one variable. The performance in the post-test was as a result of the pupils gaining confidence during the intervention in their abilities to find solutions to mathematical problems using the constructivist approach. During the intervention the pupils came up with their own ideas most of the time without relying on the directions of the research assistants to solve problems in groups. The effect size of the balance model treatment was calculated to determine the extent of the intervention. The effect size d=1.70, which is large effect size. The detail calculation is shown below:

$$d = \frac{t - value}{\sqrt{n}} = \frac{10.753}{\sqrt{40}} = \frac{10.753}{6.32} = 1.70$$

The Cohen's d of 1.70 signifies a large effect size. This means that 170% of the variance in pre-test and post-test the scores of the balance model group in linear equation with one variable were due to the intervention. Also, it means that there was massively improvement in the understanding and achievement of the concept in linear equation with one variable from pre-test and post-test. It also revealed that balance model as an instructional tool had a positive impact on Basic 9 pupils in linear equation in one variable.

Research Question 7

The results of paired samples t-test in **Table 11** showed a statistically significant difference in pre-test and posttest mean scores at t(39)=15.042, p=.000<.05 between pre-test and post-test of the algebra tiles group. The post-test recorded high mean performance (M=45.58, SD=2.926) better than the pre-test (M=31.18, SD=6.076). Hence, the null hypothesis is rejected. This finding indicates that Basic 9 pupils' performance in linear equation in one variable in post-test is better than pre-test. The pre-test scores of the algebra tiles group were within the range 25.10-37.26, while the post-test of the same group was in the range 42.65-48.51. A mean difference of 14.4 was revealed in favor of the post-test. This study is in line with Saraswati et al. (2016), where they also found that students' post-test results were far better than their pre-test. The effect size of the algebra tiles treatment was calculated to determine the extent of the intervention. The effect size d=2.38, which is large effect size. The detailed calculation is shown below:

$$d = \frac{t - value}{\sqrt{n}} = \frac{15.042}{\sqrt{40}} = \frac{15.042}{6.32} = 2.38$$

The Cohen's d of 2.38 indicates large effect size. It means 238% of the variance in the scores between the pre-test and the post-test of the algebra tile group was due to the teaching method using algebra tiles for teaching. Algebra tiles as an instructional tool had a positive impact on the Basic 9 pupils' achievement in linear equation in one variable.

Major Findings

1. There was no significant difference in Basic 9 pupils' pre-test between the control group, balance model group, and algebra tiles group in linear equation in one variable.
2. There was significant difference in Basic 9 pupils in post-test in favor of balance model group over the control group with large effect size in linear equations with one variable.
3. Also, significant difference in Basic 9 pupils in post-test favored algebra tiles group over the control group with large effect size in linear equations with one variable.
4. The balance model group achievements improved drastically in post-test better than their pre-test scores with large effect size.
5. The algebra tiles group achievements improved drastically in post-test better than their pre-test scores large effect size.
6. There was no significant difference in Basic 9 pupils' post-test between the balance model group and algebra tiles group in linear equation in one variable.

CONCLUSION

The Basic 9 pupils of the experimental groups (algebra tiles and balance model) outperformed their counterparts in the control group who did not use any manipulative in solving linear equations in one variable in post-test achievements comparison. The performance with regards to experimental groups could be due to constructivist approach used by the research assistants. This ensured pupils collaborative work and divergent thought with enhanced confidence. Thus, this minimizes their mistakes in dividing both sides of an equation by a coefficient, grouping like terms, and removing brackets in linear equations.

Recommendation

Based on the findings of this study, it is recommended that Basic School mathematics teachers should use the balance model and algebra tiles in the teaching and learning of linear equations in one variable because it will help improve performance in mathematics. Also, mathematics teachers are encouraged and expected to explore instructional strategies such as the balance model and algebra tiles in the teaching and learning of linear equations in one variable because it shifts the focus from a teacher-centred to a learner-centred. Finally, school based workshop should be organized every academic year to train teachers on how to use manipulatives effectively.

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