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The reflect-connect-apply methodology model in the mathematics classroom

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ABSTRACT

This paper dealt with reflect-connect-apply (RCA) methodology model as a teaching framework designed to enhance pre-service mathematics teachers learning through a structured process of reflection, connection and application. In the classroom, this model encourages pre-service teachers to first reflect on their prior knowledge and experiences, then connect new concepts with the existing understanding and finally apply these concepts in the real world setting. By following these stages, pre-service teachers are able to deepen their comprehension, improve critical thinking skills and retain knowledge more effectively. The design is a purposive qualitative research methodology, involving pre-service mathematics teachers in the second year of their studies with a sample size of six from a college of education in Ghana. The instruments of tasks and interviews were used for data gathering. The RCA model helped improve problem-solving skills in mathematics education in the colleges of education, which had not been documented before in the Ghanaian context. The main finding showed that pre-service teachers (T3, T4, and T6) "reached more than most expected" when they used the RCA model approach to tackle tasks. Using reflection, connection, and application between theory and practice of the Shulman content domains, they offered excellent solutions that resulted in the right answers. The RCA model supports learning by expanding the corpus of empirically grounded information, fosters deeper engagement and helps pre-service teachers bridge the gap between theoretical concepts and practical application in mathematics education.

Keywords: apply, connect, mathematics classroom, methodology model, reflect, theory-practice gap

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INTRODUCTION

One of the main challenges in mathematics education in Ghana is the lack of modern effective training methodologies for pre-service teachers at the colleges of education and teacher education in general. This calls for interventions through Transforming, Teacher Education and Learning (T-TEL, Ghana), a non-governmental organization (NGO) that designed modules to bridge this theory-practice gap. Subsequently, Right to Play, Ghana, another NGO also designed modules to bridge this theory-practice in the colleges of education with the reflect-connect-apply (RCA) methodology for general practice in all programs. The methodology was modified to bridge the theory-practice gap in mathematics education in Ghana. The RCA model has the potential to address this gap through its theoretical and practical components of the themes "reflect, connect and apply" methodology. The study's main objective is to explore the impact of the RCA model on pre-service mathematics teachers' concept development and problem-solving skills in mathematics classrooms.

In the context of teacher education in Ghana, the RCA model presents a progressive alternative to traditional teaching methods, which often rely heavily on rote memorization and passive learning. Traditional approaches in Ghanaian classrooms may struggle to foster critical thinking, problem solving and active engagement among pre-service teachers, limiting their ability to effectively teach in a dynamic real world classroom environments. The RCA addresses these weaknesses by encouraging pre-service teachers to actively reflect on their own experiences and knowledge, make meaningful connections with new content and apply. By incorporating reflection, connection and application, the limitations of traditional methods such as lack of student-centered learning, and limited development of practical teaching skills and aligns with modern educational goals of fostering critical, reflective and innovative educators in Ghana.

Mathematics educators have identified valid data collected from classroom settings and the theoretical and practical interpretation of that data as a critical research area (Schoenfeld, 1999; Steffe & D'Ambrosio, 1995). The necessity to convert the data interpretations into practical educational practice models that aid in bridging the theory-practice gap is connected to these actual data. With regard to the teaching and learning of mathematics, pre-service teachers can utilize this model of practice as a reference to help them create or design tasks based on RCA to reality (Smith, 2000). To bridge the theory-practice gap, this study presents the RCA model. Using tasks designed specifically in the context of the classroom to be used by pre-service mathematics teachers as the main instrument and being supplemented by interviews lining theory and practice for concept formation and development. An authentic and grounded description of pedagogical techniques based on pre-service teachers' lived experiences was the goal of this investigation, which was grounded in a practical classroom setting. To boost the validity of the results, the RCA methodology model was selected to "explore theoretical issues in a context that really matter" (Schoenfeld, 1999, p. 14). The reason behind the choice of mathematical tasks for this study. The pre-service teachers are also known as student teachers in the colleges of education in Ghana and are the main focus of this research. The RCA methodology model is then expected to be used in their micro-teaching to students at elementary and junior high school (JHS) levels of Ghanaian education.

From a theoretical perspective, the RCA model will be critical to pre-service teachers' mathematical preparation since it emphasizes the use of previously taught concepts. The concept of 'playing' improves newly acquired skills and involves fewer active students in the classroom while 'Right to Play' is a learning through-play model designed to make learning more meaningful and purposeful (Right to Play, 2020). The RCA methodology model in this paper is used for tasks without a play component whilst emphasizing the concepts of reflecting, connecting, and applying to the learning process.

Due to global assessment requirements, most nations have developed cutting-edge approaches that would assist pre-service teachers in developing their knowledge and using it to address real-world events. To fulfil the interests and expectations of pre-service teachers in the 21st century, many mathematics educators believe that they must reinvent their teaching strategies. Most pre-service teachers are aware that outdated techniques are ineffective today (Barana et al., 2017). Mathematics educators must acquire methodological skills to create teaching techniques that will improve learning because pre-service teachers must gain disciplinary and cross-cutting competencies (Barana et al., 2017). The cognitive processes that underline pre-service teachers' assessment, such as problem-posing and problem-solving, were some of the techniques for improving mathematics teaching that some scholars recommended (Brancaccio et al., 2015). Most of these methodologies provided pre-service teachers with teaching strategies, techniques, and tools for thinking in the 21st century, as well as technology resources, methods for collaboration and cooperative learning, and real-world projects that help them develop confidence and take ownership of their learning (Warner & Kaur, 2017).

Study's Aim and Objectives

This study's focus is to explore the impact of the RCA methodology model on pre-service mathematics teachers' concept development and problem-solving skills based on given tasks to bridge the gap in theory and practice in the mathematics classroom. This study aims to help pre-service mathematics teachers gain a better understanding of the RCA methodology model in solving mathematics tasks. Pre-service mathematics teachers are the main target because on completion they will teach students at the pre-tertiary education level where they are expected to use this methodology model to students to help shape and build their understanding and interest in mathematics. The research questions that guided the study were:

1. To what extent can pre-service mathematics teachers' content knowledge, pedagogical content knowledge, and curriculum knowledge be applied in the mathematics classroom based on the RCA model?
2. How will pre-service mathematics teachers use RCA methodology model to bridge the gap between theory and practice?

Study Contribution

This study provides insight and information into pre-service mathematics teachers' adaptation to learning using the RCA methodology model to bridge the theory-practice gap in teacher education in Ghana. The RCA model adds to knowledge by closing the theory-practice gap in the colleges of education in Ghana context specifically mathematics education. The theoretical analysis is performed with a total of six pre-service mathematics teachers at a college of education in Tamale, Ghana. Akyeampong et al. (2022) posit researchers using the RCA methodology through play-based learning as part of their evaluation report on partners in the play baseline of Right to Play and not as a model on pre-service mathematics teachers. Our study showed how pre-service mathematics teachers' perceptions of using the RCA methodology model to solve mathematical-specific tasks in the context of the classroom could enhance learning outcomes for subjects related to mathematics education.

This methodology model results in critical-reflective thinking for mathematical ideas based on relevant experiences resulting in concept development in any mathematics topic. It will help pre-service mathematics teachers impart this model to students at the elementary and junior high levels of education in Ghana and the rest of the world. This model also helps pre-service mathematics teachers to discover that there is no one solution to a mathematical task. Pre-service mathematics teachers exhibited excitement especially on discoveries of mathematical theorists through the use of the RCA methodology model.

LITERATURE REVIEW

The review of the literature relies on current topics within the globe and Ghana context, which helped analyses the implementation of the methodology in mathematics classrooms to improve concept development and provide further insight from a global viewpoint. The review also highlighted the relevance of the use of the RCA methodology model by pre-service teachers in creating concept formation in the mathematics classroom at a college to improve their comprehension abilities of mathematics topics. Right to Play (https://righttoplay.com/en/countries/ghana/) is an international NGO dedicated to empowering children in education via the use of play-based learning and the RCA approach to learn meaningful and purposeful play in the classroom. Researchers from around the globe have used play-based activities in their interventions to lessen peer violence in Pakistan public schools (Karmaliani et al., 2020).

Pre-Service Mathematics Teachers Content Knowledge, Pedagogical Content Knowledge, and Curriculum Knowledge Based on RCA Model

Shulman's (1986) knowledge domains in teaching—pedagogical content knowledge, curriculum knowledge, and subject matter content knowledge—form the theoretical basis of this study. Teacher education is placing greater focus on pre-service mathematics teachers' pedagogical content understanding. Since they can effectively teach particular subjects to particular students, this sets them apart in the classroom (Kirschner et al., 2022). According to Coe (2013), students are not necessarily learning when they are actively participating in a lesson. The notion that someone can go into a classroom and instantly sense learning is false (Aforklenu & Bukari, 2023). Engagement is sometimes a poor alternative to learning. How a teacher teaches and supports learning will determine how actively students engage in the process (Nduru & Wardhani, 2023). The pre-service teachers solved the RCA model-based tasks by utilizing these knowledge domains from Shulman (1986).

Our model is driven by the fact that it is specifically created for pre-service mathematics teachers to use with students studying mathematics at the elementary and JHS levels.

Fadillah (2019), cited in Nduru and Wardhani (2023) highlighted that children are not learning new things when they play but honing their newly learned abilities. The implementation of play-based learning, particularly for young children, has presented obstacles. These include risk-taking, going beyond boundaries, and engaging in acts of rebellion, particularly in opposition to policy-led limits (Wood, 2012). Guidelines on play were created to address the issues raised by Wood (2012) and were intended to be utilized by teachers with varying degrees of training and experience in a variety of situations. When pre-service teachers incorporate meaningful play into their lessons, our RCA methodology model becomes more effective and fits learners at the foundational level of education. This helps students at that level understand mathematical ideas, but in the implementation of the RCA model to pre-service mathematics teachers play was not incorporated. Despite the study of Ali et al. (2018) posit that play-based learning maximizes academic standards by offering more than one way to teach the same concepts. The researchers believe that the RCA methodology model could help pre-service teachers to understand same concepts through several approaches without incorporating play in the learning process.

Pre-Service Mathematics Teachers' Use of the RCA Model to Bridge Theory-Practice Gap

Play encourages creativity and invention in students even though it represents one of the best types of experience learning (Kolb & Kolb, 2010). Particularly in teacher education, it is less commonly employed explicitly in pre-service teacher training (Diaz-Varela & Wright, 2019). Experiential learning, on the other hand, gives students of all ages a framework for play and fosters cognitive spontaneity, which Lieberman (1977) suggests is characterized by curiosity, originality, imagination, and innovation. Though many of the educational strategies discussed in the research could be classified as playful learning approaches, it is less typical for those utilized with pre-service teachers to be so labeled (Whitton, 2018, p. 1). In general, despite some recent attention, there is little research on playfulness for students in teacher education (Dobson & Mckendrick, 2018). By discussing the RCA methodological paradigm to teacher preparation in teacher education, this article advances this larger goal.

To protect, educate, and enable students at elementary or foundational levels of education to harness the power of play-based learning, Right to Play International was established in 2001 and now operates in forty countries. Right to Play provides professional development training for teachers at training institutions. The program was created between 2015 and 2017 in collaboration with academic experts and practitioners in the fields of education and child development in Toronto, Canada. Just to name a few of the 40 nations, the Right to Play program has been modified, contextualized, and delivered in Benin, Ethiopia, Ghana, Lebanon, and Uganda. Three pillars make up the Right to Play package:

(1) community of practice for pre-service teachers,

(2) coaching and mentoring, and

(3) training.

Teaching materials on play-based learning, with a focus on play-based games, dance, songs, brainstorming sessions, small and large group discussions, starters/energizers, and other activities (Diaz-Varela & Wright, 2019).

In Ghana, for teacher educators to share their newly acquired skills with their peers at professional development sessions established by T-TEL, Right to Play arranged training seminars at the colleges of education. Modules created by Right to Play International aim to support educators' creativity and increase their capacity to use play as a teaching tool in the classroom (Right to Play, 2015). Teachers acquire new knowledge, abilities, and experiences through a sequence of training sessions and continuous assistance (Diaz-Varela & Wright, 2019). Most of the play-based learning activities could be used to teach different subject areas and were created with both literacy and numeracy in mind. The goal of including play as a training tool in the training sessions is to help teacher educators better understand the importance of play in their own education and, consequently, in the education of their own pre-service mathematics teachers. The goal of the learning method is to change teachers' perceptions of what effective teaching looks like—moving away from rote learning and lecturing—and to change their behavior as teacher educators—directing them toward encouraging group projects, games, and critical reflection. This argument served as the foundation for our adaptation of the RCA approach as a model to support the teaching and learning of mathematical ideas in mathematics classrooms. Pre-service teachers who receive training through play not only become more capable of implementing the RCA methodology in their classrooms but also develop the kind of skills that allow them to be creative and autonomous prospective teachers (Right to Play, 2016). Training in lesson preparation and the deliberate integration of the reflect, connect, and apply methodology are how this is accomplished.

Theoretical Background: Linking Theory to Practice

Expanding upon the literature review, the RCA model contributes significantly to fortifying the connection between theory and practice by equipping pre-service mathematics teachers with more sophisticated higher-order thinking and critical thinking abilities. The RCA model builds on the existing theory of Skemp's (1987) ideas of conceptual understanding of mathematics through a relational approach. Our model complements Skemp's ideas by promoting active learning and higher-order thinking based on reflection, connection, and application deepening conceptual knowledge and practical applications in the classroom. Studies on constructivism theory and inquiry-based learning are some of the models designed to bridge the gap between theory and practice. Despite the increased attention to inquiry-based instruction, there is still a paucity of research that directly compares inquiry-based implementation types and their effects on students learning outcomes. From a research standpoint, the assumption that all these approaches are equally effective potentially conflates the field's understanding of how much directed problem-solving students can be expected to manage. There is a lack of clarity in educators attempting to discern which inquiry-based model best supports the development of conceptual knowledge and problem-solving skills within the classroom (Tawfik et al., 2020). To address this research need, this study first distinguishes between the theoretical and empirical underpinnings of the RCA model through the lens of Skemp (1987). The RCA model provides a structured approach that balances reflections with practical application ensuring that pre-service mathematics teachers engage both cognitively and practically with the tasks. Hiebert and Carpenter (1992) emphasized understanding through problem-solving, there is a dearth of research exploring the practical steps pre-service teachers can take to integrate these concepts effectively in a real-world teaching setting. The RCA model offers a structured approach to bridge this gap by addressing 21st century skills such as critical thinking and problem-solving skills. Our model aligns with modern pedagogies that prioritize education, making it a valuable model for pre-service mathematics teachers. The RCA model not only facilitates mathematical understanding but also aligns with global educational imperatives such as developing critical thinking, problem-solving skills and fostering lifelong learning competencies.

These competencies were achieved by using tasks with multiple solutions based on reflection, connection, and application. The process involved different ways of using different representations of mathematical concepts, different mathematical methods, and theorems from different topics of mathematics (Levav-Waynberg & Leikin, 2006). One of the well-recognized ways for developing reflection, connection, and application of one's mathematical knowledge is solving tasks in different ways (House & Coxford, 1995). Stigler and Hiebert (1999) showed that multiple solutions to tasks increased the quality of mathematics lessons. Consequently, we consider integrating reflecting and connecting tasks in one's teaching critically for fostering the reflection and connectedness of pre-service teachers' mathematical knowledge both creating awareness of pre-service teachers' multiple solutions to tasks and recognizing that in a class of students, there are multiple ways in which students do solve tasks. Teachers evaluated their students' learning and guided them in their learning journeys using the RCA model. This tactic includes talks inside the guided play activities in which teachers encourage the students to think back on their experiences, relate those experiences to what they already knew, and apply their new knowledge to their everyday life and future learning (Murtagh et al., 2022). According to the findings of Murtagh et al., students in schools where teachers received training in play-based pedagogies achieved higher scores than those with traditional approaches, demonstrating the effectiveness of guided play to playful instruction (Murtagh et al., 2022), as activities are initiated by teachers with an explicit learning goal in mind and are directed by the teacher and the child depending on the activity (Zosh et al., 2018).

The RCA model fills appropriately in bridging the theory-practice gap among pre-service mathematics teachers in the Ghanaian context and could be replicated in other teacher education institutions to provide conceptual understanding and comprehension of mathematical concepts, especially for pre-service teachers in their micro-teaching at the foundation level of educational systems. The RCA methodology model made pre-service teachers value the power and beauty of mathematics for the development of students' conceptual knowledge.

METHOD

This study is a qualitative design approach, and a purposive sampling technique was employed in selecting a sample of six pre-service mathematics teachers from a college of education in Ghana. Semi-structured interviews were used for gathering data for the study after solving tasks adapted (Levav-Waynberg & Leikin, 2006). These tasks really matter in the classroom context and are suitable for the RCA methodology model and were supplemented with interviews based on the given tasks. This made the tasks very important regarding the RCA methodology model and the research question designed for the study since the play component has been excluded from our model. The research design and sampling technique in this study were to explore the experiences, ideas, views, and motivation of pre-service mathematics teachers on issues on the concept of the model (Islam & Aldaihani, 2022). A purposive sampling technique was employed in getting six pre-service mathematics teachers for the study (Patton, 2015). The sample size is suitable for exploratory research because this is a pilot study designed to evaluate the RCA model's adaptation and efficacy in a new setting in Ghana. The data was analyzed using content analysis approach with the themes of reflection, connection and application in problem-solving skills on the given tasks. The assessment of the relevance and validity of data on RCA model was based on careful consideration of both research context and methodology employed. The data was collected from teacher education setting where the RCA model is being implemented, particularly within Ghana's educational framework. This ensures that the findings are contextually relevant and reflect the challenges and opportunities within Ghanaian classrooms. The data reflect how the RCA model addresses the specific needs of pre-service teachers in Ghana such as critical reflective thinking, fostering active learning and bridging theory with practice. The data was collected from credible sources such as task adapted from literature and semi-structured interviews designed by the researchers. The validity of the data also indicated how the RCA model affects pre-service teachers learning outcomes such as knowledge retention, self-reflection and the application of teaching strategies utilizing the RCA model.

The RCA methodology helps also in developing and implementing a model for pre-service teachers' concept development which is characterized by reflection, connection, and application of pre-service teachers' content knowledge, pedagogical content knowledge, and curriculum knowledge.

The Reflect-Connect-Apply Methodology Model

In the mathematics classroom, pre-service mathematics teachers used the RCA teaching/learning model to process their experiences through reflection and discussion with their classmates. This model has been modified from the module (Right to Play, 2020). This model made learning meaningful and purposeful in the context under which it was utilized considering note of the key terms RCA. If purposeful and meaningful play is used, this practice helps pupils learn mathematical ideas. The play-based activity was not included using the RCA rather emphasis was made on how to use the "reflect-connect-apply" in posing questions and in the evaluation and answering of tasks. All six pre-service teachers were interviewed after solving the tasks. The six pre-service teachers participated in whole group discussions focusing on solving tasks using reflection, connection, and application. The report on this study is on only the six pre-service teachers who took part in the study. This sample is representative of the whole group of pre-service teachers in education at a college of education in Ghana.

Integrating Theory and Practice Through Reflection

Reflection based on theory and practice has become a major pursuit in pre-service teacher education (Laboskey, 1994; Rodgers, 2002). This implies that learning to become a reflective teacher, prospective teachers would ideally acquire competencies that transcend technical thinking about 'what to do in the classroom' and engage in trying to establish relevant connection between theory and practice (Orland-Barak & Yinon, 2007). Reasoning and reflecting on theories to solve mathematical tasks is very pertinent to pre-service teacher education.

When pre-service mathematics teachers reflect, they can recollect experiences and give them new meaning. For instance, they may arrange and describe their experiences as a straightforward chronology of events, a cause-and-effect link, or a relationship between a problem and its solution. The pre-service teacher should think about the following inquiries as reflection prompts: "What did I just go through? What ideas, emotions, deeds, or relationships took place during the lecture or activity?". "During the game or activity …" is a common opening line for reflection questions. Reflection provides insight into the activities or concepts under consideration. A reflective process involves "Aha! moment" in which something is discovered or revealed (Sánchez, 2011).

Enhancing Connection Between Theory and Practice Through Reflective Tasks

The problem of how to enhance and intensify connections between theory and practice through reflective tasks has been and still is a major challenge in teacher education (Orland-Barak & Yinon 2007). Feiman-Nemser (2001) attributes the difficulty in establishing connections between theory and practice to weak relationship between course and field experiences evidenced in the overall lack of coherence, fragmentation, weak pedagogy and lack of articulation in extant teacher preparation programs. In a similar vein, Kwo (1996) maintains that the structure of the preparation program often provide little transfer reflection and engagement in reflective tasks.

To overcome these challenges, pre-service teachers need to push more or think outside the box to more abstract connections and comprehensions of learning to teach. Such reflective thinking to connect will reduce or mitigate mere technical solutions devoid of critical scrutiny and introspective into how their belief and theoretical knowledge shape their understanding of these concepts in action.

When teachers can make connections between different types of mathematical knowledge and real-world problems and experiences, they can engage students in the learning of mathematics (Arthur et al., 2018). Connection refers to a student's ability to further organize and integrate their prior and current learning by connecting them. How does this method or approach relate to what you already know, believe, have experienced, or feel? is one of the kinds of questions the pre-service teacher might ask. Does it support or broaden your perspective? Start your connect questions with "In the past …" According to Rakes et al. (2010), students' poor performance and failure to connect mathematics to real-world situations can be attributed to the traditional teaching methods used for instruction. Since grasping prior mathematical concepts helps students acquire and master new ones, it is crucial that students study mathematics for comprehension and understanding (Arthur et al., 2018). This can be achieved through the RCA methodology model.

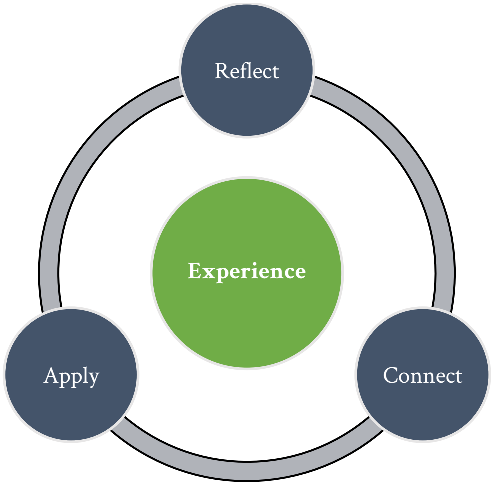

Figure 1. The RCA methodology model adapted (Right to Play, 2020)

Enhancing Applications Between Theory and Practice Through Reflection and Connection

Pre-service teachers upon the knowledge of reflection and connection on tasks need not apply to solving real-life problems but also to apply the concepts within tasks as a pedagogical knowledge to students at the elementary level of education of their practice in Ghana. These students are learners within the age group of 10–15 years from grades 4–9 in the Ghanaian education system. The pre-service teachers are also called student teachers with a mean age of 23 years who go for micro-teaching and have used the RCA methodology model concept and apply it to the students in their community of practice.

Pollak (1979) asserts a global push to increase the application of mathematics in the classroom even before the 21st century. He goes on to say that the applied point of view sometimes provides a more accurate depiction of a nation's socio-economic structure. Pollak (1969) concluded that word problems are how students engage with applications of mathematics. The ability of the pre-service teachers to apply what they have learned to events or scenarios in real life is known as application. How can you put what you have learned from this experience to use, for example, is one of the teacher's inquiries. How can you apply what you've learned to better yourself, others, and your community? "Next time you are presented with a similar situation …" might be the opening line of an apply question. See the RCA methodology model in **Figure 1**.

RESULTS AND FINDINGS

Pre-service mathematics teachers were given tasks to complete. The RCA approach model was used as the basis for this, with several solutions. The tasks' solutions were subjected to a content analysis; however, due to their length, they were not included in the paper and the journal's scope length, instead of being summarized in **Table 1** and **Table 2**. The results and findings in **Table 1** and **Table 2** addressed the research question 1: *To what extent can pre-service mathematics teachers' content knowledge, pedagogical content knowledge, and curriculum knowledge be applied in the mathematics classroom based on the RCA model?*

Table 1. Pre-service mathematics teachers' problem-solving performance on reflecting, connecting, and applying tasks (Source: Fieldwork, 2023)

Type of task	The task	T1	T2	T3	T4	T5	T6	ANST	TNDS
Curriculum specific for JHS and college mathematics on RCA model	System of linear equations	5*	4	6**	7**	5	6**	5.3	7
	Geometric task on coordinates, radius, and equation of a circle	4	1*	4**	4**	3	4**	3.3	4
	Equation of tangent of a circle	1*	1*	2**	2**	2	2**	1.7	2
	Average number of solutions per task	3.3	2.0	4.0	4.0	3.3	4.0		

** The pre-service mathematics teacher solved all tasks of this kind upon reflecting and connecting concept applications in getting multiple solutions.

* The pre-service mathematics teacher missed some approaches upon applying reflection and connection in getting multiple solutions on all tasks.

ANST: Average number of solutions per teacher; TNDS: Total number of different solutions; T1, T2, T3, T4, T5, and T6 denote pre-service mathematics teachers.

Table 2. Interpretation of level of performance on tasks (Sharp, 2006)

Scale	Range	Level of solutions	Level of performance
1	0.00–1.49	Unable to solve all	Much less than expected
2	1.50–2.49	Able to solve a few	Less than expected
3	2.50–3.49	Able to solve most	Reached expected outcome
4	4.50–5.49	Able to solve more	Reached more than expected
5	5.50–7.00	Able to solve all	Reached much more than expected

Researchers gave pre-service mathematics teachers a variety of questions to complete to get varied answers to the research questions stated in this study. How will pre-service mathematics teachers use RCA methodology to enhance concept development to bridge the gap between theory and practice? These questions guided the selection of the tasks for all three categories of interviews. Pre-service mathematics teachers discovered that certain strategies were curriculum-specific depending on the educational level at which they were working (see **Appendix A**).

Data from the six pre-service mathematics teachers who took part in the study were the only ones included in **Table 1** and **Table 2** findings presentation. All three categories of interviews were covered by our report's primary focus on three task types.

In **Table 1**, teacher 1 (T1) was unable to complete task 3 using the calculus approach, which shows that the pre-service mathematics teacher was unable to reflect on and make a connection to a subject that could have assisted in finding an alternative method for solving the tangent task's problem. Similar to task 1, teacher 2 (T2) was unable to complete task 2 and task 3 with multiple solutions. But when it came to the questions they had to complete, teacher 3 (T3), teacher 4 (T4), and teacher 6 (T6) were able to use the RCA model which resulted in a variety of problem-solving strategies that differ from traditional methods used by these pre-service mathematics teachers. The findings from T3, T4, and T6 indicated that the pre-service teachers utilized Shulman's (1986) knowledge domains and derived unique insights shaped by their cultural contexts. Their interactions facilitated the learning of concepts through interconnections, enabling them to apply these concepts to teaching using the RCA model and this answered the first research question.

The interpretation of the level of performance on the task was done using content analysis[1] of how pre-service teachers answered the tasks (see tasks in **Appendix A**). The result from pre-service teachers is presented in **Table 2** based on a similar approach to the work of Sharp (2006).

Comparing the results in **Table 1** and **Table 2**, T4 was able to solve all tasks and "reached much more than expected," this indicated how successful he used the RCA methodology model. On the other hand, T3 and T6 were only able to solve all tasks on Geometry and fall short of one or two approaches in systems of linear equations. All five pre-service teachers made effective use of the RCA methodology except for T2 who performed below average in Geometry tasks as indicated in **Table 2**. The level of performance on mathematics tasks by pre-service mathematics teachers reached more than expected testifying the level of assessment using the RCA methodology model in solving tasks.

The analysis of the data was based on content analysis of the different approaches used in answering tasks while using the RCA methodology. The tasks were necessary to see how pre-service teachers could reflect, connect theorists to application tasks using different methods or problem-solving techniques to arrive at their result. This helps identify the categories of learning that pertained to reflection, connection and application or gaps that pre-service teachers established among theoretical principles of content pedagogy and classroom discourse patterns that they identified as characteristics of their teaching and learning (Orland-Barak & Yinon, 2007). This was evident in their approach to the given tasks. In this paper, we focus on three exemplary cases of (T3, T4, and T6) used of the RCA methodology model for the given tasks. We have chosen to present these cases in depth because the three pre-service teachers were able to solve tasks and 'reached more than most expected' they provided exemplary solutions utilizing reflection, connection and application between theory and practice (Shulman, 1986). These applications were particularly rich, vivid and sophisticated exhibiting intricate levels of worked solutions between theory and practice. Specifically, we describe and interpret how each pre-service teacher reflected on theorists and connected them by conceptualizing what kind of connection exists between theory and practice by application to students at their micro-teaching. T4's results indicated complex connections between theory and practice leading to the used of grounded theories/methods of practice. We interpreted his results as using RCA methodology model to 'current theory and practice to generate grounded theories or methods of practice. T6 focused primarily on taking a critical stance towards his practice which we conceptualized as developing practical theories'.

The detailed description of T3, T4, and T6 and the interview results addressed the research question 2: *How will pre-service mathematics teachers use RCA methodology to enhance concept development to bridge the gap between theory and practice?*

T3 Learning Results: Understanding How Practice Fits the Theory

T3's analysis of her solutions as a B.Ed. Mathematics/science pre-service teacher is characterized by her constant attempt to understand how practice fits the theory through reflection that she encountered in the method used. Addressing the first task, on linear equations, she

[1] Due to constraints of length and given scope of the present paper which focuses on the analysis of pre-service teachers content work on tasks in the classroom, the worksheets of the solutions have not been included.

outlined the necessary problem-solving techniques to apply to solve the task by using the right theory based on reflection on previous knowledge through connection to solve since this was a strong class one of the things we wanted to achieve is to allow for a variety of solutions employing different approaches or methods of problem-solving to the given task outlining conceptual knowledge understanding and comprehension. We also wanted to build on reflections on prior knowledge and experiences creating maximum opportunities for using approaches or methods and for inferring from the context rather than limiting to one approach or method. She added that depending on the context will determine the approach or method to use in answering the tasks which they as pre-service teachers should take note of the contextual nature in order not to deviate from a particular task.

T3 addresses the gap successfully through good application of the RCA methodology. She was able to retrieve theorists from the reflection of previous concepts taught or practiced and move on to connect them in solving the tasks and getting the results right.

T4 Learning Result: Connecting Theory and Practice to Generate Different Methods of Practice

Like T3, T4 makes reflections, connections between theory and practice. In his results, he reflects on how practice on theory unfold his tactical approach to solving the tasks. Unlike T3, however, her reflections focus on less identified theories but due to extensive practice more new theories and methods of practice result in solving most of the tasks. In the process he uses new insights and practice to redefines unknown principles of pedagogy and approaches to generate a more informed understanding of his practice. Two main questions that guide T4 RCA methodology. The questions answering the gaps between theory and practice are: What mathematical concepts did you learn? How are these concepts you applied related? These were the interview questions addressing linking theory and practice. He stated how he used the matrix approach (inverse identity approach and Crammer's rule approach), elimination, comparison, graphical, and substitution methods, all of which resulted in the same outcome for the given task. He added that some of these approaches are related and connected to one another and knowledge of one can be related to the other to produce the same outcome. In his reflection, he showed a theoretical understanding of the need for practice to build confidence in solving tasks in mathematics.

T6 Learning Result: Using Practical Theories

Our last illustrative case considers the T6 learning result. Unlike T3 and T4, T6 focuses on her analysis in identifying gaps between theory and practice tasks. Unlike T3, however, her major concern is on the practical application that emerges from her reflections on action (Orland-Barak & Yinon, 2007). Throughout her solution, T6 focuses mostly on the problem of what his solutions revealed about his own practice and the gaps that he identified between studies (according to theory) and his action on the understanding of the theories. In his response to questions such as "During the process of solving the questions what have you noticed?" and "What have you learned from these questions?" T6 made used of "thinking outside the box" in using theories to practice and this made him solve most of the tasks with the same outcome result using different solving methods. He added using reflections draws him to previously taught concepts and theories. The gap was addressed with the idea of thinking outside the box using reflection and connecting to previous concepts of theories and solving the right learning outcome result.

To gain a deeper understanding of how the RCA methodology was applied, the researchers conducted semi-structured interviews with six pre-service mathematics teachers. Three of them have already been discussed and the other three interviews followed on how issues about reflection, connection, and application are added to the dialogue between the researchers and the pre-service mathematics teachers. The semi-structured interviews are used as a method for gathering insights and data because they offer a balanced approach between structured questions and flexibility, allowing for a deeper exploration of pre-service teachers experiences and perspectives. The semi-structured interviews allow pre-service teachers to reflect on their experiences, providing them with an opportunity to express detailed insights and personal thoughts. The flexibility of semi-structured interviews helps pre-service teachers connect their personal experiences to broader theoretical knowledge. Through semi-structured interviews, pre-service teachers can discuss how they have applied their learning and what strategies they have employed to tackle specific tasks.

> Researcher: During the process of solving the questions what have you noticed?
>
> T1: I have realized that some of the approaches are interconnected; you might have a whole inquiry where you use various ways and approaches to determine the solution.

It was clear that T1 showed "*interconnectedness of concepts*" that made him approached the geometric task with "*multiple solution methods*."

> Researcher: What have you learned from these questions?
>
> T1: I gained a lot of knowledge from answering the question since it gave me the chance to think critically and apply mathematical methods to real-world issues. To answer question 1, I used substitution, a graph, and elimination; my conclusion was the same. I applied both elimination and substitution to the second question's general formula for circles to find the solutions. While trying to solve the problem, I came across Crammer's rule. Even though I am familiar with it, I decided to apply it to question one and ended up with the same result.

T1 found multiple approaches for solving each problem, noting increased flexibility and critical thinking in applying elimination, substitutions and specific geometry techniques.

> Researcher: How are these concepts related?
>
> T1: Yes, some ideas are connected, particularly in questions one and two. To get the right answers, I employ both the elimination and substitution methods.
>
> Researcher: Will you be able to apply these concepts to your colleagues or students at junior high schools?
>
> T1: Yes, I will inform them that there are other methods of addressing mathematical problems and that they shouldn't constantly concentrate on one idea or method. I am aware that some methods do not relate to the JHS mathematics curriculum and that to help students comprehend and enjoy learning

mathematics, one must be content-specific while employing various approaches to solve problems.

T2 emphasized the need on "*learning new concepts*" using several formulas to get the same result and not relying on only one formular or approach.

Researcher: During the process of solving the questions what have you noticed?

T2: I initially believed that there was only one formula for determining the answer to a problem, but after using several formulas, I discovered other methods for doing so on "*learning new concepts.*"

Researcher: What have you learned from these questions?

T2: I didn't notice the links between the questions until you presented them to us and invited us to approach their resolution in various ways.

T2 could not apply "*interconnectedness of concepts*" and "*multiple solution methods*" to solve geometry task.

Researcher: What mathematical concepts did you learn?

T2: I was asked to identify the coordinates, radius, and equation of a circle with three points on it. I am familiar with the general formula technique, but after some analysis and research, I discovered how to approach the problem using the perpendicular bisector of the chord method.

Researcher: How are these concepts related?

T2: Since questions one and two contained simultaneous equations, I solved them using both the elimination and substitution methods.

T2 was not able to "connect different mathematical approaches" to solve some of the tasks.

Researcher: Would you be able to apply these concepts learned to your colleagues or students at JHS?

T2: According to what I've been exposed to, if I instruct my peers or JHS students, they will comprehend because I have discovered new techniques, which has increased my comprehension.

Researcher: During the process of solving the questions what have you noticed?

Teacher 5 (T5): Considering the questions, I tried a variety of approaches and concluded that the questions had given me a wealth of experience.

T5 showed how she gained a wealth of experience and this could help her "apply to teaching practice".

Researcher: What have you learned from these questions?

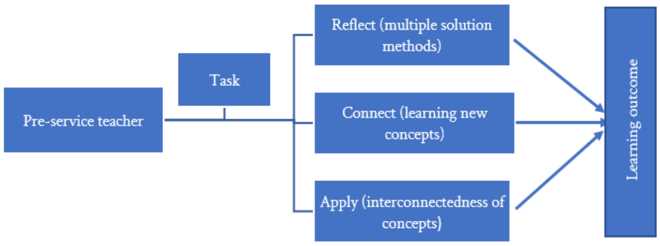

Figure 2. Concept map on pre-service teacher using RCA model approaches (Source: Authors' own elaboration, 2024)

T5: I've noticed a development in how questions are answered using many approaches to arrive at an idea. I believe that when a topic is posed to me, I won't always stick to the conventional strategy of simply considering one possible solution. Instead, I'll consider more options by employing reflections and making connections between my prior knowledge and the current circumstance.

Researcher: What mathematical concepts did you learn?

T5: By transferring from the previous to the present and then from one topic to another, I have acquired the concept of reflection. I studied the geometry question about determining the equation of the tangent to the circle and also learned about a differentiation concept called implicit differentiation, which I used for the problem and got the same answer as the geometric approach.

T5 indicated evidence of "*interconnectedness of concepts*" to problem solving.

Researcher: How are these concepts related?

T5: In terms of how the concepts and principles are applied, the concepts are connected. I am aware that the principle of multiple embodiments refers to the use of several techniques to outline a concept.

Researcher: How would you be able to apply these concepts learned to your colleagues or JHS students?

T5: Yes, I believe I can help my colleagues understand or answer these questions when given the chance. I truly learned a lot, so I can do that with assurance. Instead of always following what your teacher told you, I learned how to solve mathematical problems using a variety of methods or techniques.

Figure 2's concept map shows how a pre-service teacher used the RCA model to complete a task. Pre-service teachers can employ *"multiple solution methods"* to achieve the same goal because of their capacity to reflect on and evaluate topics from their prior knowledge. Through reflections, the pre-service teacher then relates *"learning new concepts"* to research, theoretical understanding, and more general educational ideas. This could be going over pertinent ideas or curriculum requirements that can guide their work and explain the efficacy of particular strategies. After analyzing and making connections to novel teaching scenarios, the pre-service teacher ultimately implements the *"interconnectedness of concepts"* that they have learnt. This could entail using theoretical ideas in actual classroom situations, incorporating feedback, or trying out novel teaching techniques to enhance student learning outcomes. Every theme has the potential to produce fruitful and significant learning outcomes when used effectively.

DISCUSSION

How will pre-service mathematics teachers use the RCA model to bridge the gap between theory and practice?

By applying the RCA methodology model to tasks, pre-service mathematics teachers "reached much more than expected" (T3, T4, and T6). The results also supported Orland-Barak and Yinon's (2007) findings that a pre-service mathematics teacher prioritized "practice fits theory" by retrieving theories from the reflection of previously taught or practiced topics and the capacity to connect them in problem-solving skills. Pre-service mathematics teachers acknowledged that the RCA model helped them realize that there is no one right approach to solving a mathematical problem, according to the findings of the interviews. The many approaches or techniques employed also suggest that, depending on the level being taught, one must be content-specific when putting the RCA model into practice. A pre-service mathematics teacher admitted that the RCA model provided him with "a wealth of experience" in solving mathematical problems using many approaches while still obtaining the same outcome indicating the effectiveness of the model in addressing the research questions formulated in the study.

Another finding was that applying theory to practice facilitates the generation of diverse task methods utilizing the RCA model and one's topic pedagogical and curriculum knowledge. The reflection component brought to mind the enthusiasm of aspiring mathematics educators who had just learned novel concepts that had captivated them as they approached the task. This finding aligned with (Akyeampong et al., 2022; Sánchez, 2011). Sánchez (2011) posited being unable to conceal his emotions and was forced to utilize the expression "Aha!" when he realized a thought or idea was true. The results agreed with Levav-Waynbeng and Leikin (2006), Skemp (1987), and Hiebert and Carpenter (1992) who asserted that reflection, connection, and application are crucial components of conceptual comprehension of mathematics. Connectedness is key in building mathematical concepts among pre-service teachers as the results were in consonance with (Arthur et al., 2018). This model is good when teachers pose problems using word problems that call for the use of RCA methodology to make mathematics more meaningful and understandable (Pollak, 1969). Pre-service mathematics teachers used the RCA methodology model to improve their grasp of mathematical concepts by combining facts, ideas, and generalizing to draw conclusions and synthesize their interpretations.

To what extent can pre-service mathematics teachers' content knowledge, pedagogical content knowledge and curriculum knowledge be applied in the mathematics classroom based on the RCA model?

The significance of the RCA approach model in classroom mathematics education is emphasized more. As part of their professional development, teachers at colleges of education should receive a series of training sessions to help them gain more knowledge, experience, and skills in applying this model to the teaching of mathematics in a way that aligns with (Diaz-Varela & Wright, 2019). One of the teachers mentioned in the interview that reflections enabled him to "think outside the box" and make connections to earlier concept theories to arrive at the appropriate learning outcome. A significant discovery from the interview demonstrated how a teacher applied instructional knowledge about the "principle of multiple embodiment" to complete the tasks by T5. T1 focused on broadening "mathematical methods"; T2 recognized the "interconnections between concepts" to problem solving. Most of the teachers used the RCA model and problem-solving skills to produce the right results for the given tasks. This indicated how effectively the model was utilized in achieving its purpose in problem-solving in mathematical tasks.

If this model is implemented in accordance with the Right to Play pillars—which include community practice during macro teaching, coaching and mentoring for both teachers and pre-service mathematics teachers, and effective professional development training—it will be more successful. The model can also be used successfully, particularly when teachers use insightful and good questioning styles to help students think critically about the model's main ideas rather than just relying on play-based learning activities like games, songs, and dances, as stressed by Diaz-Varela and Wright (2019).

The RCA model will be of interest to teacher education, as well, as the majority of studies in the literature highlighted how play-based learning approaches are limited for pre-service mathematics teachers. Additionally, as stated by Dobson and Mckendrick (2018), the RCA model is not exceptional and recent methodologies are of interest to pre-service mathematics teachers. In contrast, the RCA model is unrestricted, unlike some play-based learning activities that have faced opposition during adoption because of policy-led limitations (Wood, 2012). According to Nduru and Wardhani (2023), who contend that effective teaching and learning depend on how teachers implement and facilitate teaching and learning, teachers at teacher education institutions can effectively teach and facilitate learning on this model by encouraging learners to actively participate by outlining the concepts of the RCA methodology model. The only thing that stands in the way of effectively implementing this RCA methodology is the inability of mathematics teachers and pre-service teachers to formulate insightful inquiries by reflecting, connecting, and applying model concepts. In addition to this challenge, effective questioning techniques and strategies can stimulate students' thinking as they consider mathematical ideas, link them to new information, and apply these concepts to actual situations, giving mathematics in the classroom a greater sense of purpose and meaning. The limitation on the analysis method used depends on the tasks design, certain tasks may not address all facets of the model such as its application in diverse classroom settings or its effect on specific pedagogical skills. While task-based and semi-structured interviews data analysis can provide valuable insights into pre-service teachers experiences with the RCA model, the limitations of subjectivity, sample size, self-reporting and the complexity of data interpretation must be acknowledged. These limitations can affect the accuracy, generalizability, and depth of conclusions drawn from the data, suggesting the need for complementary methods and a broader data collection approach to obtain a more comprehensive understanding of the RCA model's impact.

Ethics Considerations

Research ethics are moral principles that help scientists conduct and disclose research without lying to or causing harm to the study's subjects (Dowlath, 2008). The ethical study approach and the ethical handling of research instruments are necessary for this project and should be handled cautiously. The legal and reputational protection, following the ethical requirements of Humanities and Social Sciences Research Ethics Committee (HSSREC) at University of Western Cape (UWC), and protecting those interests are all given careful consideration in this project. By keeping participant names and information private, we shall abide by the POPI Act and Data Privacy Notice in this project. They were asked non-sensitive questions, and the data they submitted was password-protected and only the researcher could access it. An ethical clearance certificate was given by HSSREC of UWC for this project. Pre-service mathematics teachers will get the study's conclusions and recommendations so they can use them in their instruction.

Implications of the Study

The RCA model has significant theoretical and practical implications for teacher education in Ghana and classroom practice. These implications can influence how pre-service teachers are trained, how they approach teaching and how they learn in classrooms.

Theoretical Implications

The RCA model aligns with constructivist theories of learning, which emphasis that knowledge is actively constructed by the learner (in this case pre-service teachers). The model encourages pre-service teachers to reflect on their prior experiences, connect new knowledge with existing frameworks and apply that knowledge in practice, thus promoting deep learning and critical thinking. This approach challenges traditional transmissive teaching models, which focus on rote memorization and passive learning. It supports the notion that learners (in this case pre-service teachers) are active participants in their own learning process.

Integrating Reflection and Practice

The RCA model emphasizes reflective practice, a central tenet in teacher education theory (e.g., Schon's reflective practice). It underscores the importance of reflection in helping pre-service teachers critically assess their teaching methods, understand their strengths and weaknesses and continuously improve their practice. The model also integrates theory and practice in a meaningful way, ensuring that what pre-service teachers learn in theory is linked to practical application in real classroom settings. This addresses a gap in traditional teacher education where theoretical knowledge often remains disconnected from practice. The RCA model contributes to the theoretical understanding of "lifelong learning" and professional development for pre-service teachers. It highlights that teaching is not a static profession but one that requires ongoing reflection, adaptation and application of evolving knowledge.

Practical Implications

The RCA model promotes active learning and student-centered teaching practices. Pre-service teachers who implement RCA based approaches can help students engage with content more deeply by encouraging reflection, making connections and applying knowledge to real world events. By applying RCA model, pre-service teachers can better address the diverse needs of students by connecting new learning to their individual experiences and allowing students to apply knowledge in various contexts, promoting inclusivity in the classroom. For pre-service teachers, the RCA model provides a more practical approach to training, equipping them with the tools to reflect on their own teaching practices, adapt their methods, and apply new pedagogical concepts in diverse educational settings. This results in more effective and adaptable pre-service teachers who can better navigate classroom challenges. Incorporating regular reflection into teacher training helps develop reflective habits, encouraging future teachers to continually assess their own performance and growth. This could lead to improved teaching techniques and more effective classroom management strategies.

By focusing on the "apply" stage, the RCA model encourages trainees and students to think critically and problem-solve within realistic, often complex scenarios. This foster a learning environment where both pre-service teachers and students practice metacognitive thinking, enhancing critical thinking skills that are essential for addressing real world classroom situations. Pre-service teachers using the RCA model are more likely to be flexible and adaptive to challenges in the classroom, whether those challenges involve diverse students' needs, curriculum changes or unexpected disruptions. The RCA model places emphasis on active learning, where students not only absorb information but also reflect on it, relate it to their own experiences and apply it in practical ways. This makes learning more relevant and engaging for students. By encouraging "reflection", the RCA model also promotes metacognition, helping pre-service teachers develop self-awareness about their learning processes. This can lead to increased motivation, self-regulation and deeper understanding of content.

CONCLUSION

The RCA model both theoretically and practically encourages a more reflective, student-centered and dynamic approach to teaching and learning. In teacher education, it prepares future teachers to be adaptive, reflective, and effective in applying pedagogical theory to real world teaching. Practically, it fosters more engaged and reflective teaching practices in the classroom, promoting a learning environment where students are actively involved in the learning process. The implication of this model suggest a shift towards teaching practices that value reflection, critical thinking and application of knowledge in diverse and meaningful contexts. The goal of mathematics education is to create effective, dedicated, efficient, and hardworking pre-service mathematicians with good content knowledge, pedagogical content knowledge, and curriculum knowledge who can teach mathematics and knowledge to students at both the elementary and secondary levels of education. The RCA model helped in filling the gap between theory and practice since pre-service mathematics teachers emphasized how "practice fits theory" through the concepts of the themes of reflection, connection, and application resulting in good problem-solving skills. Pre-service mathematics teachers underscore the RCA model application giving them rich experience of problem-solving skills in obtaining the same learning outcome result right through the principles of multiple embodiments. With its concept and methods for instilling knowledge that supports lifelong learning in students, the RCA methodology model aided pre-service mathematics teachers in achieving this through interview responses, they admitted using reflection and connection from linking theory to practice which they testify applying it to the benefit of students at the foundation stage of Ghanaian education. Pre-service mathematics teachers have the chance to revitalize their skills and engage in meaningful methodology by incorporating the RCA methodology model into their mathematics classroom. This allows them to relate to their students and facilitate lessons in the mathematics classroom more effectively.

Recommendations

The researchers strongly recommended this new RCA methodology model in the teaching and learning of mathematics for concept development at all levels of mathematics education. Students must be able to reflect on their experience or prior knowledge to discover ideas and connect with the present situation to develop new knowledge and apply it to real-life scenarios. We, therefore, recommend further research of this model across different institutions using a larger sample size to verify its adaptability and effectiveness in the teaching and learning of any mathematical topic. Further research could be conducted on how RCA models be adapted to support diverse learning styles to improve mathematical problem-solving skills across different grade levels.

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APPENDIX A: TASKS USED WITHIN THE CONTEXT OF THE STUDY

Table A1. Examples of mathematics tasks and RCA model

No	Tasks	Reflection: Types of solutions	Connections	Application
1	Solve the system of equations $5x+3y=4$ $8x+6y=10$	Substitution, elimination, linear combination, evaluative, graphing symmetry, matrices (inverse or Crammer's rule)	Different representations, techniques, topics in algebra, matrices in the field of mathematics	Prescribed solution for JHS curriculum: Substitution, elimination, linear combination, and graphing whilst all solutions apply to the college mathematics curriculum
2	A circle passes through the points A(3, 1), B(8, 2), and C(2, 6). Calculate the coordinates of the center, radius, and equation of the circle.	The general formula of the circle approach, perpendicular bisectors of the chord, using the non-collinear points approach and the gradient approach	Different representation techniques in diagrams and topics in geometry representation of the same concept	Not prescribed for JHS curriculum but rather prescribed for college mathematics curriculum
3	Find the equation of the tangent at P(5, 3) on the circle $x^2+y^2-4x+2y-20$	Geometric approach and calculus approach	Different representations in the geometric approach and differentiation topics in the calculus approach and different concepts, theorems, and definitions	Not prescribed for JHS curriculum but rather prescribed for college mathematics curriculum

Note. JHS: Junior high school.